Experiments on 2-DOF Helicopter Using Approximate Dynamic Programming

by

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Abstract

In recent years, the use of unmanned aerial vehicles has seen significant growth in commercial and military sectors. Motion control of such vehicles remains difficult due to precessional torques causing erratic movements. Conventional linear control techniques are widely used. This project aims to experiment with the use of an adaptive motion control strategy. Specifically, a model-based reinforcement learning strategy, approximate dynamic programming, was examined for use on a 2 degree-of-freedom (2-DOF) helicopter, the Quanser AERO.

The adaptive motion control strategy for a 2-DOF helicopter was implemented following electrical engineering methodologies. The proposed strategy was modified for the specific system which required modeling and mathematical analysis. Simulations and experiments were conducted in certain operating conditions. The motion control strategy was implemented to the Quanser AERO using Simulink, a Raspberry Pi, and a Raspberry Pi/smart phone. With successful implementation, a proof of concept can be attained for use in other applications using embedded systems.

Acknowledgements

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Abbreviations

2-DOF - 2 Degrees-Of-Freedom

ADP - Approximate Dynamic Programming

LQR - Linear Quadratic Regulator

PID - Proportional-Integral-Derivative

RMSE - Root Mean Squared Error

SPI - Serial Peripheral Interface

V-REP - Virtual Robot Experimentation Platform

Mathematical Symbols

$J_p(J_y)$ - Total moment of inertia about pitch (yaw) axis

$J_\theta(J_\psi)$ - Total moment of inertia about pitch (yaw) axis

$D_p(D_y)$ - Damping constant about pitch (yaw) axis

$K_{\mathbf{sp}}$ - Stiffness about pitch axis

$K_{\mathbf{pp}}(K_{\mathbf{py}})$ - Torque thrust gain acting on pitch from pitch (yaw) rotor

$K_{\mathbf{yy}}(K_{\mathbf{yp}})$ - Torque thrust gain acting on yaw from yaw (pitch) rotor

$V_p(V_y)$ - Applied voltage to pitch (yaw) motor

$V_0(V_1)$ - Applied voltage to motor #0 (#1)

$\theta(t)[\psi(t)]$ - Pitch [yaw] angle at time $t \geq 0$

Chapter 1

Introduction

The area of unmanned aerial vehicles has seen significant growth over the past few years. Popular configurations of unmanned aerial vehicles include quadcopters and helicopters. The difficulty with such unmanned aerial vehicles is the nonlinear nature of the system due to coupling between the various rotors in addition to the stochastic nature of the environment that these vehicles operate in.

One such platform for testing control techniques of such unmanned aerial vehicles is the Quanser AERO. Essentially, the Quanser AERO is a testbed developed by Quanser Inc. The Quanser AERO offers accurate measurements of the system's pitch angle, yaw angle, as well as a variety of other velocity and acceleration data.

Frequently, the Quanser AERO is used by researchers to implement test algorithms for validation on a physical system that bear resemblance to a quadcopter or helicopter. Those techniques often involve expansive linearization; however, most techniques are not adaptive. The proposed model-based reinforcement learning technique known as approximate dynamic programming, ADP, uses an approximate system model and measured state error data to update the state-feedback gain of the system.

1.1 Literature Review

The Quanser AERO has been studied extensively for a variety of control methods, but Quanser also provides instructional workbooks and lab guides. Quanser documentation uses the linearized system model to calculate LQR state-feedback gain for teaching purposes. Quanser documentation also implements the use of linear-quadratic Gaussian (LQG) control, but more advanced control techniques have been proposed as well.

More extensive control techniques have been studied outside of Quanser Inc. LQR has been used by [17], but an additional adaptive controller was implemented. Sliding mode control has been studied by [1], but sliding mode control can become difficult to implement in comparison to other control techniques.

Fuzzy model-based control was studied by [5, 12]. [4] developed a fuzzy stability controller by using a PD controller to train an adaptive neural fuzzy inference system. Fuzzy control relies on two controllers until the fuzzy control takes complete action making the task of design more intensive considering two controllers.

[6] uses an ADP approach, but the system parameters are relied upon heavily. Neural networks have been proposed in [9], but the use of high-order neural networks are computationally tedious. Reinforcement learning is used in [16], but stabilization is controlled first then parameter estimation of the system.

The problem with the aforementioned techniques is they either involve considerable mathematics not easily implemented in embedded systems or known parameters which may be difficult to calculate. That is not to say that understanding the dynamics of quadcopters and their control in general is easy. Dynamics of quadcopters were discussed in [7, 14, 11]. [14] implemented PD stabilization, and [11] developed position control on a quadcopter. [3] implemented quadcopter control via a PID controller tuned with an LQR loop. [15] developed a real-time, adaptive high-gain EKF, extended Kalman filter, which regulates system angles suitable for inertial navigation.

The Quanser AERO and quadcopters in general have seen significant research over the past few years, but not many approaches prove adaptable to the system and environment.

1.2 Problem Statement

As discussed briefly earlier, the difficulty with controlling a nonlinear system is determining the optimal state-feedback gain. ADP is one such method that hopes to make this difficulty possible. In order to implement ADP on a physical system, standard engineering practices must be followed to have successful implementation. The steps leading to implementation include analyzing the method, modeling the system, simulating the method on the system, and finally implementing the method on the physical system.

In our case, the method, ADP, was analyzed and modified to control our specific system, the Quanser AERO. Because ADP utilizes an approximate system model, the Quanser AERO's system model needed to be derived. In order to see the effectiveness of ADP, we must perform simulations verifying our method and system. These simulations were conducted in both MATLAB and V-REP. With promising simulation results, physical implementation could begin. Implementation included implementing ADP in Simulink directly, on a Raspberry Pi, and using a Raspberry Pi/Android smart phone.

1.3 Report Organization

This report is organized as follows. Chapter 1 gives an introduction to the project and problem at hand. Chapter 2 discusses the model-based reinforcement learning approach known as approximate dynamic programming, or ADP. Chapter 3 discusses the modeling of the system for ADP to be applied to, the Quanser AERO. Chapter 4 discusses the MATLAB simulations conducted to verify that the use of ADP for the Quanser AERO is possible. Chapter 5 extends the simulations conducted in Chapter 4 to provide a virtual representation of the system. Chapter 6 discusses how ADP can be physically implemented onto hardware. Finally, Chapter 7 discusses the conclusions found throughout this project.

Further resources are attached as appendices. Appendix A provides the MATLAB simulation code. Appendix B provides a tutorial and code pertaining to the real-time simulations. Appendix C provides a tutorial and MATLAB code pertaining to controlling the Quanser AERO via the QFLEX 2 USB panel. Appendix D provides a tutorial and MATLAB code pertaining to controlling the Quanser AERO via the QFLEX 2 Embedded panel and a Raspberry Pi. Appendix E provides a tutorial of implementing an Android smart phone application.

Chapter 2

Model-Based Reinforcement Learning

Reinforcement learning is a type of machine learning where an algorithm tries to determine proper actions in order to minimize or maximize some sort of performance measurement. The question is how can reinforcement learning be applied to the area of control theory? A perfect example is a simple state-feedback diagram shown in Figure 2. For most cases

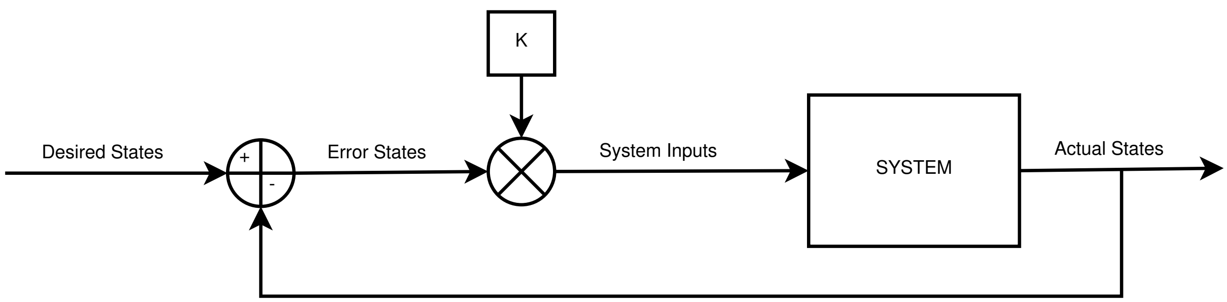


Figure 2.1: Generic state-feedback block diagram.

studied in an academic environment, the system in Figure 2 would be a linear system. The problem with this assumption is that most physical systems are in fact nonlinear systems. For a linear system, the state-feedback gain, K, is easily computed using conventional control techniques. For the nonlinear systems on the other hand, an accurate calculation for the state-feedback gain can be either difficult or impossible. This is where approximate dynamic programming, ADP, comes into play.

The main objective of ADP is to update the state-feedback gain in Figure 2 by using a model-based reinforcement learning approach. ADP accomplishes this task by utilizing measured state error data and an approximate system model. State error data is collected for T seconds at sample times of τ seconds. Every T seconds the state-feedback gain is updated by using ADP. ADP uses the measured state error data as initial conditions to the approximate system model. ADP predicts future state errors as the initial conditions would be propagated through system model. By adjusting the value of K, ADP can try to minimize or maximize a performance measurement to determine how the updated K values would affect the system. This seems like a rather simple approach, but ADP can become more complex mathematically.

The rest of this chapter is organize as follows. Section 2.1 goes into more depth concerning the mathematics of ADP. Section 2.2 discusses some situations which may deter ADP and how they can be overcame.

2.1 ADP

Let us denote the state and control variables of the system to be controlled with $\mathbf{x}^T(t)$ and $\mathbf{u}^T(t)$ at time $t \geq 0$, respectively. In order to utilize ADP, an approximate system model is needed which can be in the form of a typical state-space model seen in Equation 2.1.

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{2.1}$$

We suppose that a more exact nonlinear system model could be used, but in this project, an approximated linear system model was used in ADP. Suppose that the system is supposed to reach a predefined desired (reference) state $\mathbf{x}^{[d]}$ yielding its state error denoted by $\mathbf{e}(t) = \mathbf{x}^{[d]} - \mathbf{x}(t)$. Because all simulations and implementation must be considered in discrete-time, a sampling time must be defined for data collection and input update. Let $t = k\tau$, where $k = 0, 1, 2, \ldots$ is the discrete-time index, and $\tau > 0$ is the sampling time between data collection and model update. The sampling time in which ADP updates the state-feedback gain is T.

Using the first-order Euler integration, the discrete-time error model of the system can then be written as

$$\mathbf{e}[k+1] = \mathbf{f}(\mathbf{e}[k]) + \mathbf{G}\mathbf{u}[k] \tag{2.2}$$

where $\mathbf{f}(\mathbf{e}[k])$ and matrix $\mathbf{G}$ are given as

$$\mathbf{f}(\mathbf{e}[k]) = e[k] + \tau\mathbf{A}e[k] - \tau\mathbf{A}\mathbf{x}^{[d]} \tag{2.3a}$$

$$\mathbf{G} = -\tau\mathbf{B}. \tag{2.3b}$$

The problem is to now find the optimal $\mathbf{u}^*[k]$ such that the system's state error $\mathbf{e}[k]$ asymptotically converges to zero, *i.e.*, $\mathbf{e}[k] \to \mathbf{0}$ as $k \to \infty$.

The model-based reinforcement learning approach is based on the conventional dynamic programming technique which has the ability to determine optimal control/action inputs by considering possible future system states without actually experiencing them [8]. In other words, the task is to find the sequence of actuator inputs $\{\mathbf{u}(k)\}_{k=0}^{\infty}$ of the system that minimizes the cost function defined by

$$J(\mathbf{u}) = \sum_{k=0}^{\infty} \left(\mathbf{e}^T(k)\mathbf{Q}\mathbf{e}(k) + \mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k) \right) \tag{2.4}$$

subject to (2.2), where $\mathbb{R}^{4\times 4} \ni \mathbf{Q} = \mathbf{Q}^T \geq \mathbf{0}$ and $\mathbb{R}^{2\times 2} \ni \mathbf{R} = \mathbf{R}^T > \mathbf{0}$ penalize the system's error deviation from its desired state and its control effort, respectively. It should be noted that these dimensions are with respect to our specific system which will be discussed in the next chapter.

In addition, the cost–to–go function (value function) of the system is defined as

$$V(\mathbf{e}(k)) = \sum_{\kappa=k}^{\infty} \left(\mathbf{e}^T(\kappa)\mathbf{Q}\mathbf{e}(\kappa) + \mathbf{u}^T(\kappa)\mathbf{R}\mathbf{u}(\kappa) \right). \tag{2.5}$$

Note that the second term of the value function (2.5) takes into account the asymptotic energy consumption of the system. Following [13, Ch. 11], the value function (2.5) can be rewritten in the form:

$$V(\mathbf{e}(k)) = \mathbf{e}^T(k)\mathbf{Q}\mathbf{e}(k) + \mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k) + V(\mathbf{e}(k+1)). \tag{2.6}$$

Therefore, the optimal control input $\mathbf{u}(k)$ that drives the system using (2.1) to its desired configuration can be obtained by solving the following minimization problem:

$$\mathbf{u}^*(k) = \text{argmin}_{\mathbf{u}(k)} \Big(\mathbf{e}^T(k)\mathbf{Q}\mathbf{e}(k) + \mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k) + V^*(\mathbf{e}(k+1)) \Big). \tag{2.7}$$

A standard solution to the minimization problem (2.7) is obtained by solving the discrete–time Hamilton–Jacobi–Bellman equation given by

$$V^*(\mathbf{e}(k)) = \min_{\mathbf{u}(k)} \Big(\mathbf{e}^T(k)\mathbf{Q}\mathbf{e}(k) + \mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k) + V^*(\mathbf{e}(k+1)) \Big). \tag{2.8}$$

Following [2], calculating the gradient of the right side of (2.8) and setting it equal to zero yields

$$\frac{\partial}{\partial \mathbf{u}(k)} \left(\mathbf{e}(k)^T\mathbf{Q}\mathbf{e}(k) + \mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k) \right) + \left(\frac{\partial \mathbf{e}(k+1)}{\partial \mathbf{u}(k)} \right)^T \nabla V^*(\mathbf{e}(k+1)) = \mathbf{0} \tag{2.9}$$

where ∇ is the gradient operator which defines $\nabla V^*(\mathbf{e}(k+1)) = \frac{\partial V^*(\mathbf{e}(k+1))}{\partial \mathbf{e}(k+1)}$ as a column vector. Using (2.2), the optimal control action of the system at time instant k is given by

$$\mathbf{u}^*(k) = -\frac{1}{2}\mathbf{R}^{-1}\mathbf{G}^T\nabla V^*(\mathbf{e}(k+1)). \tag{2.10}$$

Note that the solution $\mathbf{u}^*(k)$ in (2.10) is straight forward given the fact that $\nabla V^*(\mathbf{e}(k+1))$ is defined. Since the value function (2.5) constitutes an infinite summation that takes into account the changes of the system's state-space, this function is to be approximated based on the data collected along the system trajectory (2.2).

Similar to the conventional LQR–based optimal control technique, the cost–to–go function (2.5) can be expressed as a quadratic function, *i.e.,*

$$V(\mathbf{e}(k)) = \mathbf{e}^T(k)\mathbf{P}\mathbf{e}(k) \tag{2.11}$$

for some $\mathbf{P} \in \mathbb{R}^{4\times 4}$. It should be noted that these dimensions are for our specific system which will be discussed in the next chapter. The cost–to–go function (2.11) can be approximated by

$$V(\mathbf{e}(k)) = (\text{vec}(\mathbf{P}))^T(\mathbf{e}(k) \otimes \mathbf{e}(k)) \equiv \mathbf{w}_c^T \phi(\mathbf{e}(k)) \tag{2.12}$$

where $\otimes$ is the Kronecker product operator, and the weight $\mathbf{w}_c = \text{vec}(\mathbf{P})$, where the operator vec($\mathbf{P}$) forms the vector by stacking columns of the matrix $\mathbf{P}$. The weight vector $\mathbf{w}_c$ can be approximated using the critic neural network which will be discussed shortly. The vector-valued function $\phi(\mathbf{e}(k)) = \mathbf{e}(k) \otimes \mathbf{e}(k)$ is the quadratic polynomial vector containing all possible pairwise products of the two components of $\mathbf{e}(k)$. According to the Weierstrass approximation theorem, the function $V(\mathbf{e}(k))$ can be approximated using basis functions which are polynomials of the combination of state elements $(e^{(1)}(k), e^{(2)}(k), e^{(3)}(k), e^{(4)}(k))$. It should be noted again that these dimensions are for our specific system which will be discussed in the chapter. Here we choose the basis functions up to the second order. Therefore, $\phi(\mathbf{e}(k)) = [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7, \phi_8, \phi_9, \phi_{10}, \phi_{11}, \phi_{12}, \phi_{13}, \phi_{14}]^T$ with

$$\phi_1(\mathbf{e}(k)) = e^{(1)}(k) \tag{2.13a}$$

$$\phi_2(\mathbf{e}(k)) = e^{(2)}(k) \tag{2.13b}$$

$$\phi_3(\mathbf{e}(k)) = e^{(3)}(k) \tag{2.13c}$$

$$\phi_4(\mathbf{e}(k)) = e^{(4)}(k) \tag{2.13d}$$

$$\phi_5(\mathbf{e}(k)) = (e^{(1)}(k))^2 \tag{2.13e}$$

$$\phi_6(\mathbf{e}(k)) = e^{(1)}(k)e^{(2)}(k) \tag{2.13f}$$

$$\phi_7(\mathbf{e}(k)) = e^{(1)}(k)e^{(3)}(k) \tag{2.13g}$$

$$\phi_8(\mathbf{e}(k)) = e^{(1)}(k)e^{(4)}(k) \tag{2.13h}$$

$$\phi_9(\mathbf{e}(k)) = (e^{(2)}(k))^2 \tag{2.13i}$$

$$\phi_{10}(\mathbf{e}(k)) = e^{(2)}(k)e^{(3)}(k) \tag{2.13j}$$

$$\phi_{11}(\mathbf{e}(k)) = e^{(2)}(k)e^{(4)}(k) \tag{2.13k}$$

$$\phi_{12}(\mathbf{e}(k)) = (e^{(3)}(k))^2 \tag{2.13l}$$

$$\phi_{13}(\mathbf{e}(k)) = e^{(3)}(k)e^{(4)}(k) \tag{2.13m}$$

$$\phi_{14}(\mathbf{e}(k)) = (e^{(4)}(k))^2 \tag{2.13n}$$

being the basis functions of the critic neural network, where the value function (2.12) can be approximated using a linear-in-weight critic neural network structure modeled by

$$\hat{V}(\mathbf{e}(k)) \cong \mathbf{w}_c^T \phi(\mathbf{e}(k)) \tag{2.14}$$

where $\mathbf{w}_c \in \mathbb{R}^{10\times 1}$ is the weight vector for the critic neural network. The output node of the critic neural network produces the approximate output of the cost–to–go function using (2.14). The critic neural network can be seen in Figure 2.2.

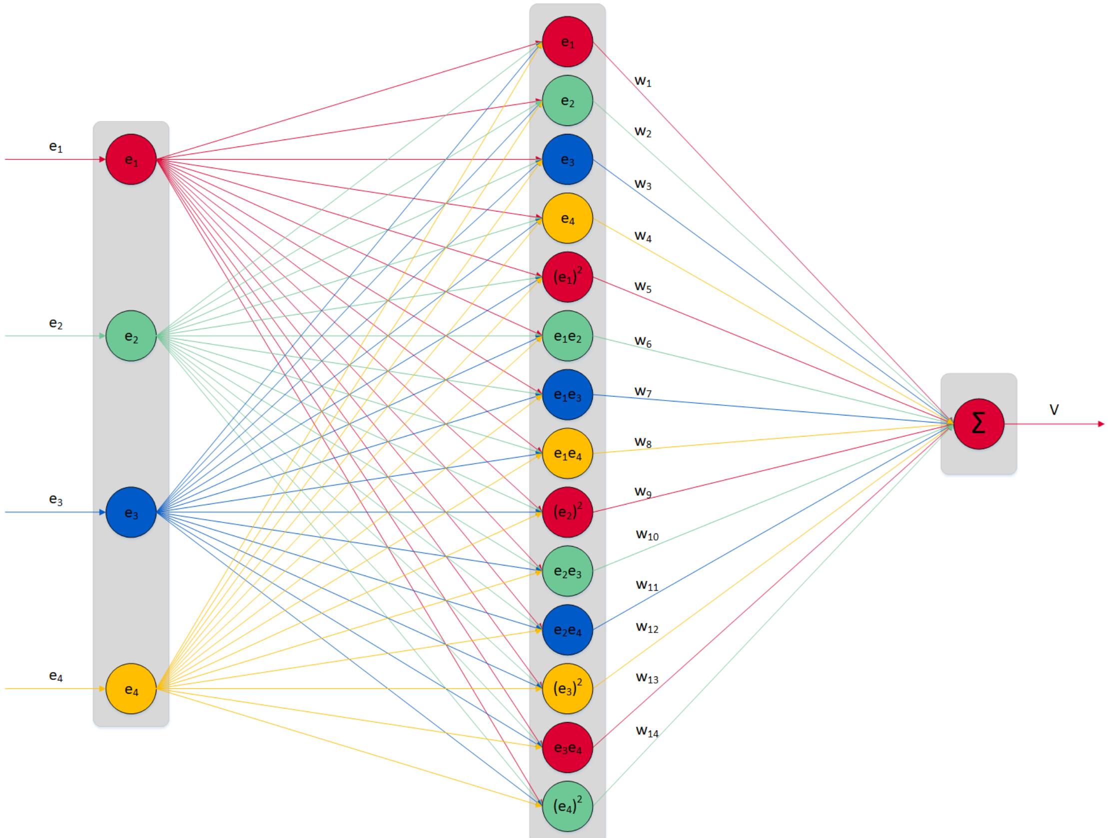


Figure 2.2: Critic neural network.

Similar to [10], the critic weight vector, $\mathbf{w}_c$, is solved using the least square technique, except that the system's data (state error) is collected online along its trajectory. For that, we introduce another sampling time τ such that the sampling time between state-feedback gain updates is $T = \bar{n}\tau$, with $\bar{n} > 14$ being a scalar parameter that quantifies the number of training samples. Note that training samples are simply a sequence of data points $\{\mathbf{e}(k+\kappa), \mathbf{e}(k+\kappa+1), \dots\}_{\kappa=0}^{\bar{n}-1}$ collected by the system along its trajectory, where $V(\mathbf{e}(k+\kappa))$ is the desired (reference) value function at time instant $k+\kappa$ with $\kappa = 0, 1, \dots, \bar{n}-1$. To solve for the weight vector $\mathbf{w}_c = [w_1, w_2, \dots, w_{14}]^T$ using the batch least square technique, the number of training samples of the critic neural network is chosen such that $\bar{n} > 14$. The target value function is determined using

$$V(\mathbf{e}(k)) = \mathbf{e}^T(k)\mathbf{Q}\mathbf{e}(k) + \mathbf{u}^T(k)\mathbf{R}\mathbf{u}(k) + \mathbf{w}_c^T\phi(\mathbf{e}(k+1)), \tag{2.15}$$

and the estimated value function is the output of the critic neural network given by (2.14). The least square error is then defined as

$$\delta_c = \frac{1}{2}\sum_{\kappa=0}^{\bar{n}-1}[V(\mathbf{e}(k)) - \hat{V}(\mathbf{e}(k))]^2. \tag{2.16}$$

The weight vector, $\mathbf{w}_c$, that minimizes the sum–of–square error δ_c is given by

$$\mathbf{w}_c = \left(\mathbf{\Lambda}^T\mathbf{\Lambda}\right)^{-1}\mathbf{\Lambda}^T\mathbf{V} \tag{2.17}$$

where the matrix $\mathbb{R}^{14\times\bar{n}} \ni \mathbf{\Lambda} = [\mathbf{a}_0, \mathbf{a}_1, \ldots, \mathbf{a}_{\bar{n}-1}]^T$ with $\mathbf{a}_\kappa = \phi^T(\mathbf{e}(k+\kappa))$ and $\mathbf{V} = [v_0, v_1, \ldots, v_{\bar{n}-1}]^T$ with $v_\kappa = V(\mathbf{e}(k+\kappa))$ for $\kappa = 0, 1, \ldots, \bar{n}-1$.

The key steps of the ADP technique implemented in this work are given in Algorithm 1. A timing flowchart of ADP can be seen in Figure 2.3.

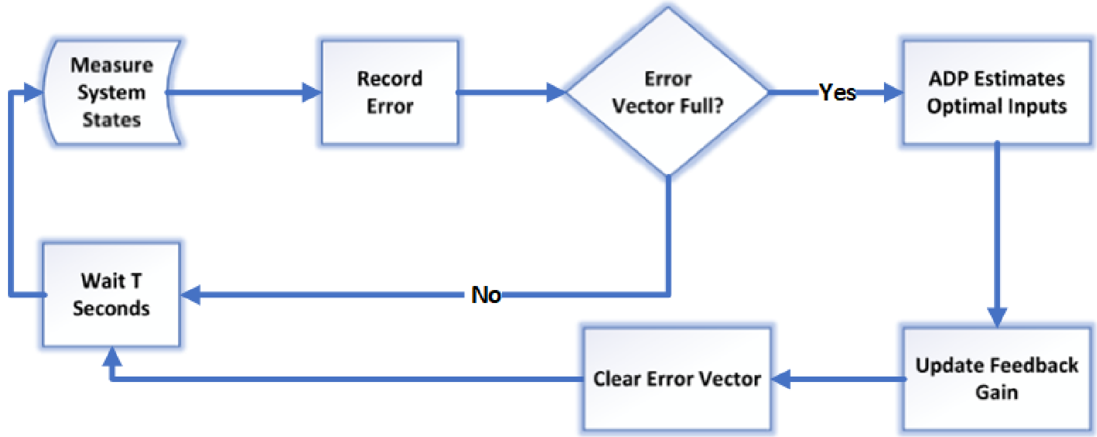


Figure 2.3: ADP timing flowchart.

Algorithm 1: ADP Outline

Input: $\mathbf{x}^{[d]}$
Output: $\mathbf{x}$

begin
  repeat
    • $t = k\tau$
    • Apply $\mathbf{u}[k]$ to system model $\rightarrow$ $\mathbf{x}[k]$
    • Apply Equation 2.1 to $\mathbf{x}[k]$ $\rightarrow$ $\mathbf{x}[k+1]$
    • Constrain ψ on $[-180°, 180°]$ and θ on $[-90°, 90°]$
    • Calculate error $\rightarrow$ $\mathbf{e}[k] = \mathbf{x}^{\text{ref}}[k] - \mathbf{x}[k]$
    • Calculate updated $\mathbf{w}_c$ if $t = T$
    repeat
      • $\mathbf{w}_{last} = \mathbf{w}_c$, $i = 0$
      repeat
        • $\mathbf{u}[i] = [0, 0]^T$, $j = 0$
        repeat
          • $\mathbf{u}_{last}[i] = \mathbf{u}[i]$
          • Find $\mathbf{e}[i+1]$ using collected error data and Equation 2.2
          • Compute new $\mathbf{u}[i]$ using Equation 2.10
          • $j = j + 1$
        until $\|\mathbf{u}[i] - \mathbf{u}_{last}[i]\| < \epsilon$ *or* $j = j_{max}$
        Output: $\mathbf{u}[i]$
        • Find $V(i)$ using Equation 2.15
        • $\Lambda(i) = \phi(\mathbf{e}(i))$
        • $i = i + 1$
      until $i = \bar{n} - 1$
      Output: Λ, V
      • Update $\mathbf{w}_c$ using Equation 2.17
    until $\|\mathbf{w}_c - \mathbf{w}_{last}\| < \epsilon_c$
    Output: $\mathbf{w}_c$
    • Calculate state-feedback gain using Equation 2.10 and Equation 2.11
    • Constrain optimal inputs on $[-18, 18]$
    • $k = k + 1$
  until $t = t_f$

2.2 ADP Conditions

ADP utilizes a critic neural network to approximate the value function. The weights of this critic neural network are determined recursively. The problem with this approach, and ADP in general, is that these weights can diverge. It was initially suspected that the weights would converge on almost all instances, but it turns out this is not the case. Initial MATLAB simulations, that will be discussed Chapter 4, showed that the weights always seemed to diverge for a linear model. It appears that the neural network cannot handle the error data of a linear system. The cause may be due to the error data not being random enough because of a truly linear system. This divergence of the weights make ADP useless if it cannot support simple linear systems.

After much research and trial-and-error with the MATLAB simulations, it was determined that several conditional cases should be considered when performing the algorithm. Research showed that if random error data was inputted to ADP, the weights would converge on almost all instances. This findings still baffle us as to why the random error data causes convergent weights while the linear system error data causes divergent weights. Further research will be needed to determine the cause of this phenomenon.

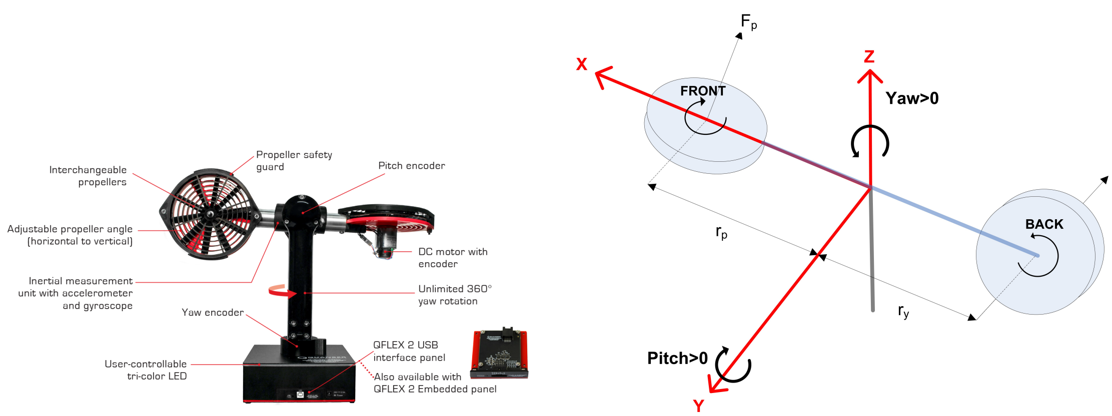
The conditional cases implemented in ADP utilize the fact that the weights converge for random error data. Before ADP is used, it is ran with random error data to obtain convergent weights; these weights are then saved. When ADP is called again, several types of errors can cause divergent weights. Because the algorithm is recursive, we can reach the end of one of the recursive loops. If this is the case, the initial convergent weights should be used to save time instead of continuing for further loop iterations. The weights may also not be found because the least-squares regression calculation cannot be performed. If this is the case, the initial convergent weights should be used again as well. The last possible failure would be if the weights actually begin to diverge to infinity. If this is the case, the weights will be assigned a very large number which is user-defined in the algorithm.

These conditional cases in ADP algorithm have been shown effective, and they guarantee that ADP will output some type of weights instead of failing.

Chapter 3

Quanser AERO System Modeling

In order to test the effectiveness of the ADP algorithm, the ideal testing platform would be a nonlinear system that is very difficult to control. This almost chaotic system would then highlight the effectiveness of ADP in its adaptive learning capabilities. Our problem is we do not have such a system readily available for testing, and it is also difficult to simulate such a system. The closest physical system we have is the Quanser AERO (https://www.quanser.com/products/quanser-aero/) seen in Figure 3.1(a).



(a) (b)

Figure 3.1: Quanser AERO. (a) Quanser AERO configured as a 2-DOF helicopter. (b) A simplistic view of the Quanser AERO's free body diagram and its orientation.

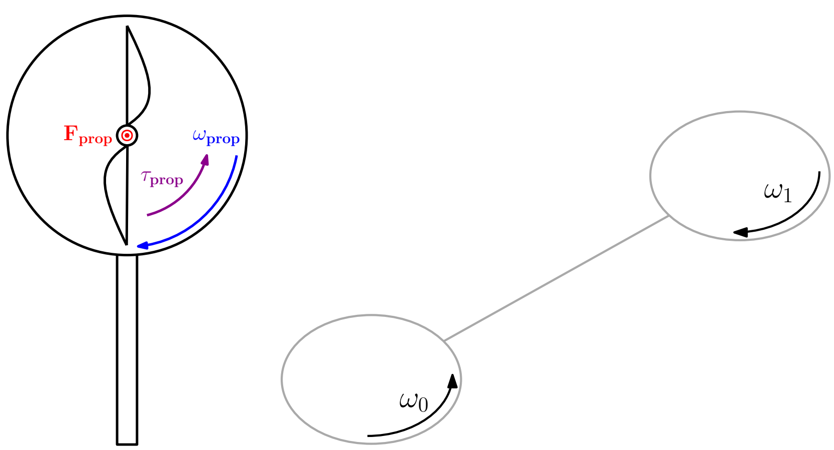
The Quanser AERO is a configurable testing platform that utilizes two fans to mimic a 2-DOF helicopter. It should be noted that this system does not actually model a physical helicopter, but it does model something similar. The Quanser AERO can either be configured as a 2-DOF helicopter or a half-quadcopter both of which will be discussed. This reconfiguration can be done by unscrewing various screws and rotating the fan assemblies. Control of the Quanser AERO can be achieved by applying the proper input voltages V_p and V_y to the main and tail DC motors, respectively. Determining the values of these motor input voltages is the task of ADP.

In order to be able use ADP, we need to know the system model of our particular Quanser AERO configuration. This system model can be approximated, but a relatively accurate model compared to that of the actual system model is needed for ADP. In order to utilize state-space modeling, we need to determine what states we will be considering. The Quanser AERO utilizes many sensors for measurement, but we will only focus on the sensors measuring pitch (θ) and yaw (ψ). Let us denote the state and control (DC motor input voltages) variables of the state-space models as $\mathbf{x}^T(t) \equiv [\theta(t), \psi(t), \dot{\theta}(t), \dot{\psi}(t)]$ and $\mathbf{u}^T(t) \equiv [V_p(t), V_y(t)]$ at time $t \geq 0$, respectively. It should be noted that $\psi > 0$ when the Quanser AERO moves in a counter-clockwise direction, and $theta > 0$ when the Quanser AERO raises above the horizontal plane.

This chapter examines the state-space models of the Quanser AERO configured as a half-quadcopter and 2-DOF helicopter. This chapter is organized as follows. Section 3.1 shows the derivation of the Quanser AERO configured as a half-quadcopter. Section 3.2 shows the derivation of the Quanser AERO configured as a 2-DOF helicopter.

3.1 Half-Quadcopter

The propeller-arm system comprises half of the Quanser AERO, shown in Figure 3.2. The movement of the arm is characterized by a thrust generated by the propeller and a resultant force, perpendicular to the thrust, generated as a result of Newton's Third Law by virtue of the motor accelerating the propeller and the propeller driveshaft.



(a) (b)

Figure 3.2: (a) Rotor arm dynamics (top view) and (b) blade rotations ($V_0 = V_1$).

We'll assume that each rotor produces an upward force, or *thrust*. T_i, that is proportional to the square of the angular velocity of the propeller times an empirically derived constant, c_t. c_t is a constant empirically derived from various characteristics of the surrounding air

[14].

$$F_{prop_i} = c_t \omega_i^2, \qquad \text{for } i = 0, 1 \tag{3.1}$$

The thrust force will be applied at radius, R, creating a torque on the body in the θ plane.

$$\tau_{\theta_i} = RF_{prop_i}, \qquad \text{for } i = 0, 1 \tag{3.2}$$

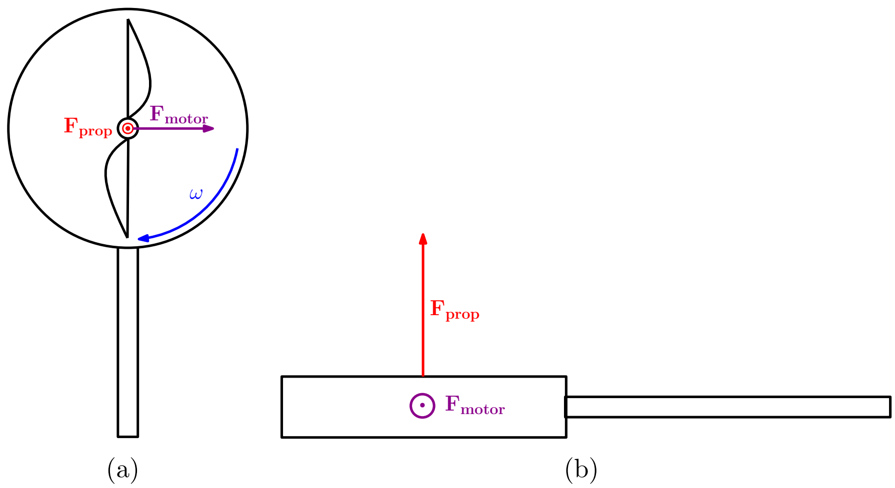


Figure 3.3: (a) Force diagram (top view) and (b) force diagram (side view).

Gyroscopic effects from the propeller on the body and torque on body from the drag of the propeller generate torque opposite of the direction of ω_{prop}. The torque will be a function of the squared angular speed of the propeller multiplied by a constant, b. b is empirically determined and understood to be a function of the propeller cross sectional area and surrounding air density.

The gyroscopic effects on the body are defined as torques τ_i and will articulate the Quanser AERO perpendicular to the thrust causing movement in the ψ plane:

$$\tau_{\psi_i} = b\omega_i^2 + J_{prop}\dot{\omega}_i, \qquad \text{for } i = 0, 1 \tag{3.3}$$

Damping torques exist in the Quanser AERO in the form of bearing stiffness. D_θ and D_ψ are constants empirically determined for the system. These torques,τ_{θ_d} and τ_{ψ_d}, are expressed as follows

$$\tau_{\theta_d} = -D_\theta \dot{\theta} \tag{3.4a}$$
$$\tau_{\psi_d} = -D_\psi \dot{\psi} \tag{3.4b}$$

Finally, in the pitch plane, a gravitational torque exists on the Quanser AERO. The gravitational torque is found at the center of mass which is R_c from the pivot. The gravitational force is then the mass of the body and the two propellers. g is the acceleration due to gravity.

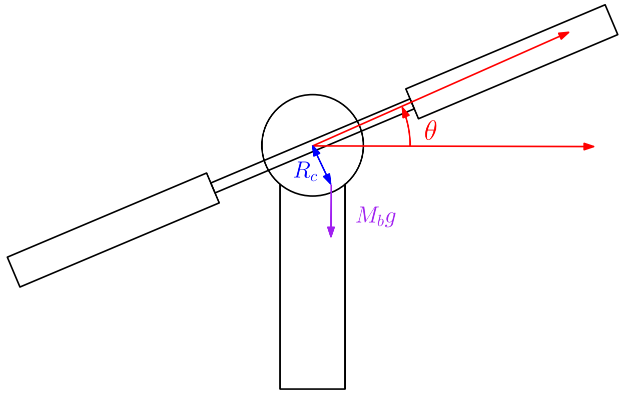


Figure 3.4: Gravitational torque exerted on Quanser AERO in θ plane.

$$\tau_{\theta_g} = R_c M_b g sin(\theta) \tag{3.5}$$

Next, we use a definition of torque, τ:

$$\sum_n \tau = J_{\text{total}}\alpha_{\text{total}} = J_\theta \ddot{\theta} + J_\psi \ddot{\psi} \tag{3.6}$$

We use components of torque to generalize:

$$\tau_\theta = \tau_{\theta_0} + \tau_{\theta_1} + \tau_{\theta_d} + \tau_{\theta_g} \tag{3.7a}$$

$$\tau_\psi = (\tau_{\psi_0} + \tau_{\psi_1} + \tau_{\psi_d}) cos(\theta) \tag{3.7b}$$

Next, we consider the coordinates of our system to finalize the equations of motion:

$$\tau_\theta = J_\theta \ddot{\theta} = Rc_t(\omega_0^2 - \omega_1^2) - R_c M_b g sin(\theta) - D_\theta \dot{\theta} \tag{3.8a}$$

$$\tau_\psi = J_\psi \ddot{\psi} = b\omega_0^2 - b\omega_1^2 + J_{prop}\dot{\omega}_0 - J_{prop}\dot{\omega}_1 - D_\psi \dot{\psi} \tag{3.8b}$$

The linearized form of the controls are made with the following assumptions:

$$R_c M_b g sin(\theta) \approx K_{sp}\theta \tag{3.9a}$$

$$cos(\theta) \approx 1 \tag{3.9b}$$

$$J_{prop}\dot{\omega}_{0,1} \approx 0 \tag{3.9c}$$

$$Rc_t\omega_{0,1}^2 \propto K_{pp}V_{0,1} \tag{3.9d}$$

$$b\omega_1^2 \propto K_{yy}V_{0,1} \tag{3.9e}$$

where K_{yy} and K_{pp} are thrust gains, and K_{sp} is the stiffness constant.

$$\tau_\theta = J_\theta \ddot{\theta} = K_{pp}V_0 - K_{pp}V_1 - K_{sp}\theta - D_\theta\dot{\theta} \tag{3.10a}$$

$$\ddot{\theta} = \frac{K_{pp}V_0 - K_{pp}V_1 - R_c M_b g\theta - D_\theta\dot{\theta}}{J_\theta} \tag{3.10b}$$

$$\tau_\psi = J_\psi \ddot{\psi} = K_{yy}V_0 - K_{yy}V_1 + -D_\psi\dot{\psi} \tag{3.11a}$$

$$\ddot{\psi} = \frac{K_{yy}V_0 - K_{yy}V_1 + -D_\psi\dot{\psi}}{J_\psi} \tag{3.11b}$$

Next, the linearized equations of motion are realized in a state-space representation with Table 3.1 representing the constant values:

$$\begin{bmatrix}\dot{\theta}\\ \dot{\psi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -K_{\text{sp}}/J_\theta & 0 & -D_\theta/J_\theta & 0\\ 0 & 0 & 0 & -D_\psi/J_\psi\end{bmatrix}\begin{bmatrix}\theta\\ \psi\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ K_{\text{pp}}/J_\theta & -K_{\text{pp}}/J_\theta\\ K_{\text{yy}}/J_\psi & -K_{\text{yy}}/J_\psi\end{bmatrix}\begin{bmatrix}V_0\\ V_1\end{bmatrix} \tag{3.12}$$

Table 3.1: Parameters calculated and measured by Quanser.

Parameter	Value	Unit
J_θ	0.0215	[kg m^2]
J_ψ	0.0237	[kg m^2]
D_θ	0.00711	[N m s rad^{-1}]
D_ψ	0.0220	[N m s rad^{-1}]
K_{sp}	0.0375	[N m rad^{-1}]
K_{pp}	0.0011	[N m V^{-1}]
K_{yy}	0.0022	[N m V^{-1}]

Using Equations 3.12 and Table 3.1, the model can explicitly be stated as:

$$\begin{bmatrix}\dot{\theta}\\ \dot{\psi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -1.7442 & 0 & -0.3307 & 0\\ 0 & 0 & 0 & -0.9283\end{bmatrix}\begin{bmatrix}\theta\\ \psi\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 0.05116 & -0.05116\\ 0.09282 & -0.09282\end{bmatrix}\begin{bmatrix}V_0\\ V_1\end{bmatrix} \tag{3.13}$$

3.2 2-DOF Helicopter

In order to utilize ADP for the 2-DOF helicopter, the state-space model of the system must be derived. Quanser documentation has provided this derivation, but we started from the beginning in order to verify their results and better understand the system. It should be noted that the system model derived here contains many assumptions and linearizations. These same assumptions and linearizations were assumed by Quanser in their documentation. It should also be noted that this derivation is very different from the one derived in Section 3.1 where Quanser documentation was not provided.

To begin the state-space model derivation, let us draw a free body diagram of the Quanser AERO which can be seen in Figure 3.2. We can then look at the free body diagram in terms of the two propellers seen in Figures 3.6(a) and 3.6(b). This also divides the forces in terms of horizontal and vertical planes.

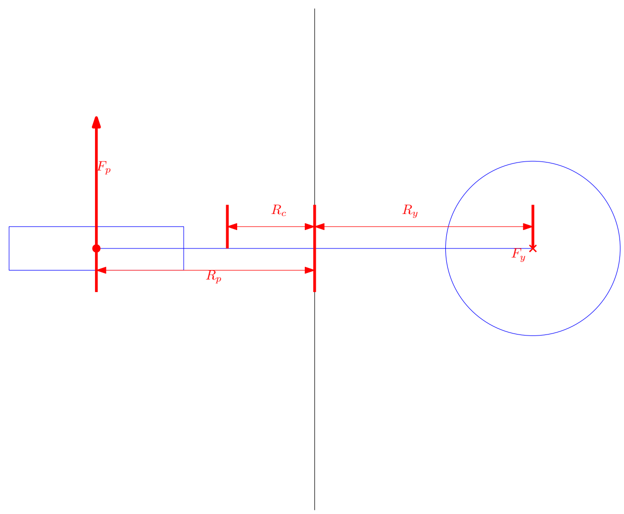
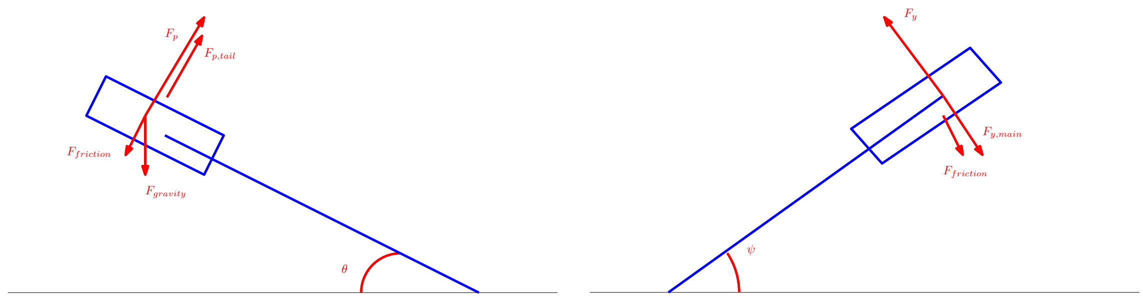


Figure 3.5: General free body diagram of the Quanser AERO.

The symbols represented in Figure 3.2 can be described as follows.

- R_p is the distance from the fork of the Quanser AERO to the center of the main propeller.
- R_y is the distance from the fork of the Quanser AERO to the center of the tail propeller.
- R_c is the distance from the fork of the Quanser AERO to the center of mass of the body. The body is the structure connecting and including the main and tail propellers.
- F_p is the force produced by the main rotor causing lift. We will assume that when the helicopter is rising, F_p is positive.
- F_y is the force produced by the tail rotor causing thrust. We will assume that when the helicopter is rotating counter-clockwise, F_y is positive.



(a) (b)

Figure 3.6: (a) Forces acting on the propeller controlling the pitch of the Quanser AERO. (b) Forces acting on the propeller controlling the yaw of the Quanser AERO.

We can look at the forces specifically associated with the main rotor as in Figure 3.6(a). The symbols represented in Figure 3.6(a) can be described as follows.

- F_p is the force produced by the main rotor causing lift. We will assume that when the helicopter is rising, F_p is positive.
- $F_{p,tail}$ is the coupled force generated by the tail rotor. To visualize this force, think of the torques on the tail rotor. As the tail propeller spins, the propeller is causing torque on the actual motor shaft. This torque is translated to the pitch axis. We assume this coupling force is aiding the main rotor force as in [1].
- $F_{friction}$ is the frictional force opposing pitch.
- $F_{gravity}$ is the gravitational force exerted on the main rotor. This force only exists in the vertical plane.

We can look at the forces specifically associated with the main rotor as in Figure 3.6(b). The symbols represented in Figure 3.6(b) can be described as follows.

- F_y is the force produced by the tail rotor causing thrust. We will assume that when the helicopter is rotating counter-clockwise, F_y is positive.
- $F_{y,main}$ is the coupled force generated by the main rotor. Again, we used the directions used in [1].
- $F_{friction}$ is the frictional force opposing yaw.

We know that the forces in each of the planes are tangential to the path of motion except the force of gravity. This means we can find the torques associated with each of the forces. Let the torques be defined as follows.

- $\tau_{p,main}$ is the torque associated with the force generated by the main rotor on the main rotor.

- $\tau_{p,tail}$ is the torque associated with the force generated by the tail rotor on the main rotor.
- $\tau_{p,friction}$ is the frictional torque opposing a change in pitch.
- $\tau_{p,gravity}$ is the torque associated with the force of gravity on the main rotor.
- $\tau_{y,tail}$ is the torque associated with the force generated by the tail rotor on the tail rotor.
- $\tau_{y,main}$ is the torque associated with the force generated by the main rotor on the tail rotor.
- $\tau_{y,friction}$ is the frictional torque opposing a change in yaw.
- $\tau_{p,net}$ is the net torque on the main rotor.
- $\tau_{y,net}$ is the net torque on the tail rotor.

We also know that the net torque is the rotational inertia times the tangential angular acceleration. Let the rotational inertia for the the main rotor be J_p, and let the rotational inertia for the tail rotor be J_y. The angular acceleration for the pitch and yaw are $\ddot{\theta}$ and $\ddot{\psi}$, respectively.

$$\tau_{p,net} = J_p\ddot{\theta} \tag{3.14a}$$

$$\tau_{y,net} = J_y\ddot{\psi} \tag{3.14b}$$

Now we can calculate the individual torques because we know that torque is the tangential force times the distance from the pivot. Let us also make one assumption here. Let us assume that the torque generated on the main (tail) rotor from the main (tail) rotor is directly proportional to V_p and V_y, respectively. The proportional gains here will be classified as thrust gains.

$$\tau_{p,main} = R_pF_p = K_{p,main}V_p \tag{3.15a}$$

$$\tau_{p,main} = G_p \tag{3.15b}$$

$$\tau_{p,main} = R_p\beta_p\dot{\theta} \tag{3.15c}$$

G_p is the torque coupling which we have yet determined. The tangential frictional force is the equal to a damping coefficient, β_p, times the velocity, $\dot{\theta}$.

$$\tau_{p,main} = R_yF_y = K_{y,tail}V_y \tag{3.16a}$$

$$\tau_{p,main} = G_y \tag{3.16b}$$

$$\tau_{p,main} = R_y \beta_y \dot{\psi} \tag{3.16c}$$

$$\tau_{p,gravity} = R_c m_{body} g sin(\theta) \tag{3.16d}$$

G_y is the torque coupling which we have yet determined. The tangential frictional force is the equal to a damping coefficient, β_y, times the velocity, $\dot{\psi}$. The gravitational torque is found at the center of mass which is R_c from the pivot. The gravitational force is then the mass of the body and the two propellers. g is the acceleration due to gravity.

We can now combine Equations 3.14, 3.15, and 3.16 using the same sign conventions as used in Figures 3.6(a) and 3.6(b).

$$J_p \ddot{\theta} = K_{p,main} V_p + G_p - R_p \beta_p \dot{\theta} - R_c m_{body} g sin(\theta) \tag{3.17a}$$

$$J_y \ddot{\psi} = K_{y,tail} V_y - G_y - R_y \beta_y \dot{\psi} \tag{3.17b}$$

We can rewrite Equations 3.17a and 3.17b with the second derivative state variables set alone.

$$\ddot{\theta} = \frac{K_{p,main} V_p}{J_p} + \frac{G_p}{J_p} - \frac{R_p \beta_p \dot{\theta}}{J_p} - \frac{R_c m_{body} g sin(\theta)}{J_p} \tag{3.18a}$$

$$\ddot{\psi} = \frac{K_{y,tail} V_y}{J_y} - \frac{G_y}{J_y} - \frac{R_y \beta_y \dot{\psi}}{J_y} \tag{3.18b}$$

We can now write Equation 3.18 in state-space form.

$$\begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{R_p \beta_p}{J_p} & 0 \\ 0 & 0 & 0 & -\frac{R_y \beta_y}{J_y} \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{p,main}}{J_p} & 0 \\ 0 & \frac{K_{y,tail}}{J_y} \end{bmatrix} \begin{bmatrix} V_p \\ V_y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{G_p}{J_p} - \frac{R_c m_{body} g sin(\theta)}{J_p} \\ -\frac{G_y}{J_y} \end{bmatrix} \tag{3.19}$$

At this point, we shall make a few more assumptions to obtain a linearized model of the 2-DOF helicopter. ADP, in theory, should be able to withstand the inconsistencies between the linearized model and the physical system. Let us assume that we do not the directions of the coupling forces acting on each of the rotors. This means we do not know the direction of G_p and G_y. Let us also assume that these torques are proportional to the voltages that generate them, V_p and V_y. This means we can set up two more equations in terms of thrust gains as we did in Equations 3.15a and 3.16a.

$$G_p = K_{p,tail} V_y \tag{3.20a}$$

$$G_p = K_{y,main} V_p \tag{3.20b}$$

Let us also use the small angle approximation method to get rid of the sinusoidal term. This means $sin(\theta) = \theta$ for small angles. Combining assumptions, we can rewrite the state-space model in a linear form.

$$\begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{R_c m_{body} g}{J_p} & 0 & -\frac{R_p \beta_p}{J_p} & 0 \\ 0 & 0 & 0 & -\frac{R_y \beta_y}{J_y} \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{p,main}}{J_p} & \frac{K_{p,tail}}{J_p} \\ \frac{K_{y,main}}{J_y} & \frac{K_{y,tail}}{J_y} \end{bmatrix} \begin{bmatrix} V_p \\ V_y \end{bmatrix} \tag{3.21}$$

We can calculate the rotational inertias because we know the Quanser AERO configuration.

$$J_p = I_{body} + 2I_{prop} = \frac{m_{body} L_{body}^2}{12} + 2m_{prop} r_{prop}^2 \tag{3.22a}$$

$$J_y = I_{body} + 2I_{prop} + I_{yoke} = \frac{m_{body} L_{body}^2}{12} + 2m_{prop} r_{prop}^2 + \frac{m_{yoke} r_{fork}^2}{2} \tag{3.22b}$$

Finally, we can add some notations to make the state-space model a little more concise. Let us define the stiffness about the pitch axis as $K_{sp} = R_c m_{body} g$. Let us define the damping constant about the pitch axis as $D_p = R_p \beta_p$. Let us define the damping constant about the yaw axis as $D_y = R_y \beta_y$.

$$\begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_{sp}}{J_p} & 0 & -\frac{D_p}{J_p} & 0 \\ 0 & 0 & 0 & -\frac{D_y}{J_y} \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{p,main}}{J_p} & \frac{K_{p,tail}}{J_p} \\ \frac{K_{y,main}}{J_y} & \frac{K_{y,tail}}{J_y} \end{bmatrix} \begin{bmatrix} V_p \\ V_y \end{bmatrix} \tag{3.23}$$

For conciseness, we will also rename the thrust gains. The first term of the subscript will be the rotor that is affected [pitch (main) or yaw (tail)], and the second term of the subscript will be the rotor causing the effect [pitch (main) or yaw (tail)].

$$\begin{bmatrix} \dot{\theta} \\ \dot{\psi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_{sp}}{J_p} & 0 & -\frac{D_p}{J_p} & 0 \\ 0 & 0 & 0 & -\frac{D_y}{J_y} \end{bmatrix} \begin{bmatrix} \theta \\ \psi \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_p} & \frac{K_{py}}{J_p} \\ \frac{K_{yp}}{J_y} & \frac{K_{yy}}{J_y} \end{bmatrix} \begin{bmatrix} V_p \\ V_y \end{bmatrix} \tag{3.24}$$

The derived state-space model is the same as the one derived by Quanser documentation provided in the licensed software package resources. The only item left is to determine the constants used in the state-space model. Some of the constants can be calculated and others need to be measured. The rotational inertias and the stiffness can be be calculated by provided product features, but the damping constants and the thrust gains need to be measured. Quanser documentation provides experimental procedures to measure these values including the signs for the thrust gains. The constants calculated and measured by Quanser for the Quanser AERO can be seen in Table 3.2. With the parameters known, we

Table 3.2: Parameters calculated and measured by Quanser.

Parameter	Value	Unit
J_p	0.0215	$[\mathrm{kg\,m^2}]$
J_y	0.0237	$[\mathrm{kg\,m^2}]$
$D_p = R_\mathrm{p}\beta_\mathrm{p}$	0.00711	$[\mathrm{N\,m\,s\,rad^{-1}}]$
$D_y = R_\mathrm{y}\beta_\mathrm{y}$	0.0220	$[\mathrm{N\,m\,s\,rad^{-1}}]$
$K_\mathrm{sp} = R_\mathrm{c} m_\mathrm{body} g$	0.0375	$[\mathrm{N\,m\,rad^{-1}}]$
K_pp	0.0011	$[\mathrm{N\,m\,V^{-1}}]$
K_py	0.0021	$[\mathrm{N\,m\,V^{-1}}]$
K_yy	0.0022	$[\mathrm{N\,m\,V^{-1}}]$
K_yp	-0.0027	$[\mathrm{N\,m\,V^{-1}}]$

can calculate the numeric state-space model of the Quanser AERO configured as a 2-DOF helicopter as defined the Quanser measurements.

$$\begin{bmatrix}\dot{\theta}\\ \dot{\psi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -1.7442 & 0 & -0.3307 & 0\\ 0 & 0 & 0 & -0.9283\end{bmatrix}\begin{bmatrix}\theta\\ \psi\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\\ 0.0512 & 0.0977\\ -0.1139 & 0.0928\end{bmatrix}\begin{bmatrix}V_p\\ V_y\end{bmatrix} \tag{3.25}$$

Chapter 4

MATLAB Simulations

In any electrical engineering project, one must perform theoretical research which includes a mathematical analysis before they can continue to implementation. That research was performed in the previous chapters. The next step is combining the theoretical aspect with the defined system, the Quanser AERO. We cannot go straight to implementation because we need to verify that the ADP algorithm does, in fact, behave as intended for the Quanser AERO. In our case, the most viable option was to perform MATLAB simulations using the derived models discussed in Chapter 3 and the ADP algorithm discussed in Chapter 2.

The MATLAB simulation code can be seen in Appendix A.1. The MATLAB simulation can be ran for both the half-quadcopter and the 2-DOF helicopter configurations. Both ADP and LQR are ran in a single simulation to compare the results of the two control methods. The desired trajectory can be set, and then both control approaches are ran for the state-space model.

A few assumptions where made when writing the MATLAB simulation code, and these assumptions can be seen in the code provided. Parameters discussed in Chapter 3 were used to describe the state-space models. No measurements were used from our physical Quanser AERO. Quanser documentation also provided reasonable matrices for $\mathbf{R}$ and $\mathbf{Q}$ used in the LQR control approach. These matrices were adjusting slightly to make them positive semi-definite and then used for both ADP and LQR. Sampling times of $\tau = 0.01$ and $T = 0.2$ seconds were used for timing. These constraints were provided by Dr. Miah at the beginning of the project.

The disadvantage of using MATLAB is the state-space models used for the system are linear. We derived linear state-space models in Chapter 3. We cannot simulate a truly nonlinear system for the Quanser AERO because the coupling is described as linear. In essence, the MATLAB simulations only provide a comparison of how ADP and LQR control a linear system. One would suspect both of them to perform well given the fact that LQR is designed for linear systems. The real question is whether ADP will out-perform LQR for a linear system. To try to mimic a nonlinear system, the MATLAB simulation code has a nonlinear adjustment factor. This factor adds a random few degrees to the output of the system every sampling time. The idea of this factor is to try to mimic some pseudo-random noise or nonlinearity to the system. This factor is adjustable, but it is currently set to zero in Appendix A.1.

For both the half-quadcopter and 2-DOF helicopter, three MATLAB simulations are provided. The initial conditions are the same for all of the MATLAB simulations. The first MATLAB simulation is setting the desired pitch and yaw to a constant value. The second MATLAB simulation imposes time-varying constant trajectories for both the desired pitch and yaw. The last MATLAB simulation provided imposes sinusoidal desired trajectories for both the pitch and yaw. The desired angular velocities are always zero.

This chapter is organized as follows. Section 4.1 provides and analyzes the MATLAB simulation results for the half-quadcopter. MATLAB simulation results for the 2-DOF helicopter are provided and analyzed in Section 4.2.

4.1 Half-Quadcopter

4.1.1 Results

The results of the MATLAB simulations for the half-quadcopter can be seen in Figures 4.1, 4.2, and 4.3.

4.1.2 Analysis

The MATLAB simulation results for constant desired pitch and yaw can be seen in Figure 4.1. The angular positions for both ADP and LQR can be seen in Figure 4.1(a). The MATLAB simulation shows that both control techniques have convergent pitch and yaw values. It does appear that the ADP technique does oscillate initially slightly more than LQR, but ADP does reach the desired values. The same results can be seen with the angular velocities shown in Figure 4.1(b). The voltage inputs seen in Figure 4.1(e) show similar traits for ADP and LQR, but no technique has a distinct advantage.

For the time-varying constant desired trajectories, the MATLAB simulations can be seen in Figure 4.2. The angular position seen in Figure 4.2(a) shows faster convergence for LQR and large initial oscillations for ADP. The angular velocities seem to have difficulty converging to zero because of the changing trajectories, but both ADP and LQR have similar traits as seen in Figure 4.2(b). Notice that the voltage inputs of opposite motors mirror each other. Again for the voltage inputs, there is no clear advantage for either ADP or LQR.

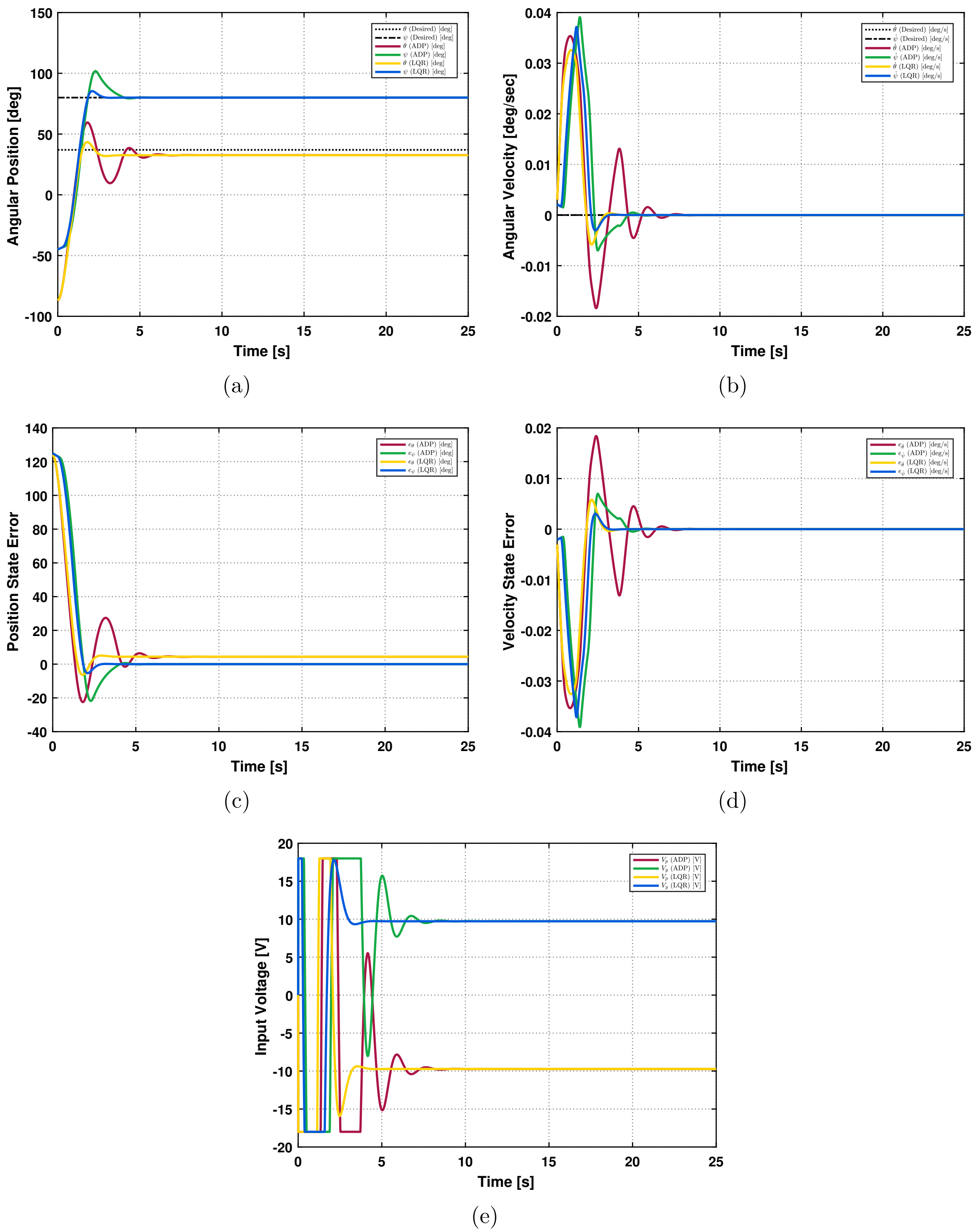
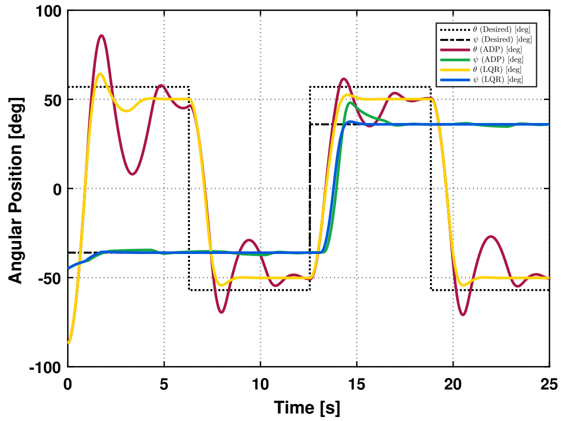
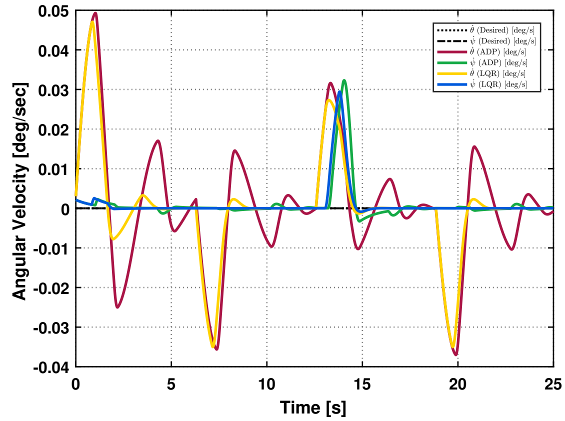


Figure 4.1: Constant desired pitch and yaw for half-quadcopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

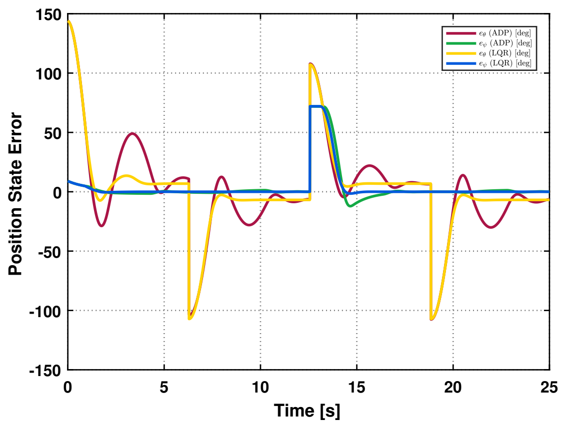
The most surprising MATLAB simulation is the sinusoidal desired trajectory as seen in Figure 4.3. For the angular position, we see the initial oscillations from ADP, while LQR



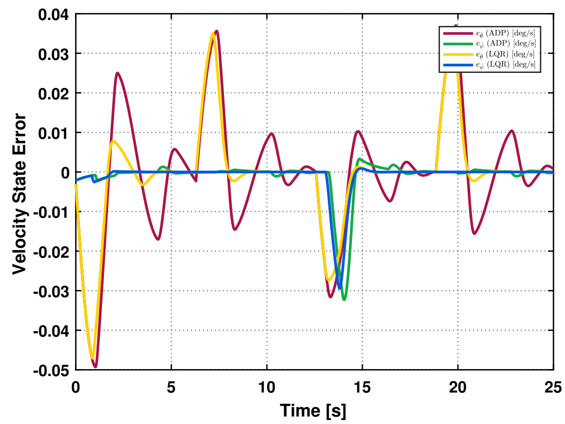
(a)



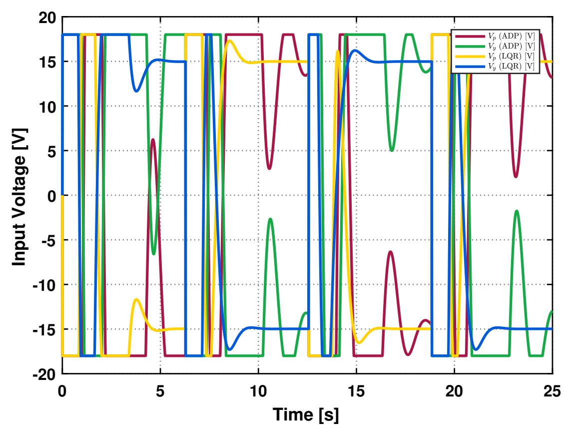
(b)



(c)



(d)



(e)

Figure 4.2: Time-varying constant desired pitch and yaw for half-quadcopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

seems to converge quicker which can be seen in Figure 4.3(a). The difference with this MATLAB simulation is that ADP actually appears closer to the desired trajectory than

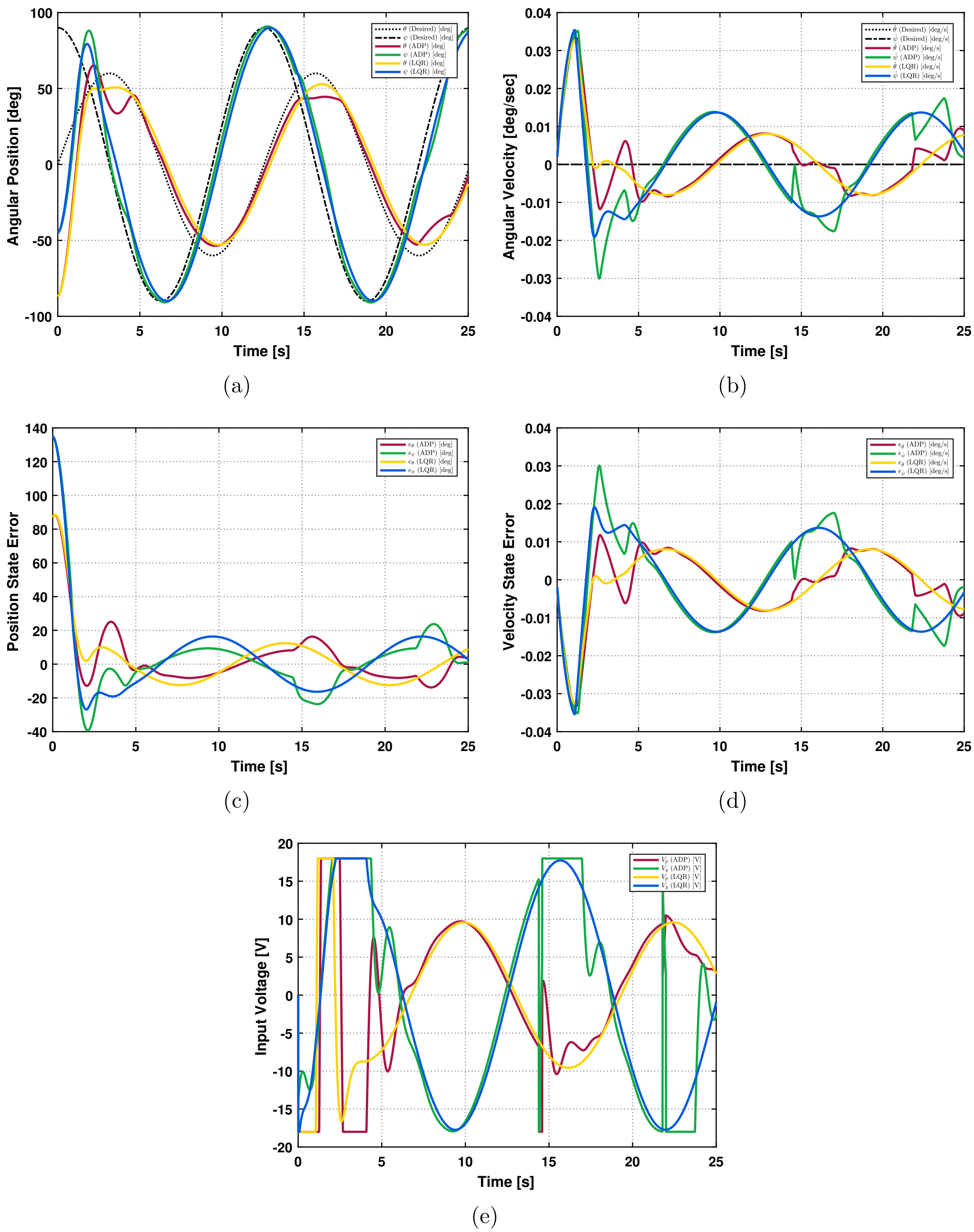


Figure 4.3: Sinusoidal desired pitch and yaw for half-quadcopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

LQR. This can be verified in the angular position error plot seen in Figure 4.3(c). As expected, the angular velocity is no where close to zero because we have a changing desired trajectory. Also, there is again no clear advantage for either ADP or LQR in terms of input voltage as seen in Figure 4.3(e).

Overall, both ADP and LQR perform well for the linear system of the half-quadcopter as seen in the MATLAB simulation results. We usually see more initial oscillation from ADP than LQR, but this might be the algorithm learning the system. Both control techniques, however, converge to the desired trajectories. On the down side, there is no clear advantage to ADP over LQR for a linear system as hoped for in the beginning of this project.

4.2 2-DOF Helicopter

4.2.1 Results

The results of the MATLAB simulations for the 2-DOF helicopter can be seen in Figures 4.4, 4.5, and 4.6.

4.2.2 Analysis

The MATLAB simulation results for constant desired pitch and yaw can be seen in Figure 4.4. The angular positions for both ADP and LQR can be seen in Figure 4.4(a). The MATLAB simulation shows that both control techniques have convergent pitch and yaw values. It does appear that the ADP technique does oscillate initially much more than LQR, but ADP does reach the desired values. The same results can be seen with the angular velocities shown in Figure 4.4(b). The voltage inputs seen in Figure 4.4(e) show similar traits for ADP and LQR, but no technique has a distinct advantage.

For the time-varying constant desired trajectories, the MATLAB simulations can be seen in Figure 4.5. The results of the time-varying trajectories are very similar to those of the constant desired trajectories. The angular position seen in Figure 4.5(a) shows faster convergence for LQR and slight initial oscillations for ADP. This is exactly what we saw in the constant desired trajectories. The angular velocities seem to have difficulty converging to zero because of the changing trajectories, but both ADP and LQR have similar traits as seen in Figure 4.5(b). Again for the voltage inputs, there is no clear advantage for either ADP or LQR.

The most surprising MATLAB simulation is the sinusoidal desired trajectory as seen in Figure 4.6. For the angular position, we see the initial oscillations from ADP, while LQR seems to converge quicker which can be seen in Figure 4.6(a). The difference with this MATLAB simulation is that ADP actually appears closer to the desired trajectory than LQR. This can be verified in the angular position error plot seen in Figure 4.6(c). As expected, the angular velocity is no where close to zero because we have a changing desired trajectory. Also, there is again no clear advantage for either ADP or LQR in terms of input voltage as seen in Figure 4.6(e).

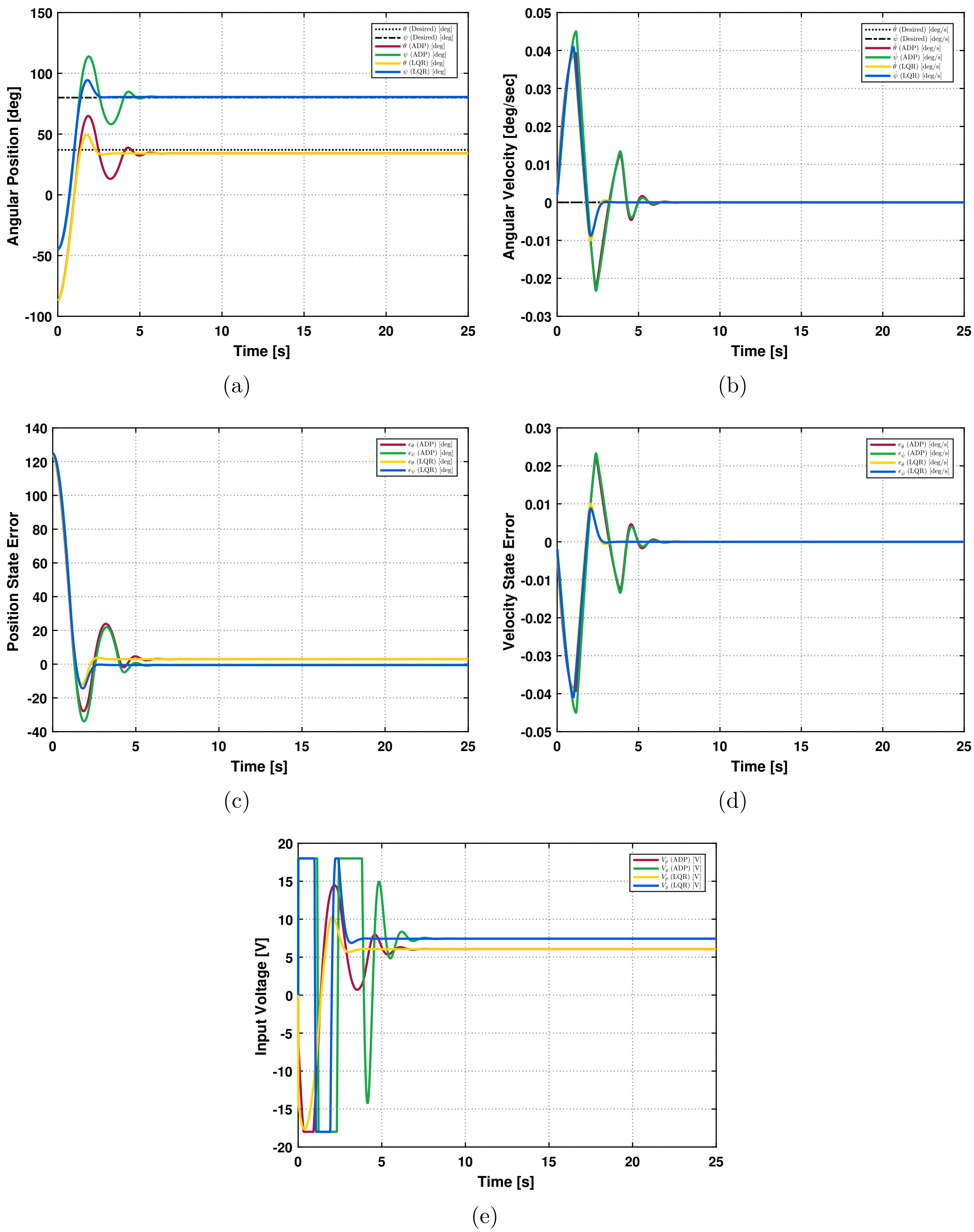
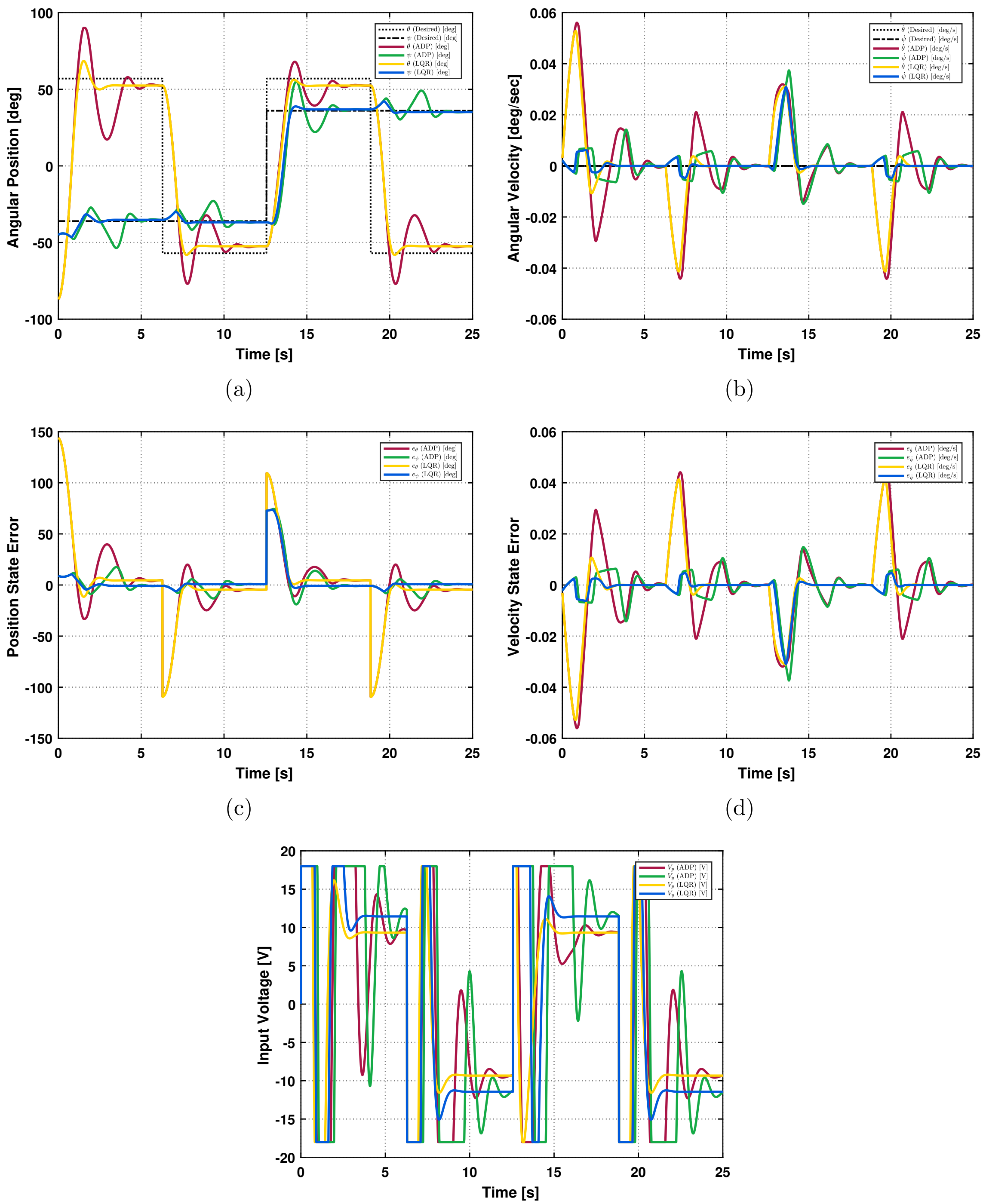


Figure 4.4: Constant desired pitch and yaw for 2-DOF helicopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

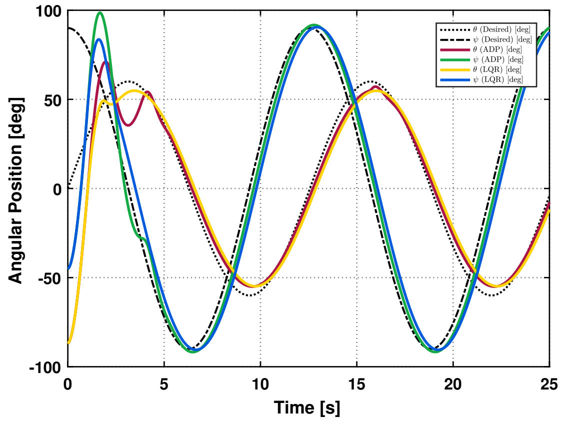
Overall, both ADP and LQR perform well for the linear system of the 2-DOF helicopter as seen in the MATLAB simulation results. We usually see more initial oscillation from ADP



(e)

Figure 4.5: Time-varying constant desired pitch and yaw for 2-DOF helicopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

than LQR, but this might be the algorithm learning the system. Both control techniques, however, converge to the desired trajectories. On the down side, there is no clear advantage



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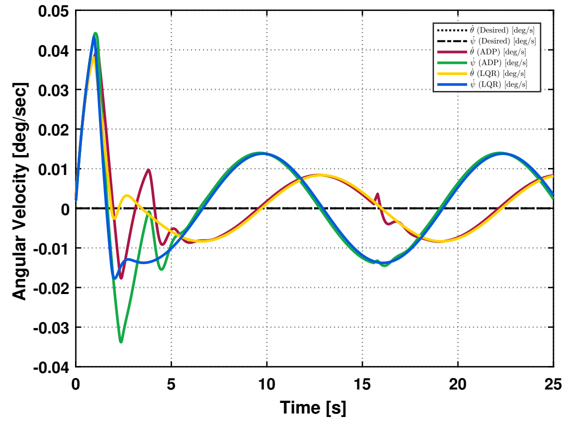

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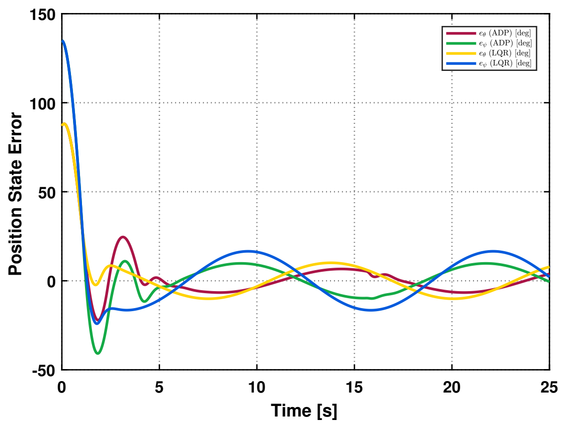

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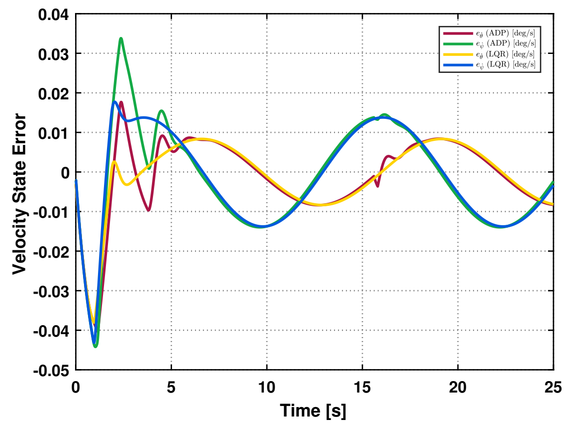

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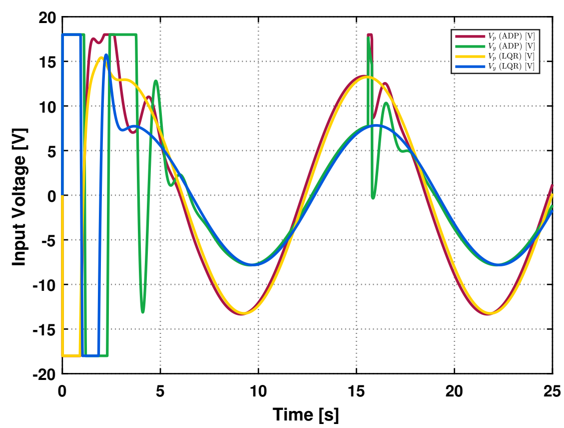


(e)

Figure 4.6: Sinusoidal desired pitch and yaw for 2-DOF helicopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

to ADP over LQR for a linear system as hoped for in the beginning of this project.

Chapter 5

Real-Time Simulation

V-REP, Virtual Robot Experimentation Platform, is a program made by Coppelia Robotics that offers an integrated development environment for prototyping robots and testing control algorithms on different robots among many other capabilities.

On the most basic level, V-REP offers an environment, called a "scene," in which the user places components of a robot, such as joints, sensors, grabbers, and basic shapes. When implemented, these items are called "scene objects." Each scene object can be assigned code called by V-REP at each time step in a simulation. The code, written in the LUA language, allows the user to use V-REP's built-in API to have scene objects interact, governed by the different physics engines available within V-REP.

V-REP can be run solely by internal LUA code, and many of the provided robots come with some short script attached to each robot demonstrating functionality. One of the deliverables for this project is to have V-REP be controlled remotely by MATLAB.

This chapter is organized as follows. Section 5.1 discusses the software used throughout this project pertaining to V-REP. Section 5.2 discusses the data transfer between MATLAB and V-REP. Section 5.3 discusses the design of the Quanser AERO in V-REP. Section 5.3.1 discusses the design of a working Quanser AERO model, and Section 5.3.2 discusses a model still under development. Section 5.4 discusses the results including RMSE measurements and the plots from simulations performed throughout the project.

5.1 Required Software

At the time of this publication, the software simulations were performed using V-REP PRO EDU V3.5.0.4 and MATLAB 2016B. It is not required, but it is recommended to use the Vortex Physics engine to control the V-REP simulation.

NOTE: Different versions of V-REP may affect model functionality.

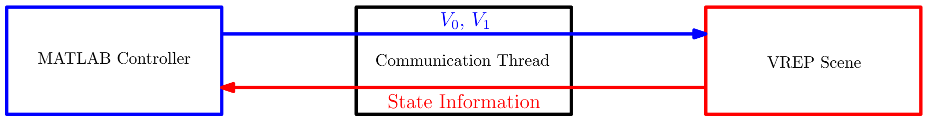


Figure 5.1: Communication scheme between MATLAB (acting as controller) and V-REP (acting as environment).

5.2 Communication

V-REP and MATLAB communicate through a communication thread defined at port 19999. This process is shown in Figure 5.1. Next, the V-REP API offers a function to package a MATLAB vector and send that through the port for V-REP to read. Once V-REP reads the signal, it clears the signal, reading it for the next MATLAB assertion. A simple MATLAB and LUA setup are provided in Appendices B.2.1 and B.2.2.

5.3 Designing Quanser AERO

The V-REP design started with considering the deliverables. The initial design was to use the V-REP quadcopter model provided. To adapt this model to the Quanser AERO system, the center was to be set at a fixed point, while two of the four propellers were to be used to actuate the system. This plan was modified to a two pronged approach at designing and implementing a Quanser AERO base, each utilizing different API function calls within V-REP. The first design will make use of the linearized model and exert control in V-REP via API calls concerning the pitch and yaw joints, configured as motors. The second model will make use of the motor circuit to calculate thrust and resultant torques.

Clearly, factors such as the weight, size, and location of each shape that comprise the V-REP model are important to accurately define the **A** and **B** matrices of the state-space model. The model uses only cylinders and rectangular prisms such that algebraic moments of inertia could be utilized for convenience. To compute the moment of inertia of a cylinder, Equation 5.1 is used. To compute the moment of inertia of a rectangular prism, Equation 5.2 is used. To compute the moment of inertia of a cylinder or rectangular prism that is off of its axis of rotation by a distance d, the Parallel Axis Theorem, Equation 5.3, is used.

$$I_{cyl} = m * r^2 \tag{5.1}$$

$$I_{prism} = \frac{m}{12}(a^2 + b^2) \tag{5.2}$$

$$I_{PAX} = I + md^2 \tag{5.3}$$

Table 5.1 displays the similarity of the moment of inertia about θ and ψ for V-REP and the Quanser documentation values. Code to calculate the moment of inertia for the V-REP Quanser AERO in yaw and pitch is included in Appendix B.3.

Table 5.1: Comparing V-REP and Quanser AERO Moments of Inertia

Parameter	Source	Value	Unit
J_θ	Quanser	0.215	$[\text{kg}\,\text{m}^2]$
J_θ	V-REP	0.0214	$[\text{kg}\,\text{m}^2]$
J_ψ	Quanser	0.237	$[\text{kg}\,\text{m}^2]$
J_ψ	V-REP	0.0244	$[\text{kg}\,\text{m}^2]$

5.3.1 Joint Control

In this approach, the V-REP model makes use of the model constants provided by Quanser. MATLAB is configured to send the optimized motor voltages decided by the control algorithm. V-REP then receives this signal, and sets the corresponding motor's input voltage in the simulation. LUA script then estimates the current angular velocities numerically by trapezoidal integration. The angular velocities are applied to the scene joints. The design of the LUA script attached to the Quanser AERO is shown in Figure 5.2.

In this design, the joints are configured as motors capable of receiving angular velocities from the V-REP API's setJointTargetVelocity function. In this configuration, V-REP will command the physics engine running the scene to apply torque to the selected joint until the target velocity is reached. V-REP will continue adjusting this torque to maintain the target velocity.

For the 2-DOF helicopter, we can write the angular accelerations as:

$$\ddot{\theta} = \frac{-K_{\text{sp}}}{J_\theta}\theta + \frac{-D_\theta}{J_\theta}\dot{\theta} + \frac{K_{\text{pp}}}{J_\theta}V_0 + \frac{K_{\text{py}}}{J_\theta}V_1 \tag{5.4a}$$

$$\ddot{\psi} = \frac{-D_\psi}{J_\psi}\dot{\psi} + \frac{K_{\text{yp}}}{J_\psi}V_0 + \frac{K_{\text{yy}}}{J_\psi}V_1 \tag{5.4b}$$

For the half-quadcopter, we can write the angular accelerations as:

$$\ddot{\theta} = \frac{-K_{\text{sp}}}{J_\theta}\theta + \frac{-D_\theta}{J_\theta}\dot{\theta} + \frac{-K_{\text{pp}}}{J_\theta}V_0 + \frac{K_{\text{pp}}}{J_\theta}V_1 \tag{5.5a}$$

$$\ddot{\psi} = \frac{-D_\psi}{J_\psi}\dot{\psi} + \frac{-K_{\text{yy}}}{J_\psi}V_0 + \frac{K_{\text{yy}}}{J_\psi}V_1 \tag{5.5b}$$

The LUA script for the half-quadcopter is included as Appendix B.4.1, and the 2-DOF helicopter script is included as Appendix B.4.2. Each of these scripts are attached to the base of the Quanser AERO in the V-REP scene. Table 5.2 provides the reader with information about the MATLAB script that runs each V-REP scene.

Table 5.2: File Sets

Model	MATLAB File	V-REP Scene
Half-quadcopter	B.4.3	AeroHalfquadcopter
2-DOF Helicopter	B.4.4	AeroHelicopter
Half-quadcopter	B.4.5	AeroHalfquadcopter
2-DOF Helicopter	B.4.6	AeroHelicopter

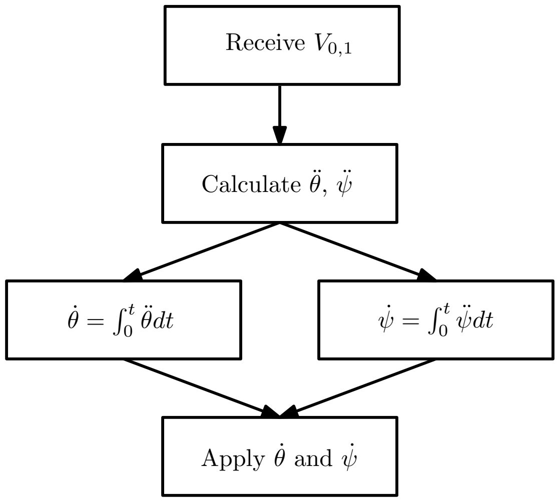
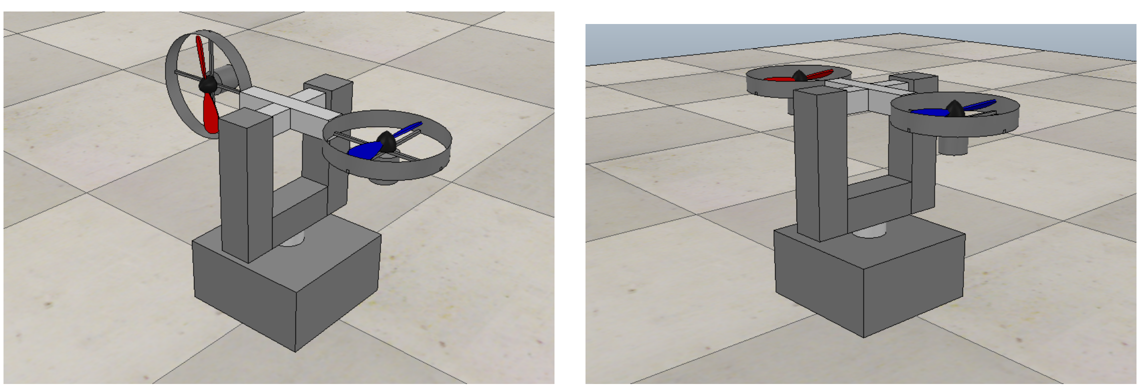


Figure 5.2: V-REP joint model flowchart.



(a) (b)

Figure 5.3: Quanser AERO developed for joint control. (a) 2-DOF helicopter mode. (b) Half-quadcopter mode.

5.3.2 Motor Control

For the motor control approach, the LUA script applies a force and a torque on the Quanser AERO at the location of the motor. This will simulate the propeller's lift and the motor's counter-torque on the Quanser AERO through V-REP's selected physics engine.

V-REP joints in this model must be configured as free spinning connections and contain no internal friction. Therefore, $D_\theta = D_\psi = 0$. Further, the model is balanced about the pitch joint, therefore, $K_{sp} = 0$. Using these values, the state-space matrices for the 2-DOF helicopter become:

$$\begin{bmatrix}\dot{\theta}\\ \dot{\psi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}\theta\\ \psi\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0.0515&0.0983\\-0.1105&0.0901\end{bmatrix}\begin{bmatrix}V_0\\V_1\end{bmatrix} \tag{5.6}$$

And for the half-quadcopter, the state-space matrices become:

$$\begin{bmatrix}\dot{\theta}\\ \dot{\psi}\\ \ddot{\theta}\\ \ddot{\psi}\end{bmatrix} = \begin{bmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}\theta\\ \psi\\ \dot{\theta}\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\-0.0450&0.0450\\-0.0901&0.0901\end{bmatrix}\begin{bmatrix}V_0\\V_1\end{bmatrix} \tag{5.7}$$

Figure 5.5 shows the modeling circuit of the electric motors driving each propeller in the Quanser AERO. Both the ADP and LQR will decide an optimal voltage to regulate the error states to zero, so the goal will be to calculate the lift and torque from equations provided in Quanser literature.

First, we calculate the lifting force created by the propeller. We express τ_θ, assuming 90˙ application:

$$\tau_\theta = R \cdot F_0 \sin 90 - R \cdot F_1 \sin 90 = K_{\text{pp}}V_0 - K_{\text{pp}}V_1 \tag{5.8}$$

Therefore:

$$F_{prop_{0,1}} = \frac{K_{\text{pp}}}{R}V_{0,1} \tag{5.9}$$

Next, the torque generated against the motor by the propeller dynamics is considered. V-REP provides a local coordinate system for the motor, and the torque is applied about the Z-axis. The model defines Motor#0 as having upwards thrust with a counterclockwise spin, and Motor#1 as having upwards thrust with a clockwise spin. Likewise, when the thrust changes direction as the polarity of voltage changes, the torque vector has to switch directions.

Quanser literature expresses drag torque as:

$$\tau_{drag} = k_d\omega_m \tag{5.10}$$

And gyroscopic torque as:

$$\tau_{gryo} = k_m i_m \tag{5.11}$$

Creating a total torque about the motor's body frame Z-axis as:

$$\tau_z = k_d\omega_m + k_m i_m \tag{5.12}$$

Using Equation 5.12, the input voltage from the control algorithm, and the motor current, i_m, the torque created by the motor can be modelled. However, we don't have an expression for the motor current, i_m, or the propeller speed, ω_m.

To model the current in V-REP, a sweep was performed on the actual Quanser AERO with the Simulink model developed in Section 6.1. For this test, we enabled the yaw and pitch locks on the Quanser AERO, applied voltages in steps of 2V from -22 to 22 Volts, and logged the currents. A cubic function estimating ($R^2 = 0.997$) the motor current, given the input voltage, was generated shown in Equation 5.13. The recorded data points and the current plot are shown in Figure 5.4.

$$i(v_{0,1}) = 2.77 \times 10^{-5} v^3 + -9.66 \times 10^{-6} v^2 + 2.13 \times 10^{-2} v + -3.19 \times 10^{-3} \tag{5.13}$$

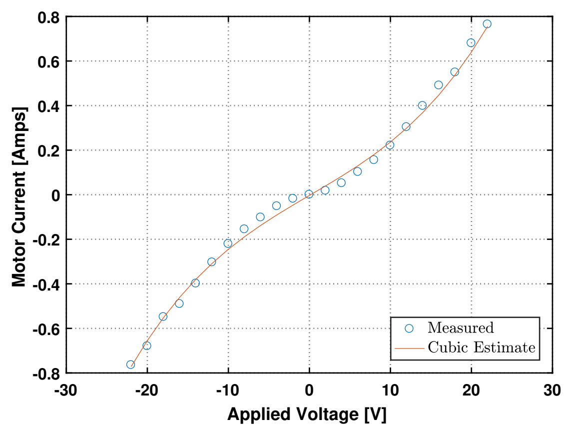


Figure 5.4: Measured currents and estimate function.

Using the current model allows for calculation of the torque from the drag force of the propeller and from the motor accelerating the propeller. Referencing Quanser literature, Equation 5.14 represents the back EMF, a relationship between the voltage across the motor and the motor speed.

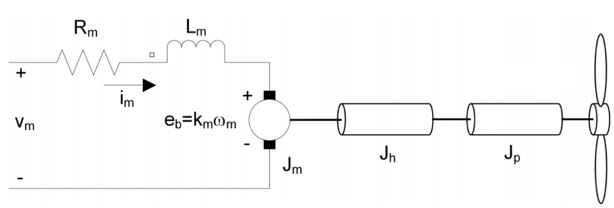


Figure 5.5: Circuit modeling Quanser AERO motors.

$$e_b = k_m \omega_m \tag{5.14}$$

Using Figure 5.5 and KVL:

$$v_m - R_m i_m - L_m \frac{di_m}{dt} - k_m \omega_m = 0 \tag{5.15}$$

Table 5.3: Parameter values shown in Figure 5.5

Parameter	Description	Value	Unit
R_m	Terminal Resistance	8.4	$[\Omega]$
k_t	Torque Constant	0.042	$[\mathrm{N\,m\,A^{-1}}]$
k_m	Back EMF Constant	0.042	$[\mathrm{V\,rad^{-1}\,s^{-1}}]$
J_m	Rotor Inertia	4×10^{-6}	$[\mathrm{kg\,m^2}]$
L_m	Rotor Inductance	0.00116	[H]
k_d	Drag Coefficient	1×10^{-5}	$[\mathrm{N\,m\,rad^{-1}\,s^{-1}}]$

Leading to:

$$\omega_m = \frac{v_m - R_m i_m - L_m \frac{di_m}{dt}}{k_m} \tag{5.16}$$

Equation 5.16 represents the model to find the propeller speed, given the input voltage, and the current. Therefore, using Equations 5.13, 5.15, 5.10, and 5.11 allows for estimated forces and torques to be calculated. Each propeller's LUA script is described by the flowchart in Figure 5.3.2.

The LUA script is implemented as follows. The Quanser AERO base receives Appendix B.5.1, then each motor gets its own script, Motor#0: Appendix B.5.2, Motor#1: Appendix B.5.3. For now, the only implementation of MATLAB code is provided as an untuned PID controller, provided as Appendix B.5.4. Thus far, LQR and ADP algorithms have not been applicable to the system.

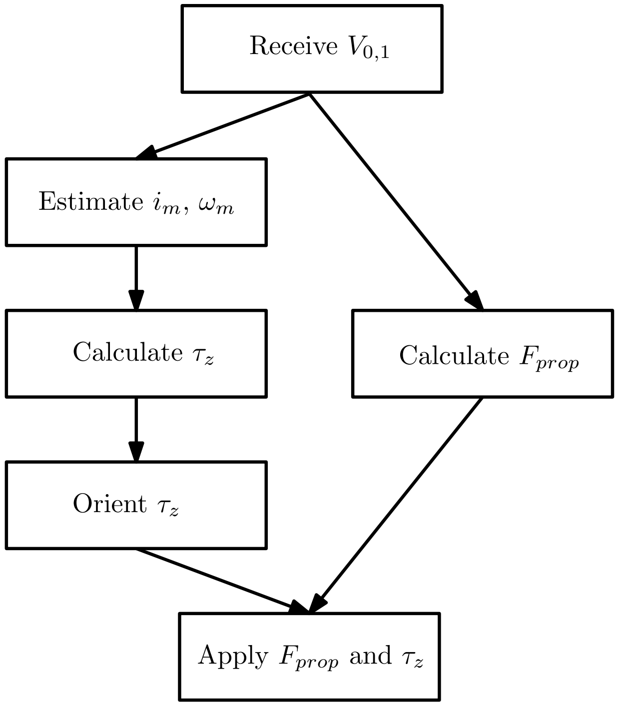
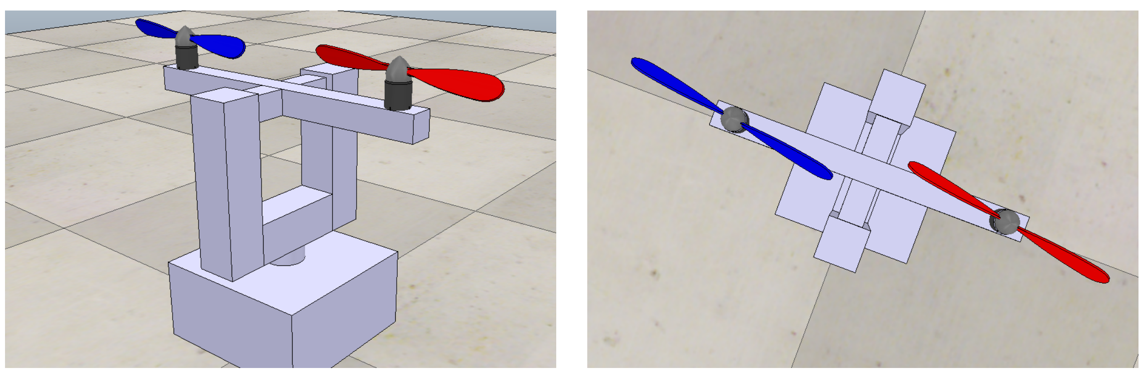


Figure 5.6: V-REP motor model flowchart.



(a) (b)

Figure 5.7: Quanser AERO developed for motor control. (a) Half-quadcopter mode (side). (b) Half-quadcopter mode (top).

5.4 Simulation Results

Figures 5.8, 5.9, 5.10, 5.11, 5.12, and 5.13 are presented to demonstrate the V-REP joint control model performing in both 2-DOF helicopter and half-quadcopter mode with the LQR algorithm acting as the controller. The results of the RMSE of the measurements are summarized in Table 5.4.

Figures 5.14, 5.15, 5.16, and 5.17 are presented to demonstrate the V-REP joint control model performing in both 2-DOF helicopter and half-quadcopter mode with ADP acting as the controller.

We can see that LQR performs well for this particular system. ADP under-performs LQR most notably in smoothness; however, ADP does track the referenced angular trajectory. The results are similar to the MATLAB simulations, but there are more disturbances. The results of the RMSE measurements are summarized in Table 5.5.

Figures 5.18, 5.19 and 5.20 are provided as proof of concept for the motor modeled V-REP scene. In these simulations, $K_p = 8$, $K_i = 10$, and $K_d = 3$.

5.4.1 LQR

Table 5.4: Experimental V-REP LQR angular RMSE measurements.

Figure	Configuration	θ LQR [deg]	ψ LQR [deg]	$\dot{\theta}$ LQR [deg /s]	$\dot{\psi}$ LQR [deg /s]
5.8	2-DOF Helicopter	3.60063	1.42257	0.00034832	0.00082065
5.9	2-DOF Helicopter	0.984427	1.75581	0.000235371	0.000358396
5.10	2-DOF Helicopter	0.433482	0.309782	0.000207048	0.000369757
5.11	Half-quadcopter	5.03313	3.18276	0.000334748	0.00081096
5.12	Half-quadcopter	1.25609	1.15357	0.000226184	0.000364947
5.13	Half-quadcopter	0.643793	0.170658	0.000205815	0.000367207

5.4.2 ADP

Table 5.5: Experimental V-REP ADP angular RMSE measurements.

Figure	Configuration	θ ADP [deg]	ψ ADP [deg]	$\dot{\theta}$ ADP [deg /s]	$\dot{\psi}$ ADP [deg /s]
5.14	2-DOF Helicopter	3.94776	1.655	0.000159409	0.00091622
5.15	2-DOF Helicopter	0.449811	1.58411	0.000406199	0.00139641
5.16	Half-quadcopter	6.58232	3.27046	0.00096847	0.00108425
5.17	Half-quadcopter	0.742075	1.95259	0.000410448	0.00106001

5.4.3 PID

5.4.4 Analysis

From the simulation results provided in Sections 5.4.1, 5.4.2, and 5.4.3, we can see that the V-REP models seem to be working correctly. There is, however, one difficulty associated with ADP compared to both LQR and PID control. Because of the computational

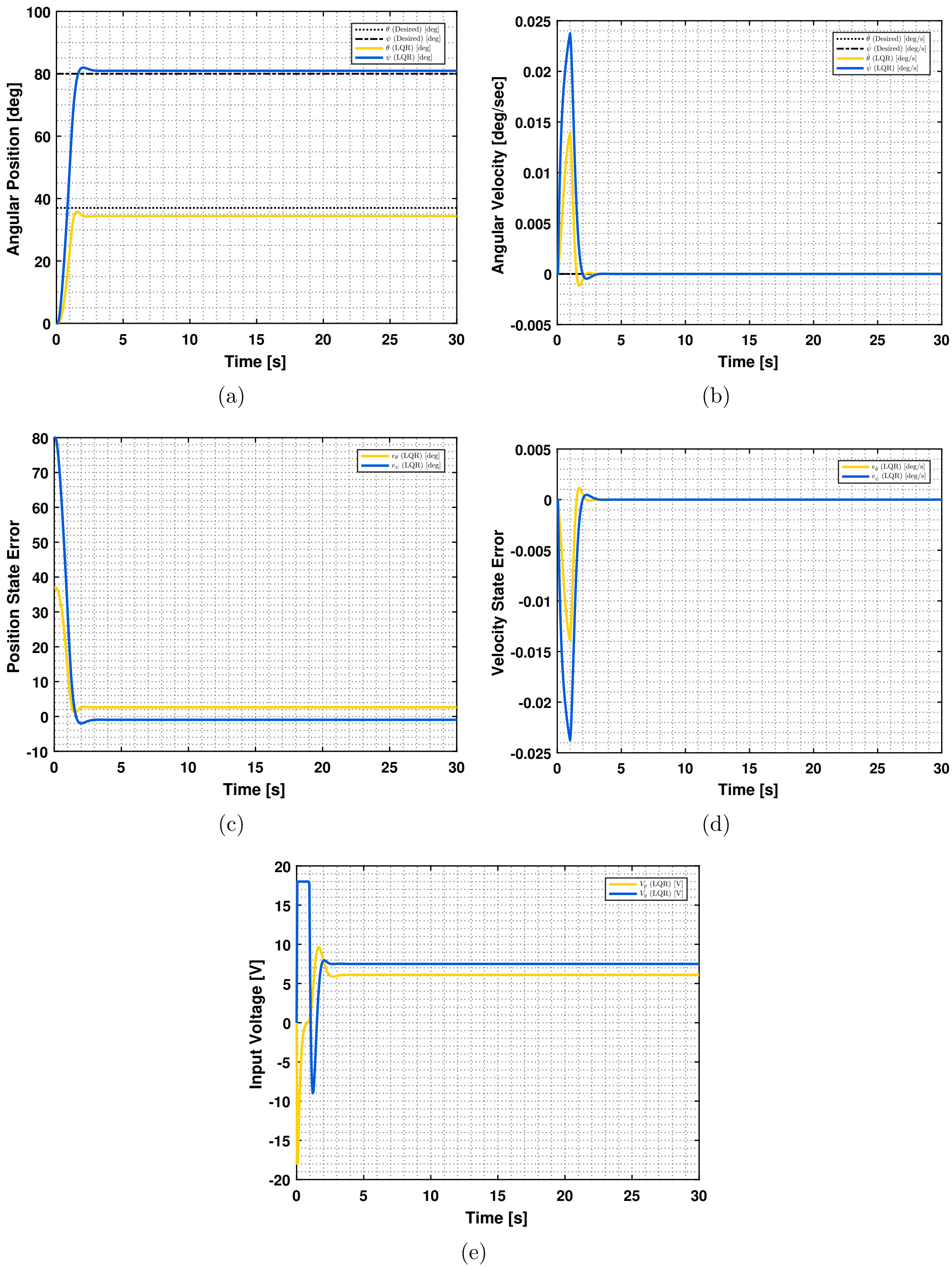


Figure 5.8: Time-varying constant desired pitch and yaw for 2-DOF helicopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

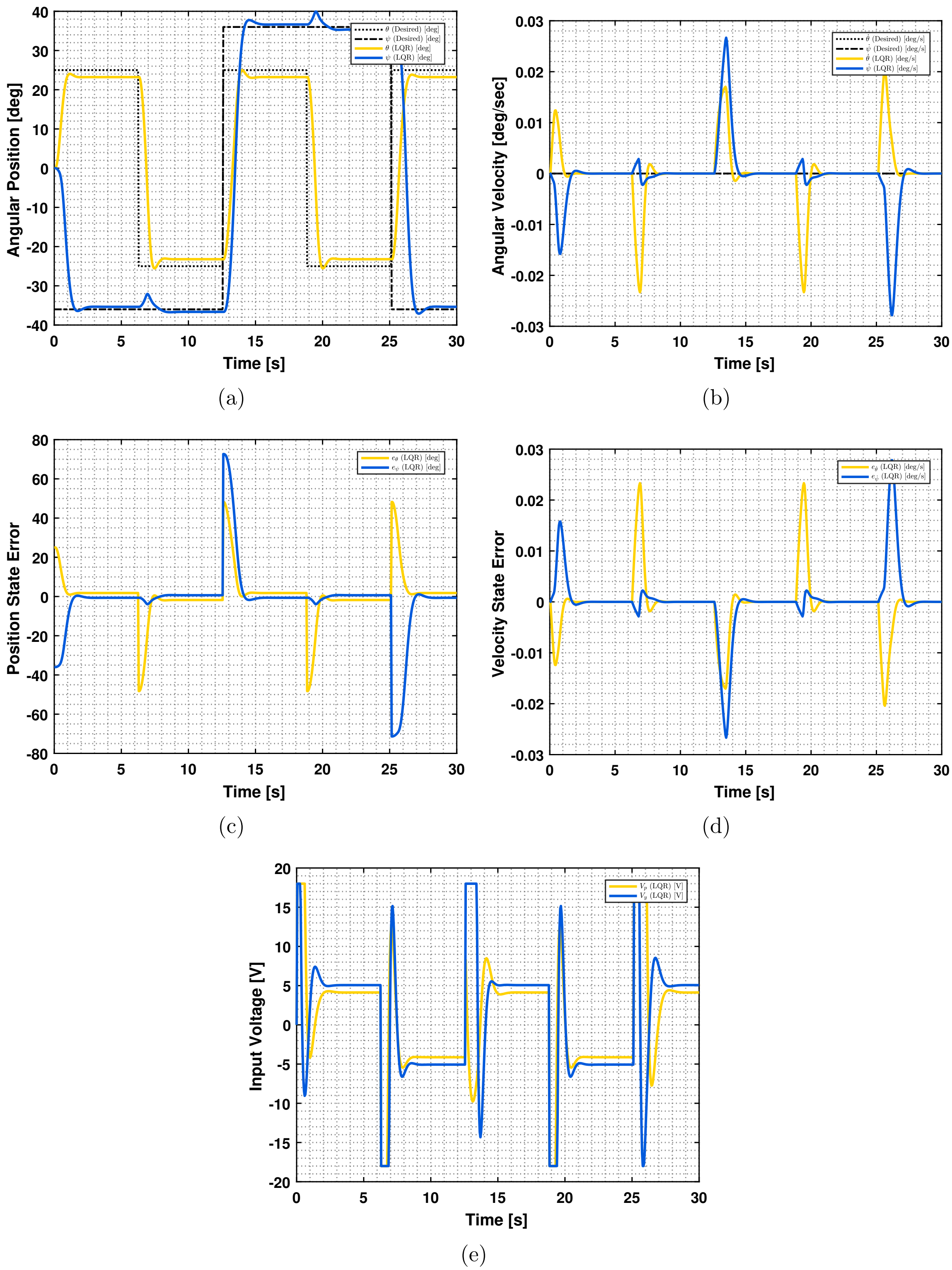


Figure 5.9: Time-varying constant desired pitch and yaw for 2-DOF helicopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

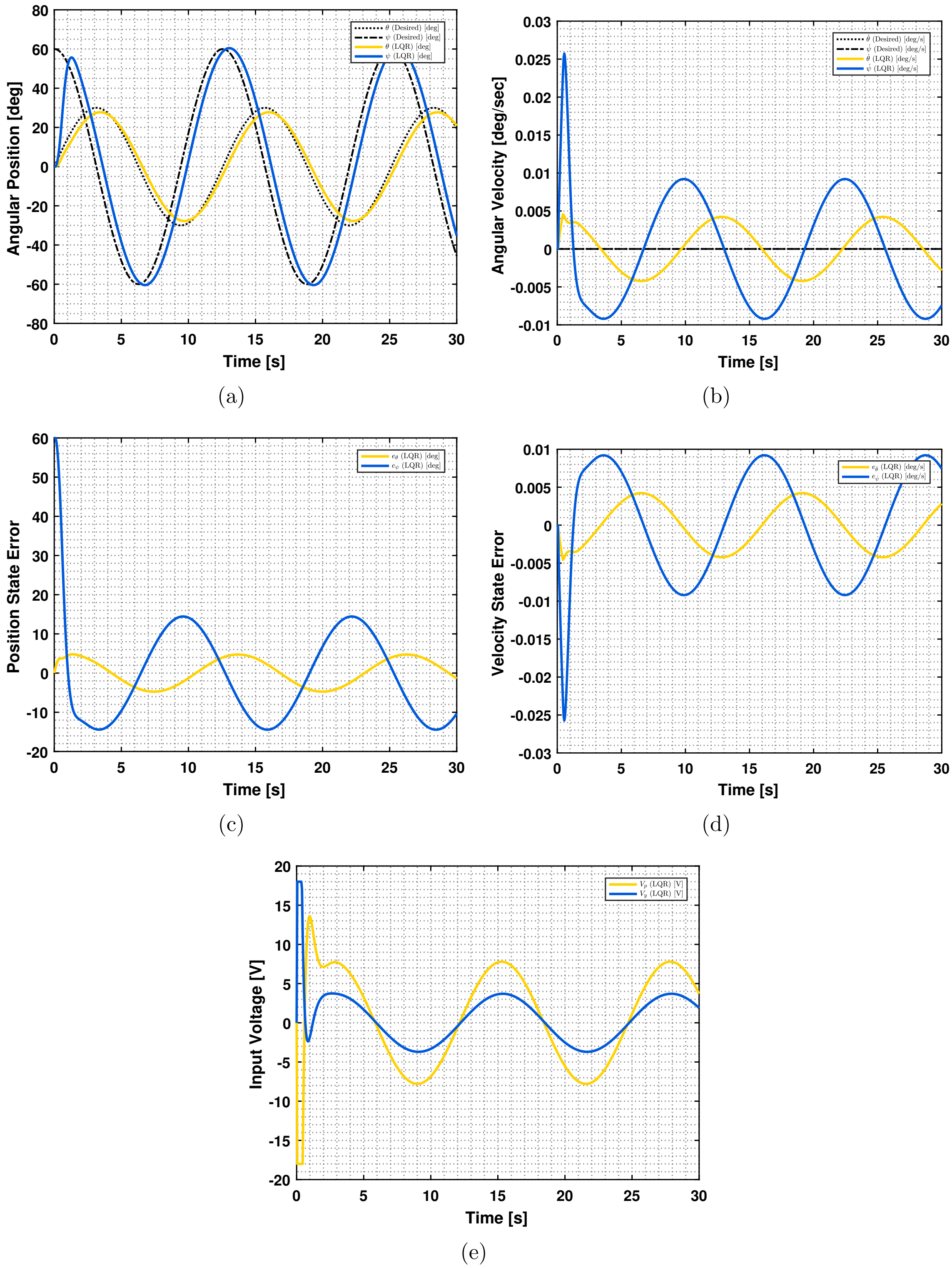
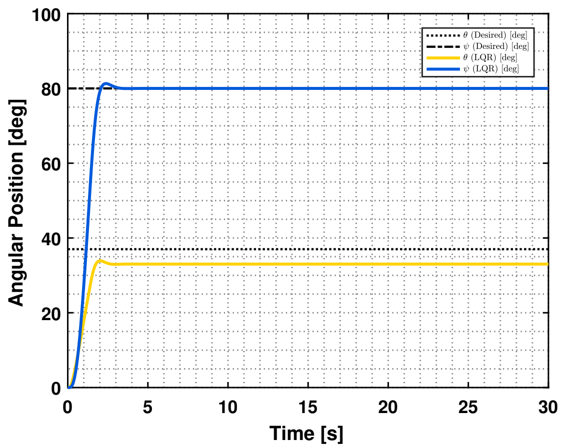


Figure 5.10: Time-varying constant desired pitch and yaw for 2-DOF helicopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.



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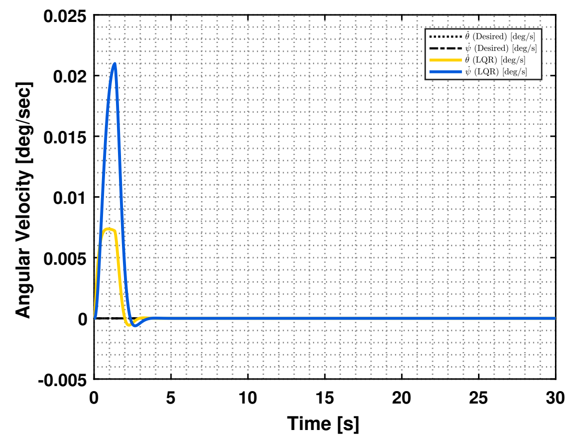

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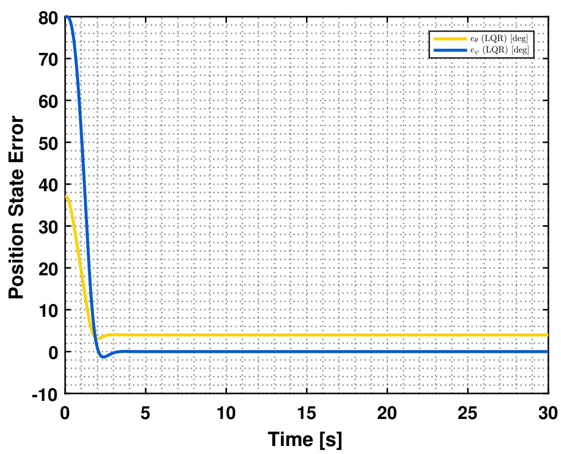

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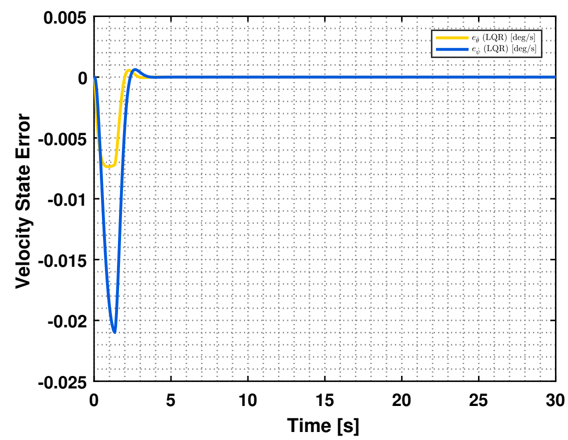

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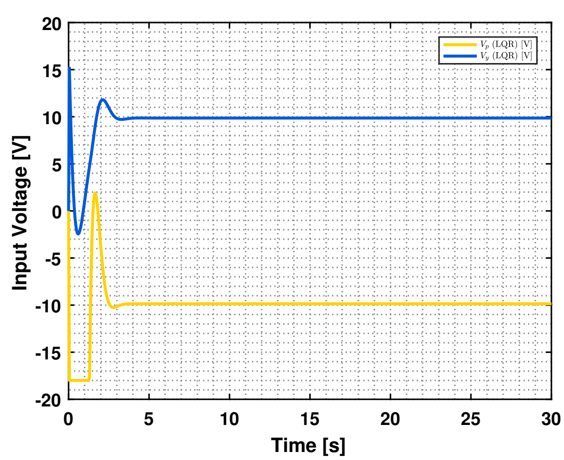


(e)

Figure 5.11: Time-varying constant desired pitch and yaw for half-quadcopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

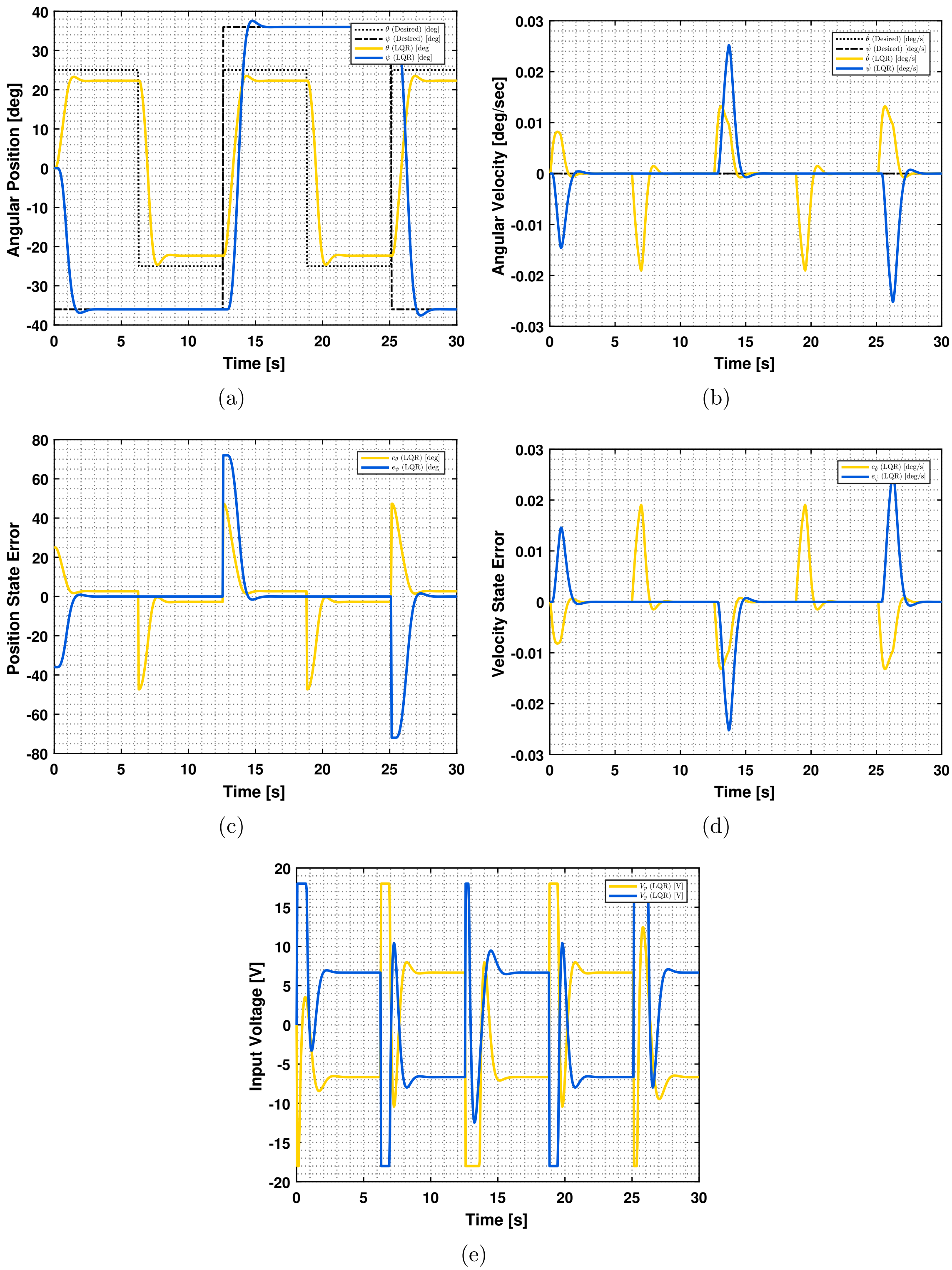


Figure 5.12: Time-varying constant desired pitch and yaw for half-quadcopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

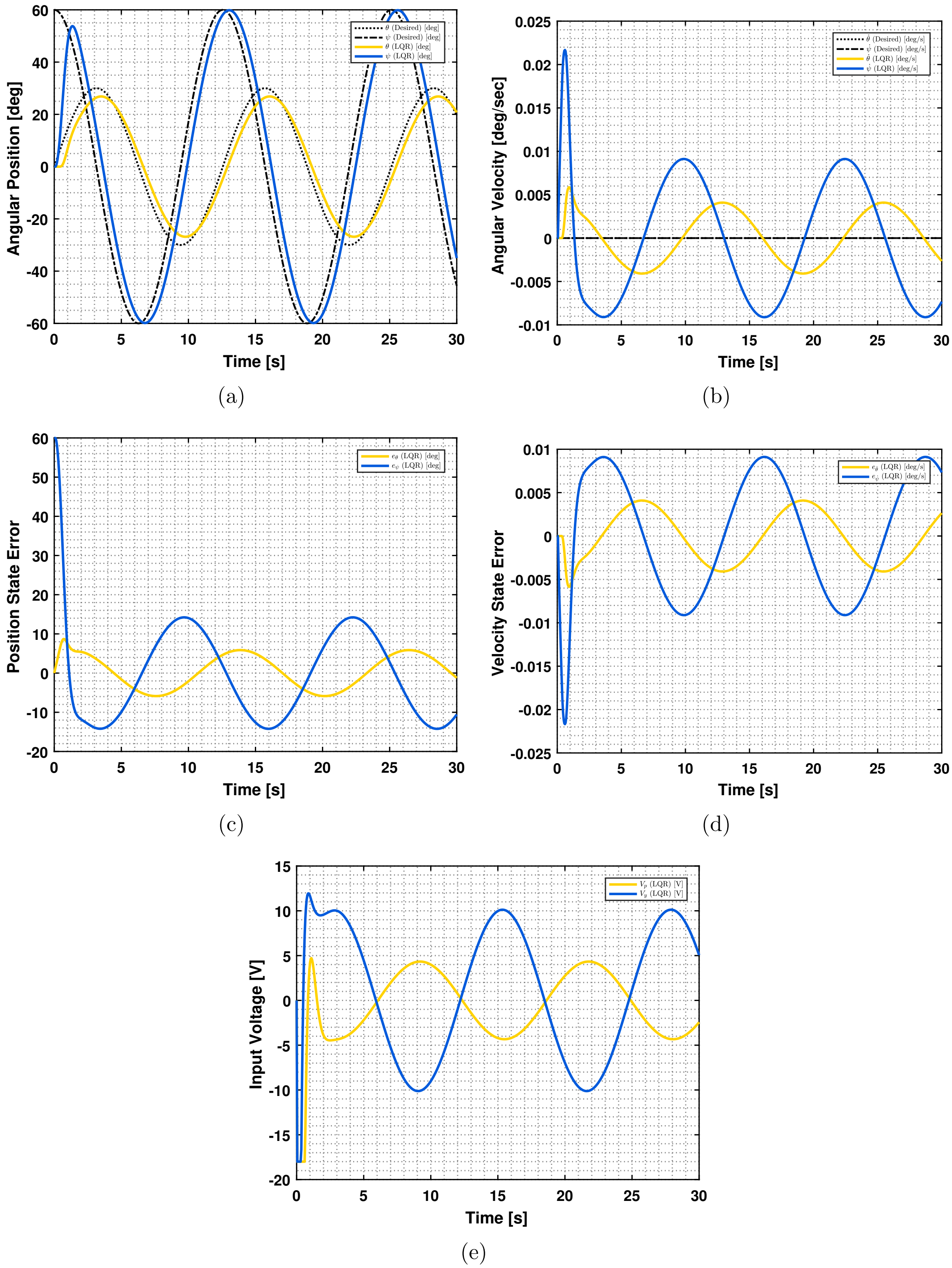
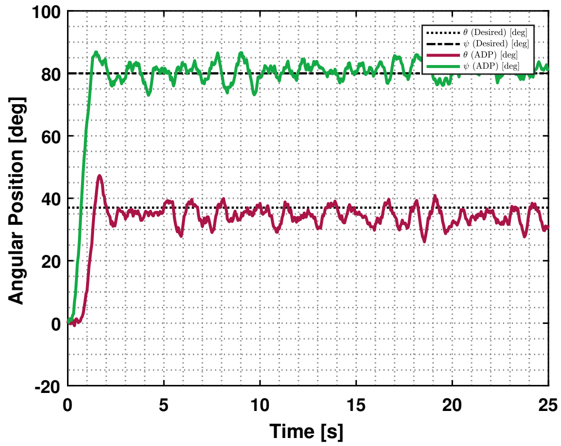
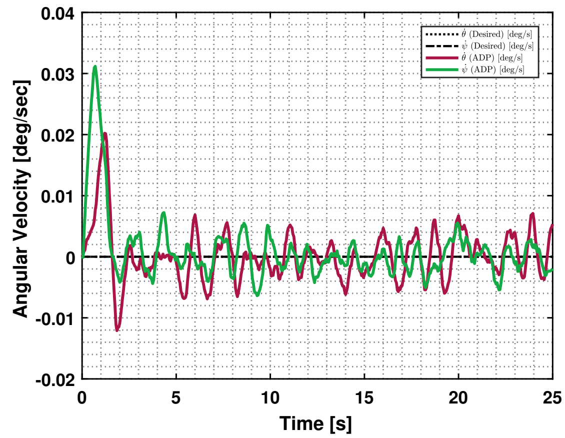


Figure 5.13: Time-varying constant desired pitch and yaw for half-quadcopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.



(a)



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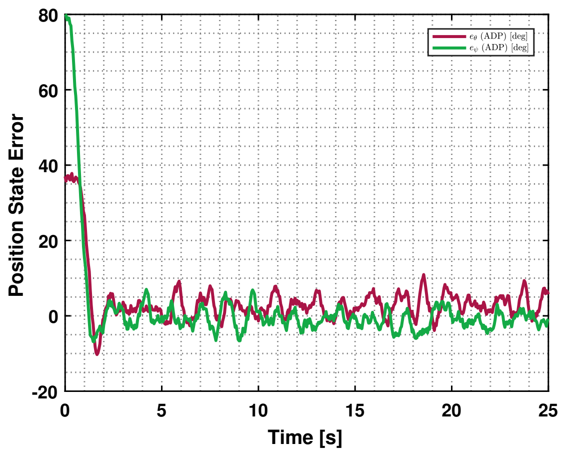

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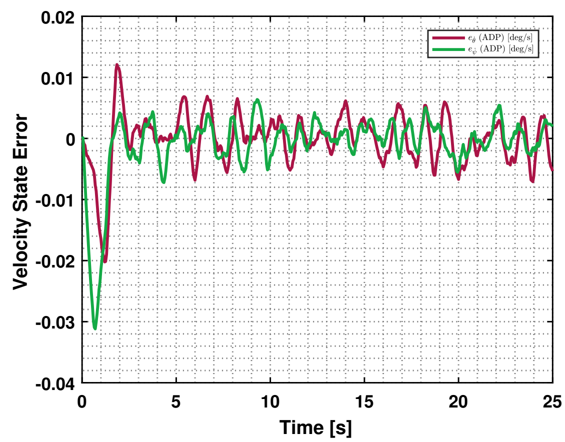

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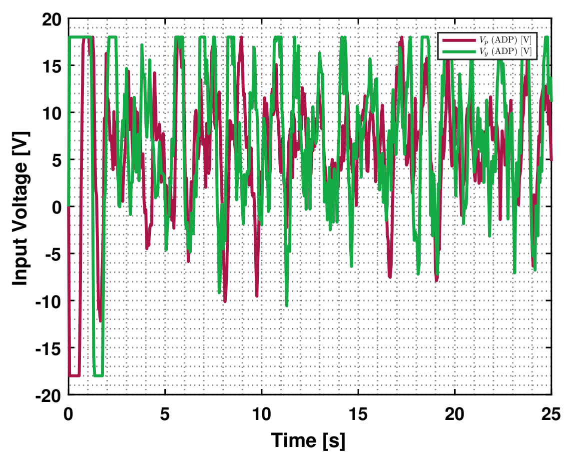


(e)

Figure 5.14: Time-varying constant desired pitch and yaw for 2-DOF helicopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

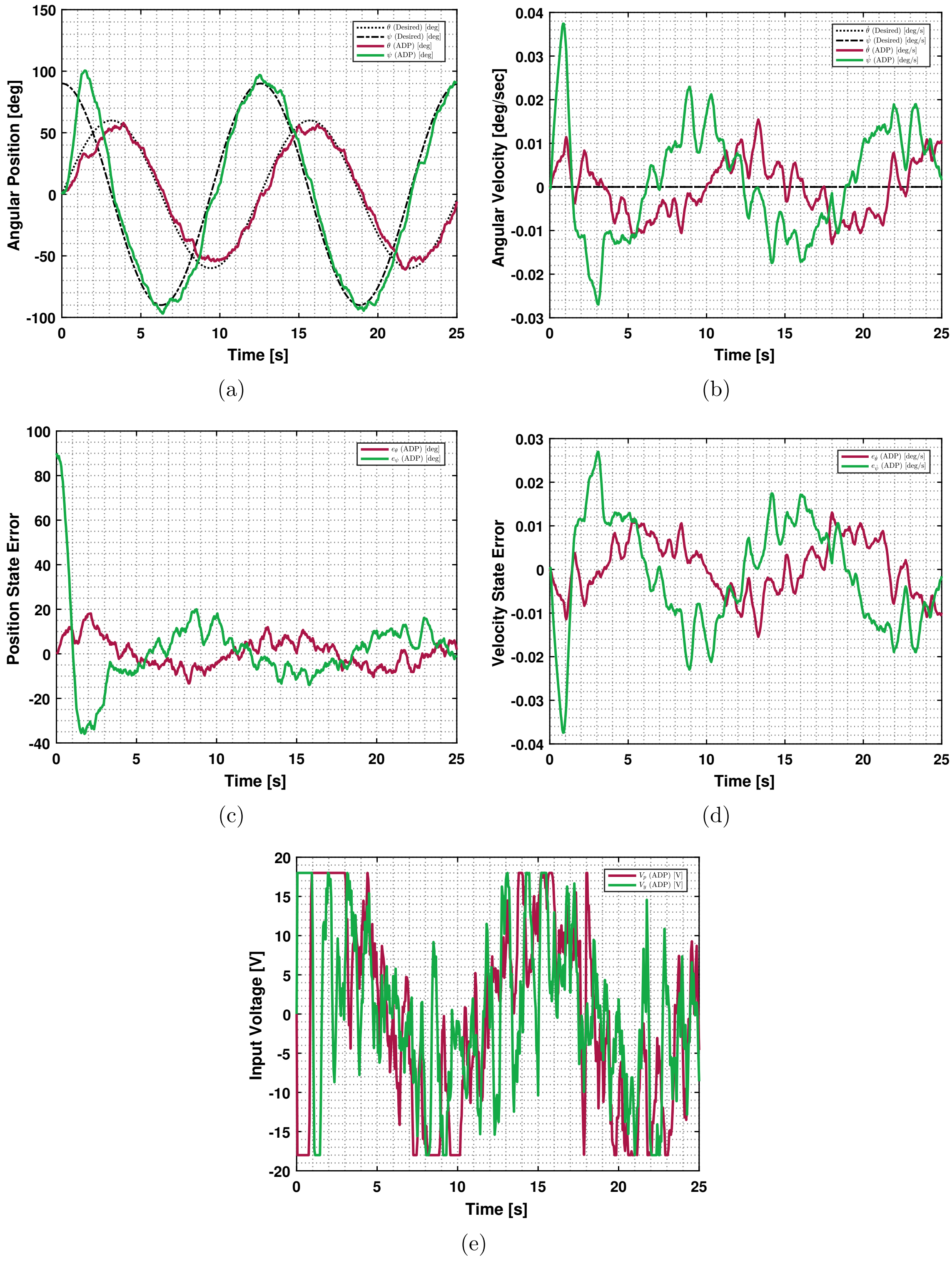


Figure 5.15: Time-varying constant desired pitch and yaw for 2-DOF helicopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

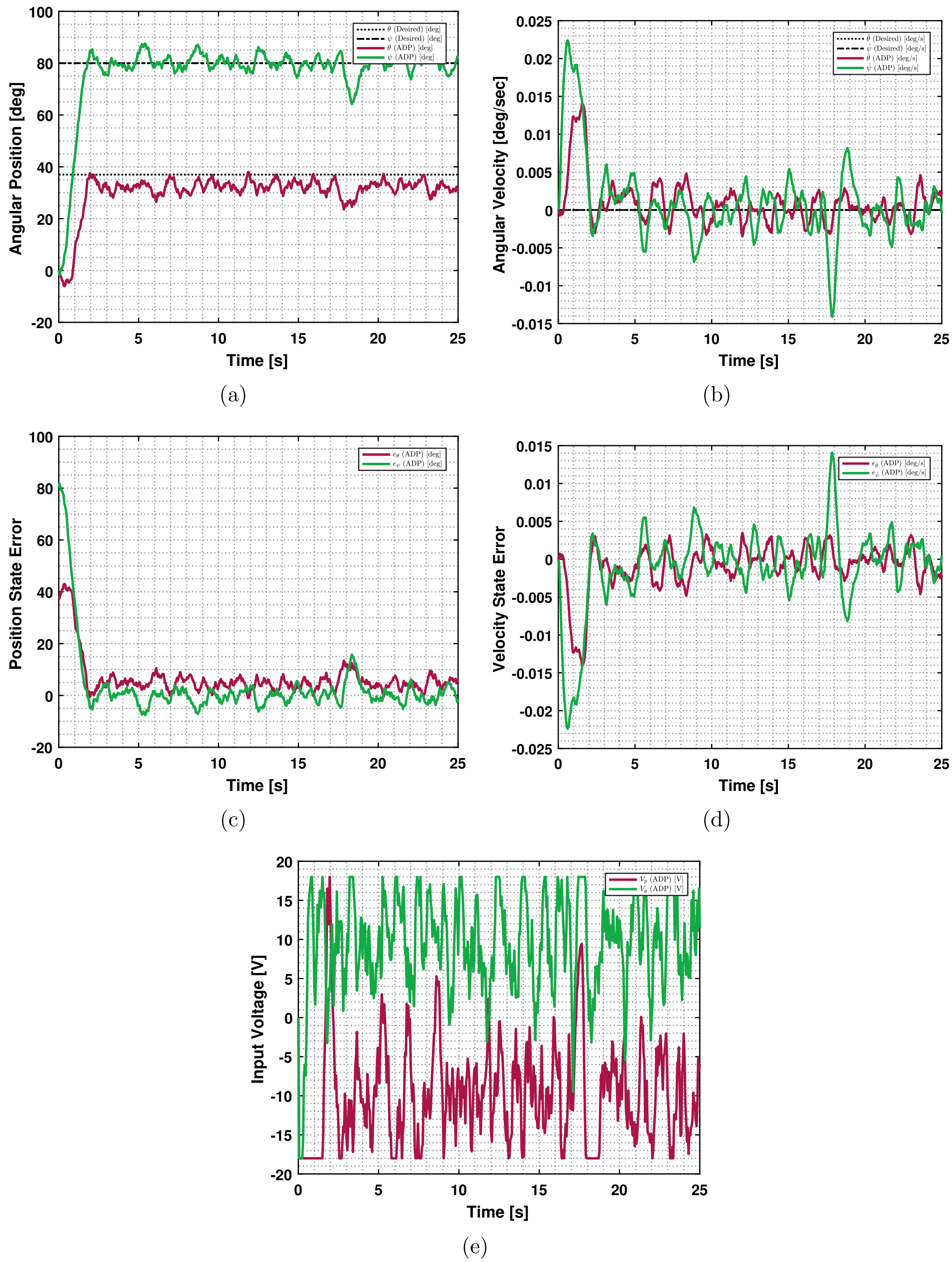


Figure 5.16: Time-varying constant desired pitch and yaw for half-quadcopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

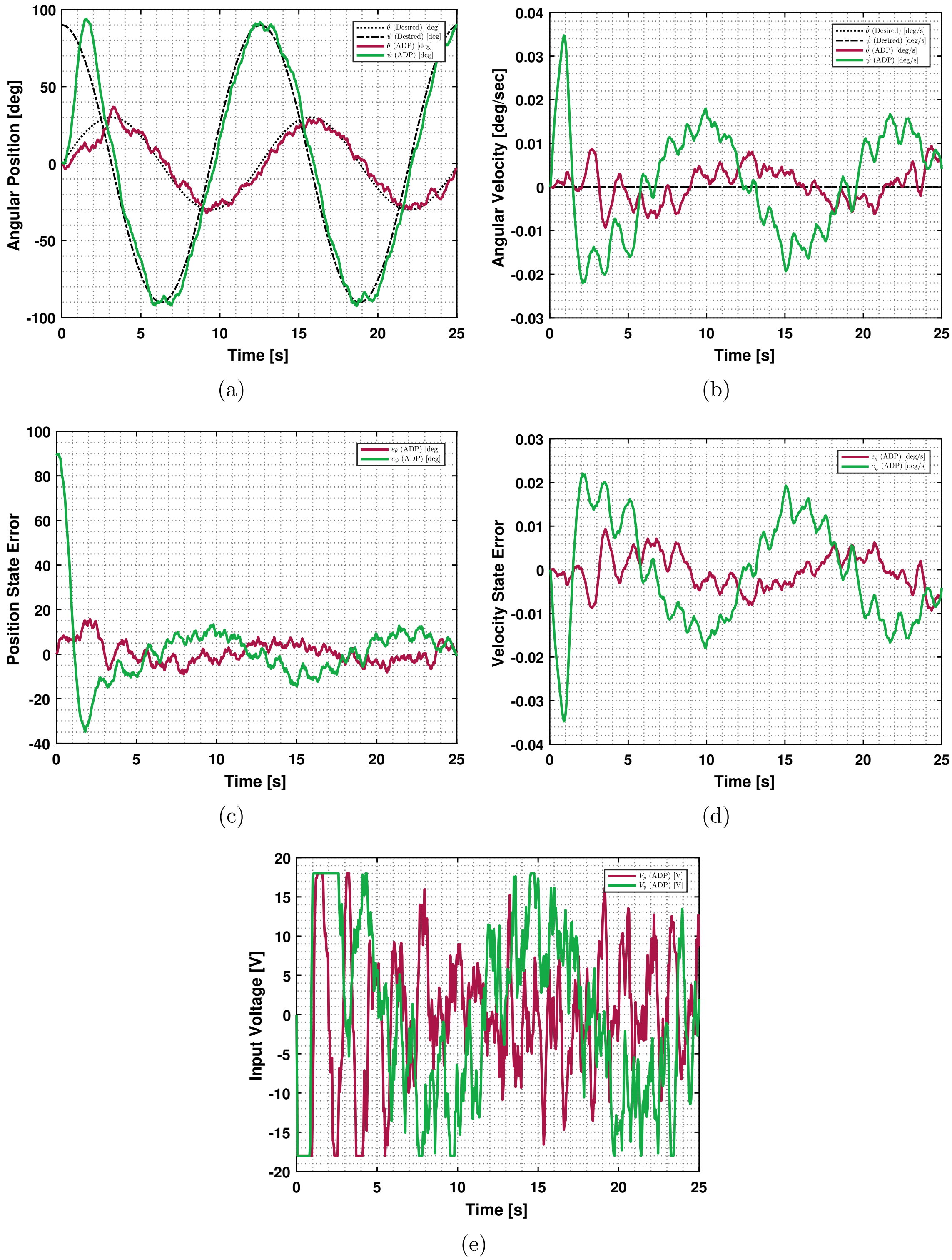
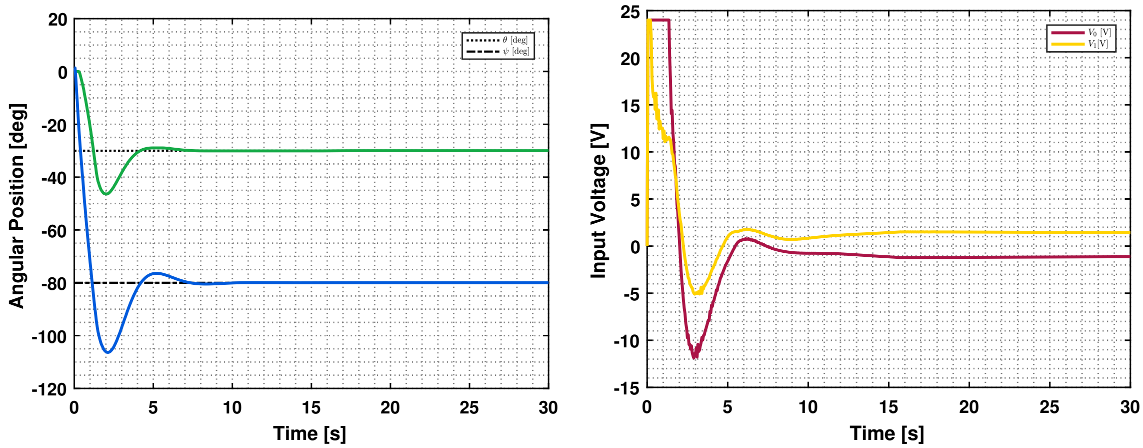
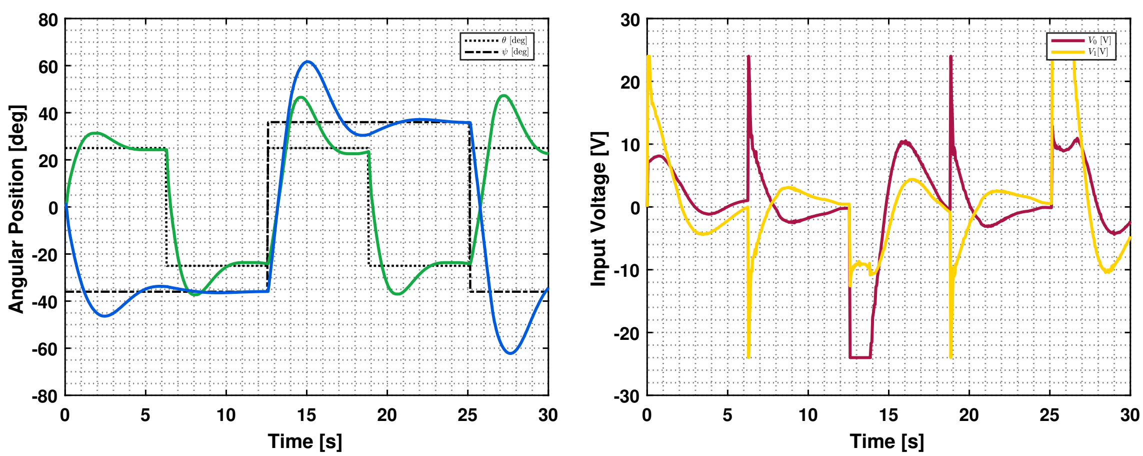


Figure 5.17: Time-varying constant desired pitch and yaw for half-quadcopter. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.



(a) (b)

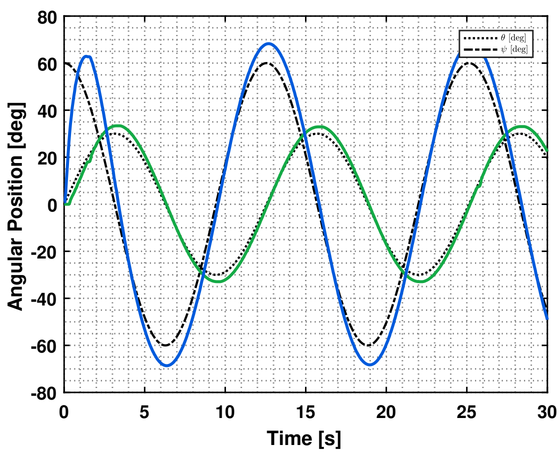
Figure 5.18: Time-varying constant desired pitch and yaw for half-quadcopter with PID controller. (a) Angular position. (b) Voltage inputs.



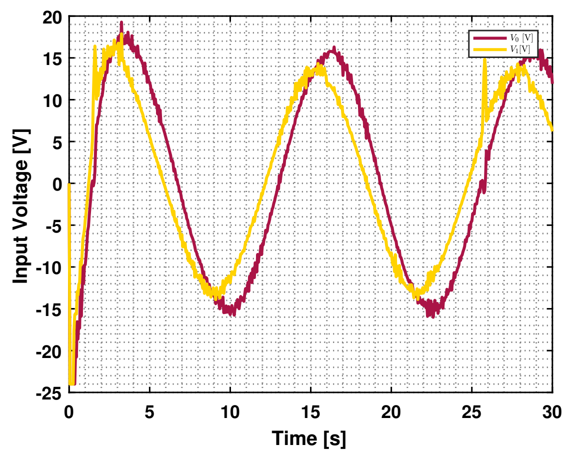
(a) (b)

Figure 5.19: Time-varying constant desired pitch and yaw for half-quadcopter with PID controller. (a) Angular position. (b) Voltage inputs.

complexity of ADP, latency issues arise between MATLAB and V-REP. MATLAB is not able to perform fast enough for ADP to simulate in real-time in V-REP. Options to fix this could either be optimizing ADP or using a faster language besides MATLAB. For now, it suffices to run V-REP not in real-time mode, made possible by the MATLAB calling function. The results predict that ADP will work in the V-REP simulations, but that optimization could greatly be advanced. Further research needs to be conducted to address this issue.



(a)



(b)

Figure 5.20: Time-varying constant desired pitch and yaw for half-quadcopter with PID controller. (a) Angular position. (b) Voltage inputs.

Chapter 6

Hardware Implementation

Once MATLAB simulations showed that ADP can be used as a controller for the Quanser AERO, ADP could be implemented to control the physical Quanser AERO. To accomplish this hardware implementation, we used the graphical programming language Simulink which is recommended by Quanser. Simulink is an industry-standard graphical programming language used for the purpose of C-code generation from the graphical interface. This is exactly why it has been implemented by Quanser. Our initial task was to implement ADP into a Simulink model. This is actually more difficult than it sounds because of the complexity of ADP.

The Quanser AERO can be configured with two different interface panels, the QFLEX 2 USB and the QFLEX 2 Embedded. The QFLEX 2 USB is the panel that allows the Quanser AERO to be controlled via USB from a personal computer. This panel requires specialized Quanser software which will be discussed. Quanser also provides plenty of documentation and Simulink examples for the QFLEX 2 USB panel.

The other panel provided by Quanser is the QFLEX 2 Embedded. This panel allows communication to the Quanser AERO via SPI communication. The QFLEX 2 Embedded is designed to be used with embedded devices such as a Raspberry Pi. We implemented this panel using a Raspberry Pi 3 (`https://www.raspberrypi.org/products/raspberry-pi-3-model-b/`) which can be seen in Figure 6.1. The problem with the QFLEX 2 Embedded panel is that Quanser does not have much documentation or any examples for this panel. All of the results that will be discussed about the Raspberry Pi 3 were made from our own research and technical experiences.

One advantage of starting from the beginning with the Raspberry Pi 3 is that we had the freedom to expand its applications once we were able to get the basics under control. This can be seen with the application of the Android smart phone application which will be discussed as well.

Our initial goal for the project was to be able to implement ADP on the Quanser AERO through various media. The options can be seen in Figure 6.2. Not all media have been achieved, but the ones completed will be described in this chapter.

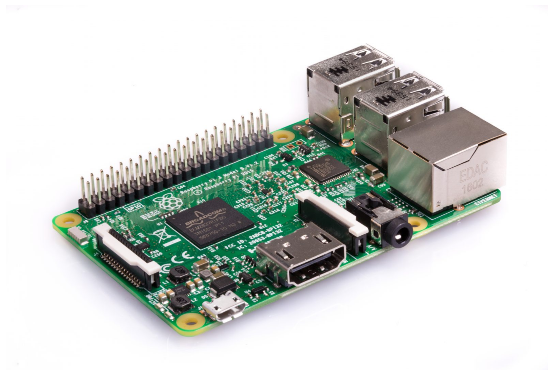


Figure 6.1: Raspberry Pi 3.

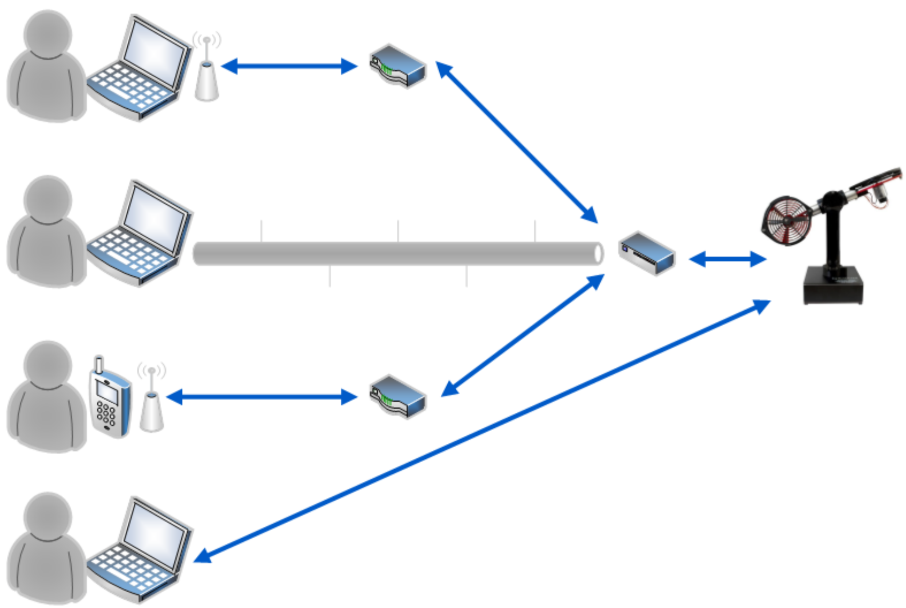


Figure 6.2: From top to bottom. 1) User controls the Quanser AERO from their laptop via Wi-Fi through the Raspberry Pi 3. 2) User controls the Quanser AERO from their laptop via Ethernet through the Raspberry Pi 3. 3) User controls the Quanser AERO from their cell phone via Wi-Fi through the Raspberry Pi 3. 4) User controls the Quanser AERO via a USB connection using the QFLEX 2 USB panel.

This chapter is organized as follows. Section 6.1 discusses the Simulink model of ADP. Section 6.2 discusses the QFLEX 2 USB panel and its use in running a Simulink model on the Quanser AERO. The use of the QFLEX 2 Embedded panel is discussed in Section 6.3 along with the use of the Raspberry Pi 3 and its complexities. Section 6.4 showcases the development of a Android smart phone application created by Simulink to control the Quanser AERO via the Raspberry Pi 3.

NOTE: MATLAB 2016b and its associated Simulink version were used for the entirety of this project. Different versions may affect some of the Simulink models.

6.1 ADP Algorithm in Simulink

In order to utilize ADP on the Quanser AERO, we have to convert the ADP MATLAB code to a Simulink model. The Simulink model is essentially a block diagram of ADP. We need to use Simulink to model ADP because the QUARC software, which will be discussed later, generates C-code from the Simulink model. This C-code is what drives the QFLEX 2 USB panel.

The Simulink model of ADP can be seen in Figure 6.3. The Simulink model seen in

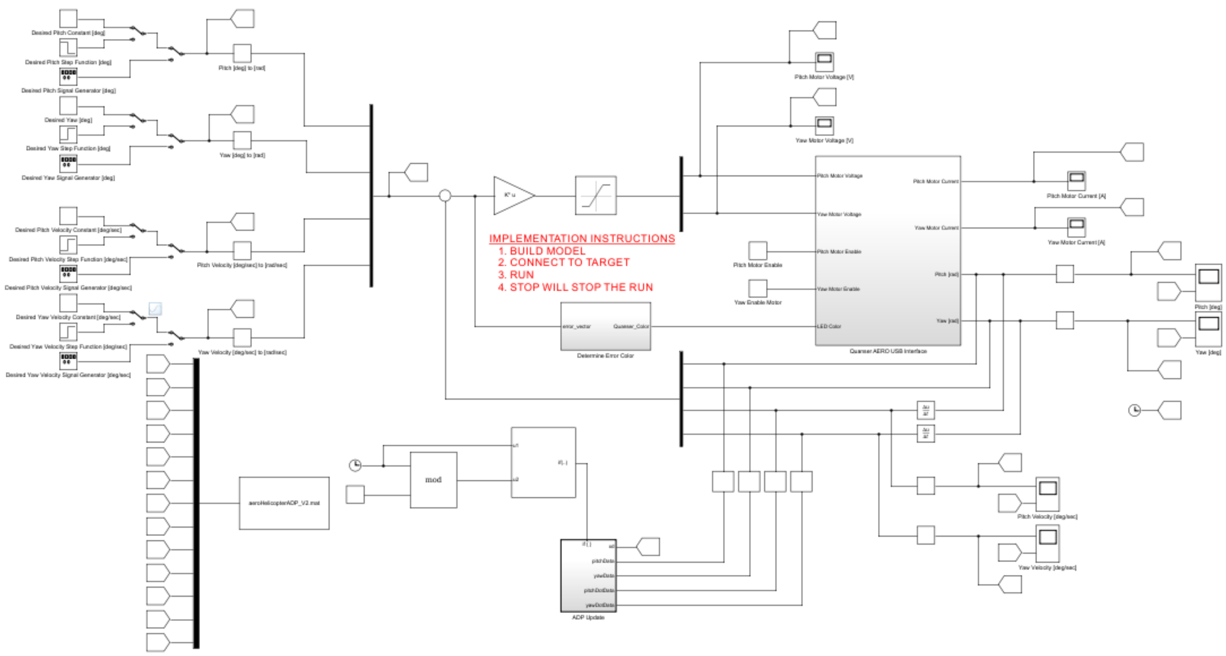


Figure 6.3: ADP algorithm in the form of a Simulink model.

Figure 6.3 is actually combined with the communication necessary for the QFLEX 2 USB, but we will only focus on the ADP aspects in this section.

To analyze the Simulink model shown in Figure 6.3, we can briefly describe the model clockwise starting in the top-left. Before we describe the model, it should be noted that the state-feedback gain needs to be initialized before ADP can be ran. This is accomplished in Simulink by calling an "init" function which is declared in the Model Properties. This initialization function essentially calculates the state-feedback gain using random error data. It then saves the state-feedback gain and other important values such as sampling times in the workspace. The specific initialization function can be seen in Appendix C.2. The MATLAB code is very similar to that of the MATLAB simulations because MATLAB executes this script before executing the Simulink model.

Now, observing the Simulink model in Figure 6.3, we can begin analyzing it starting with the inputs. The inputs to the Simulink model are the desired pitch and yaw and their corresponding desired angular velocities. The inputs are setup such that they can be easily switched. Moving to right, the desired configurations are combined into a vector with the mux block. The vector then enters the state-feedback system. We can see the state-feedback gain and the system which is the the largest block in the Simulink model. The inputs are the two motor voltages which are saturated between -24 and 24 Volts by the saturation block. We saturated the voltage inputs to 18 Volts in the MATLAB simulations, but Quanser Simulink examples use a saturation level of 24 Volts, so we will implement that method.

Once the system uses the input voltages to update the position, we use feedback to find the pitch, yaw, and angular velocity error used to find new inputs. Because we are not measuring the angular velocity, we use the derivative block to find the angular velocity. The model thus far essentially represents a state-feedback system, but we have not discussed ADP yet.

ADP is the subsystem in the lower center of the Simulink model. This subsystem is only triggered when the sampling is a multiple of T. The subsystem contains the user-defined function in Figure 6.4. This user-defined function essentially contains another version of

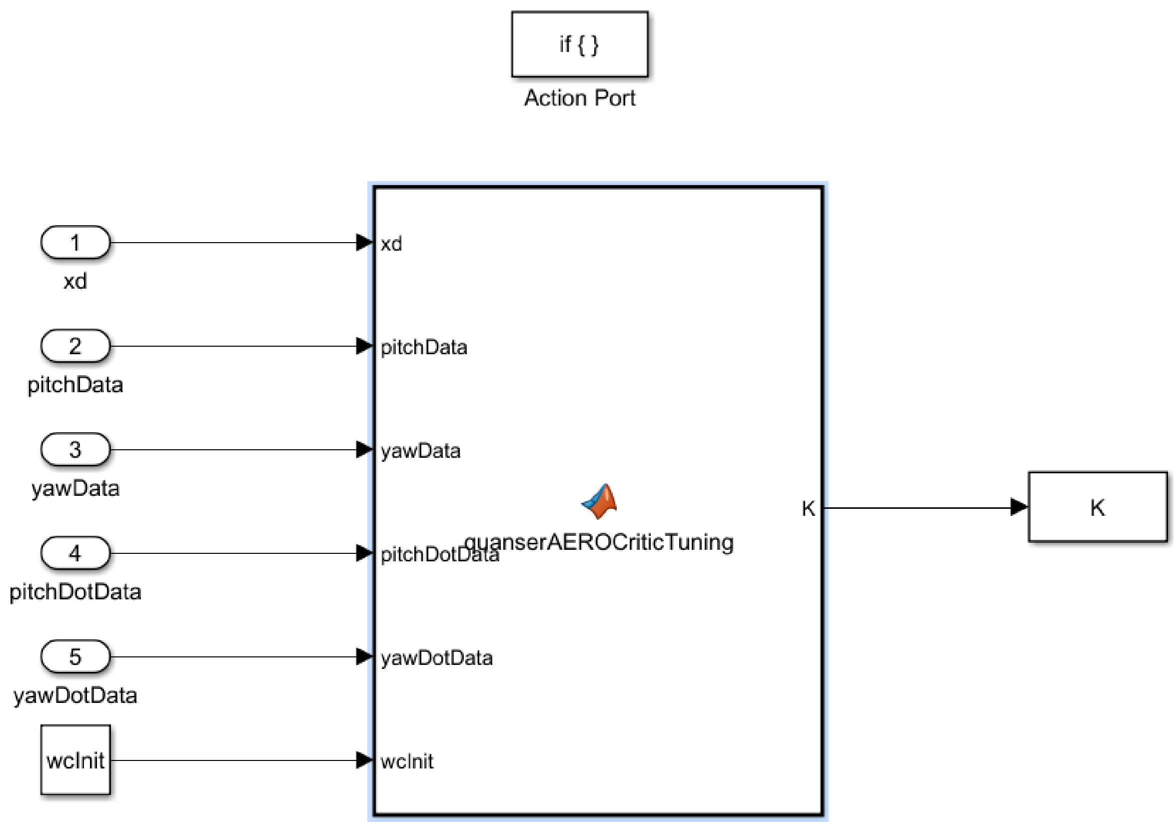


Figure 6.4: User-defined function to update the state-feedback gain in the ADP Simulink model.

ADP in MATLAB code. When designing the ADP Simulink model, it was determined that it would be easier to utilize ADP MATLAB code rather than converting it to a model itself. The disadvantage is the MATLAB code must be compatible for C-code generation. The reason for this will be discussed in later sections, but for now, the ADP update MATLAB code can be seen in Appendix C.3. One can notice that certain functions used in the original MATLAB simulations had to be changed to overcome this C-code generation challenge. One example is the use of anonymous functions. It should also be noted that the error data needed to be saved for use in the ADP subsystem. To save the error data, a tapped delay block was used for each of the four states. The tapped delay allows you to specify the number of delays you would like stored in the block saved as an array.

The last subsystem yet to be discussed in the Simulink model in Figure 6.3 is the subsystem in the center. This subsystem will be used later for the QFLEX 2 USB, but it can be discussed now. The subsystem changes the color of the base LED. In the subsystem is another user-defined function based on the magnitude of the error seen in Figure 6.5. The

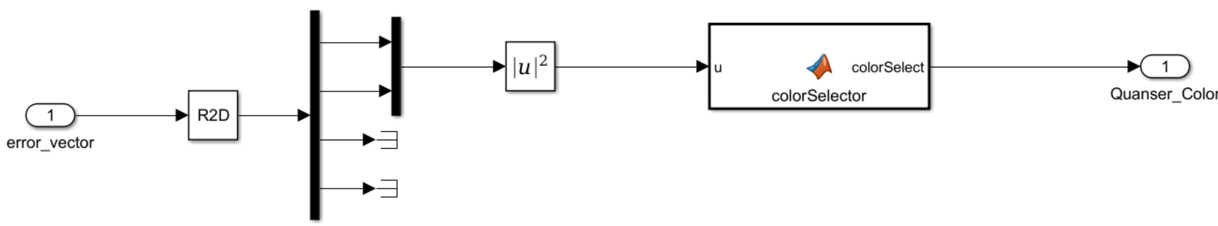


Figure 6.5: User-defined function to change the base LED color.

user-defined MATLAB function code can be seen in Appendix C.4. The output of the MATLAB code is the intensities of red, blue, and green values to be sent to the Quanser AERO.

The items mentioned here were the main components of ADP in the form of a Simulink model. It is actually a rather simple model since we already had the MATLAB code ready from the simulations. The model could have been made a lot more difficult if the user-defined MATLAB functions were not used. As for the other components of the model, there are multiple scopes and gotos. The gotos make signal routing much cleaner. They are also used to save data to a .mat file. This data can be used for plotting and comparing experiments.

6.2 QFLEX 2 USB

As mentioned before, the QFLEX 2 USB panel provides an interface between the Quanser AERO and a computer running QUARC which will be discussed next. The QUARC software is the software that uses the C-code generated by Simulink and makes it compatible to run on the QFLEX 2 USB panel. We have a control method set up, so the only item left is to send the actuator commands to the Quanser AERO with the use of the QFLEX 2

USB panel. Quanser provides the necessary Simulink blocks for this communication which will be discussed in the following sections.

NOTE: A tutorial is provided in Appendix C.1 on how to run a complete Simulink model for the Quanser AERO. The Simulink model must contain a control and communication method to send the actuator commands to the Quanser AERO.

6.2.1 Required Software

In order to utilize the QFLEX 2 USB panel, some extra software is needed. To obtain and use the Simulink blocks provided by Quanser, a licensed version of QUARC must be obtained. QUARC is the proprietary software of Quanser that installs the Simulink QUARC package that provides communication blocks for the QFLEX 2 USB panel. QUARC also provides additional Simulink blocks such as filters and interfaces in Simulink.

Additional MATLAB and Simulink packages must also be obtained in order for QUARC to operate correctly. The QFLEX 2 USB panel cannot understand a Simulink model; some form of C-code is needed for the Quanser AERO to operate correctly. Because we are using Simulink, we need to convert the graphical model to C-code. To do this, we must have the MATLAB Coder (`https://www.mathworks.com/products/matlab-coder.html`) and Simulink Coder (`https://www.mathworks.com/products/simulink-coder.html`) packages installed. These packages can be obtained by going to one's MATLAB. In the Home tab, click on Add-Ons. You can either use Manage Add-Ons or Get Add-Ons to find the MATLAB Coder and Simulink Coder. It should be noted that these add-ons are not free, so a MathWorks account is needed.

6.2.2 Quanser AERO Simulink Model

With the QUARC software, communication between Simulink and the Quanser AERO is made much easier. As discussed earlier, the QUARC software installs Simulink libraries and blocks supported by the Quanser AERO. For our case, only three additional blocks were needed to communicate ADP to the Quanser AERO. The three blocks can be seen in Figure 6.6. Figure 6.6 is the Simulink subsystem that depicts the physical system in Figure 6.3.

All three new blocks correlate to each other. The HIL Initialize block defines the platform we are implementing once the proper settings are entered. With the proper settings entered in the HIL Initialize block, the QUARC software knows what platform to send the Simulink signals to and from.

The HIL Write block does exactly what its name implies. The HIL Write block writes our provided signals to the Quanser AERO. For our purposes, we are only concerned with motor voltages, enabling motors, and the base color. These inputs to the block can be

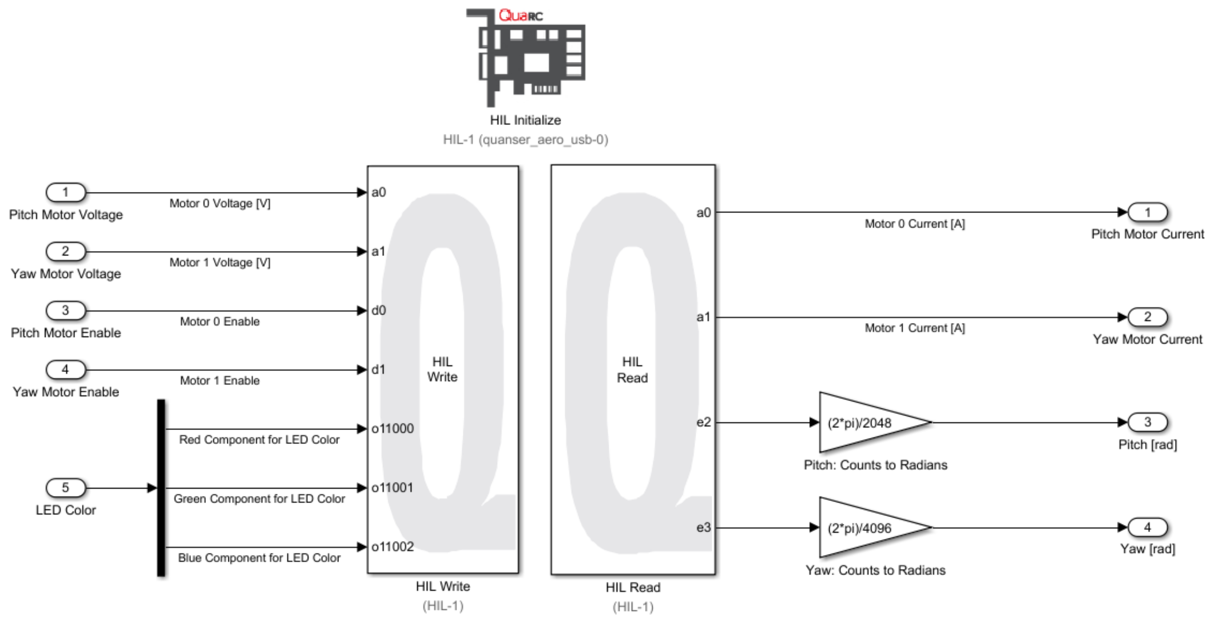


Figure 6.6: Simulink subsystem used to communicate with the QFLEX 2 USB panel.

changed by updating the settings inside the block. It is recommended to also refer to the help documentation associated with this block when selecting inputs as the names are abbreviated.

The HIL Read block also does exactly what its name implies. The HIL Read block reads our specified signals from the Quanser AERO. For our purposes, we are only concerned with the pitch, yaw, and motor currents. These outputs to the block can be changed by updating the settings inside the block. It is again recommended to also refer to the help documentation associated with this block when selecting inputs as the names are abbreviated.

With these three Simulink blocks implemented in your model and with the QUARC software installed, communication between Simulink and the Quanser AERO is much simpler. A tutorial on how to run your Simulink model on the Quanser AERO is provided in Appendix C.1.

6.2.3 Results

In order to see the effectiveness of ADP, we needed to see physical measurements from the implementation results. Because we ran ADP through Simulink, we could record various measurements from the Quanser AERO in a .mat file. in Section 4. The plots we generated can be seen in Figures 6.7, 6.8, 6.9, 6.10, 6.11, 6.12, 6.13, 6.14, 6.15, 6.16, 6.17, 6.18, 6.19, and 6.20. For each of the experiments, we ran the experiment with a saturation level of either 18 or 24 Volts. The motors are rated at 18 Volts, but Quanser Simulink models run them at 24 Volts, so we tried both voltages. In order to have a baseline comparison, we compared ADP to that of LQR. Quanser provides a Simulink model for LQR which we modified slightly to save the measurements. We then ran each test using both ADP and LQR.

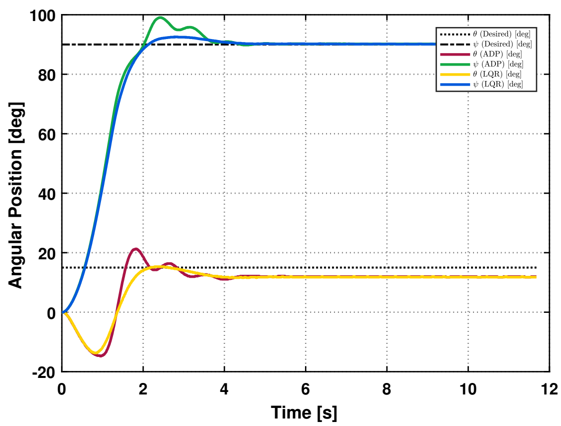
It should be noted that ADP needs the **B** matrix of the state-space model to update the state-feedback gains accurately. To make sure we had an accurate **B** matrix, we measured the thrust gains. We measured the thrust gains by running the experiments provided in Quanser documentation. The documentation went through measuring and calculating the thrust gains. We then used the constants provided by Quanser to find the **B** matrix. We did not use any experiments to find the **A** matrix for our system; we used the constants provided. ADP should be able to correct if the **A** matrix is slightly off, but we could see drastic changes in the performance of ADP using the calculated **B** matrix and the one provided in Quanser documentation.

From Figures 6.7, 6.8, 6.9, 6.10, 6.11, 6.12, 6.13, 6.14, 6.15, 6.16, 6.17, 6.18, 6.19, and 6.20 we can see that both ADP and LQR perform well for this particular system. The results are similar to the MATLAB simulations, but that is not enough evidence. To see if there are any bigger differences, we calculated the root-mean-squared-error of the measurements and also the root-mean-squared values of the power of the motors. These values can be seen in Tables 6.1, 6.2, and 6.3. It should also be noted that these experiments were conducted using the inefficient propellers provided with the Quanser AERO.

Table 6.1: Experimental Simulink angular RMSE measurements.

Figure	θ ADP [deg]	θ LQR [deg]	ψ ADP [deg]	ψ LQR [deg]
6.7	8.794	23.135	8.565	23.308
6.8	8.463	21.205	7.777	21.219
6.9	23.088	21.612	21.144	21.289
6.10	21.478	19.727	18.944	17.942
6.11	9.414	14.833	9.595	19.207
6.12	4.661	8.424	5.925	13.371
6.13	3.787	9.728	3.375	14.155
6.14	3.143	8.885	2.964	13.622
6.15	6.162	14.185	7.116	14.171
6.16	5.973	12.910	6.533	12.812
6.17	14.496	14.939	14.220	15.049
6.18	13.816	14.074	13.368	13.594
6.19	32.126	72.588	30.188	73.744
6.20	30.408	65.609	27.501	65.348

From Table 6.1, we can see that ADP performs better at minimizing the error with respect to the desired pitch and yaw compared to LQR. From Table 6.2, we can see that ADP



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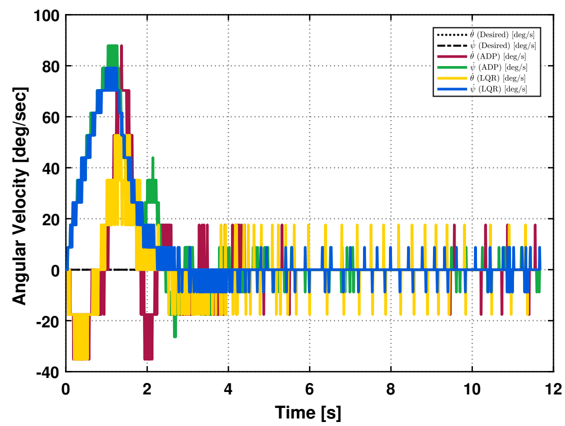

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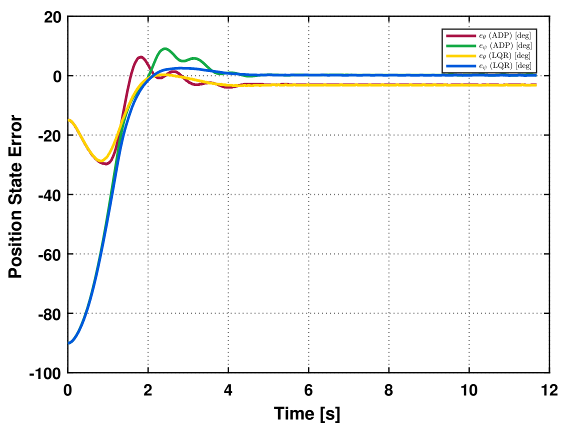

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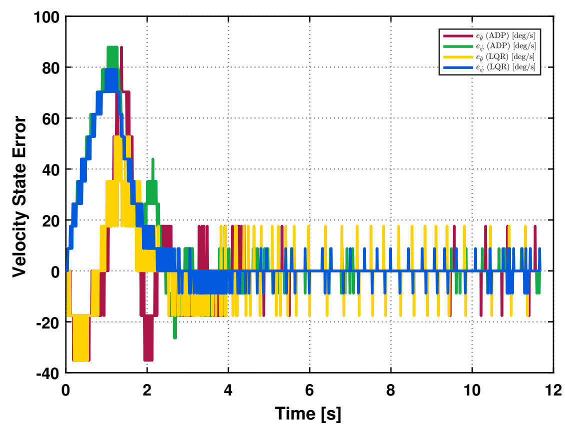

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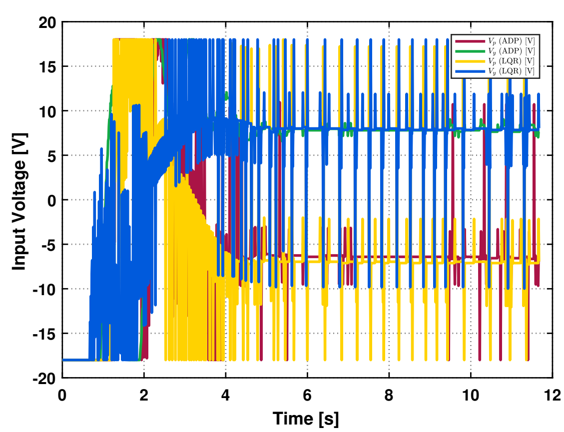
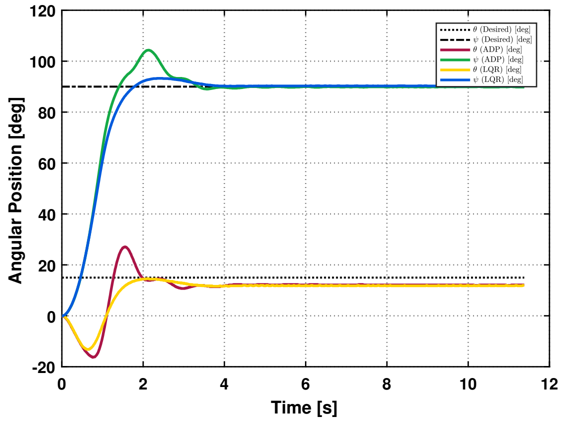


(e)

Figure 6.7: Constant desired pitch and yaw with inputs saturated to 18 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

does a somewhat decent job in minimizing the error with respect to the desired angular velocities. ADP only performs slightly better, but this measurement can be difficult to



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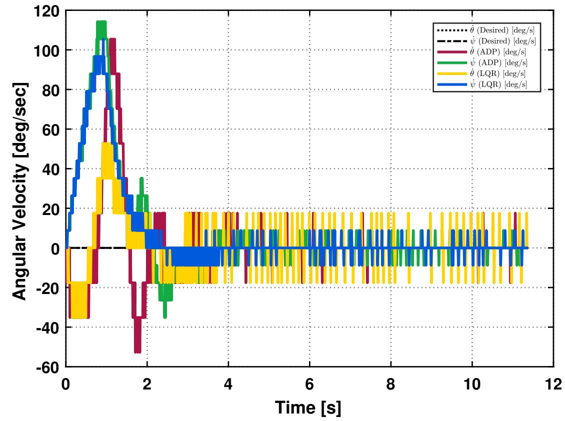

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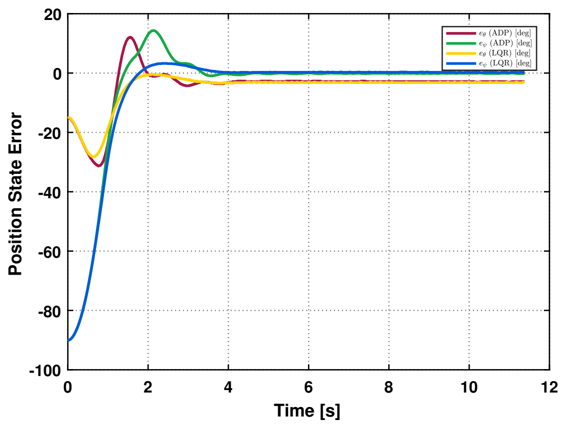

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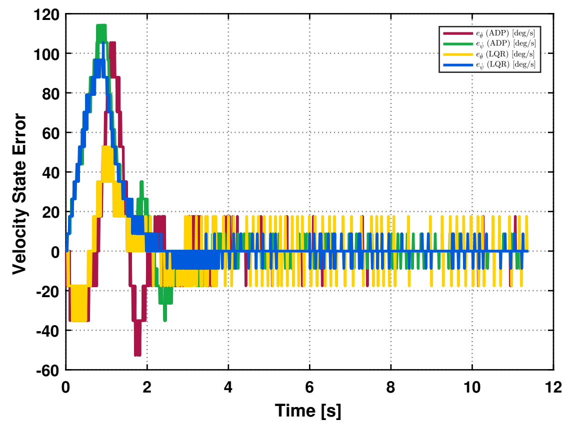

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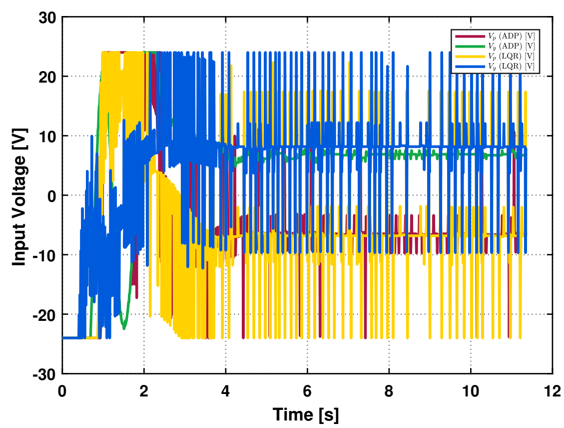


(e)

Figure 6.8: Constant desired pitch and yaw with inputs saturated to 24 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

interpret. We are changing the desired trajectory, so the desired velocities cannot always be zero as we intended. As for the power of the motors, Table 6.3 shows that there is no

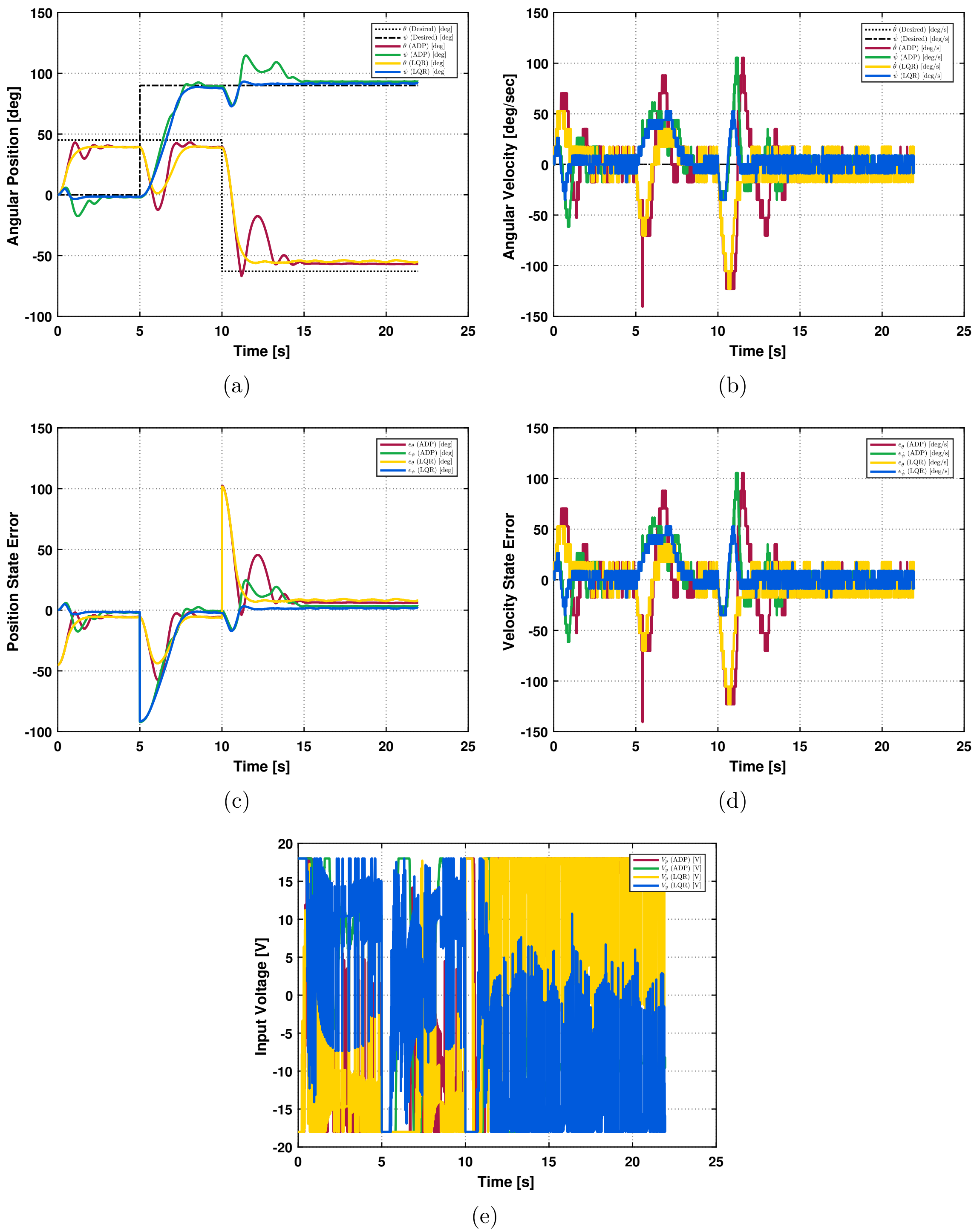


Figure 6.9: Step desired pitch and yaw with inputs saturated to 18 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

clear advantage to either ADP or LQR. The results seem to be evenly split in favor of either. This is not great news because we were hoping that ADP would yield control that

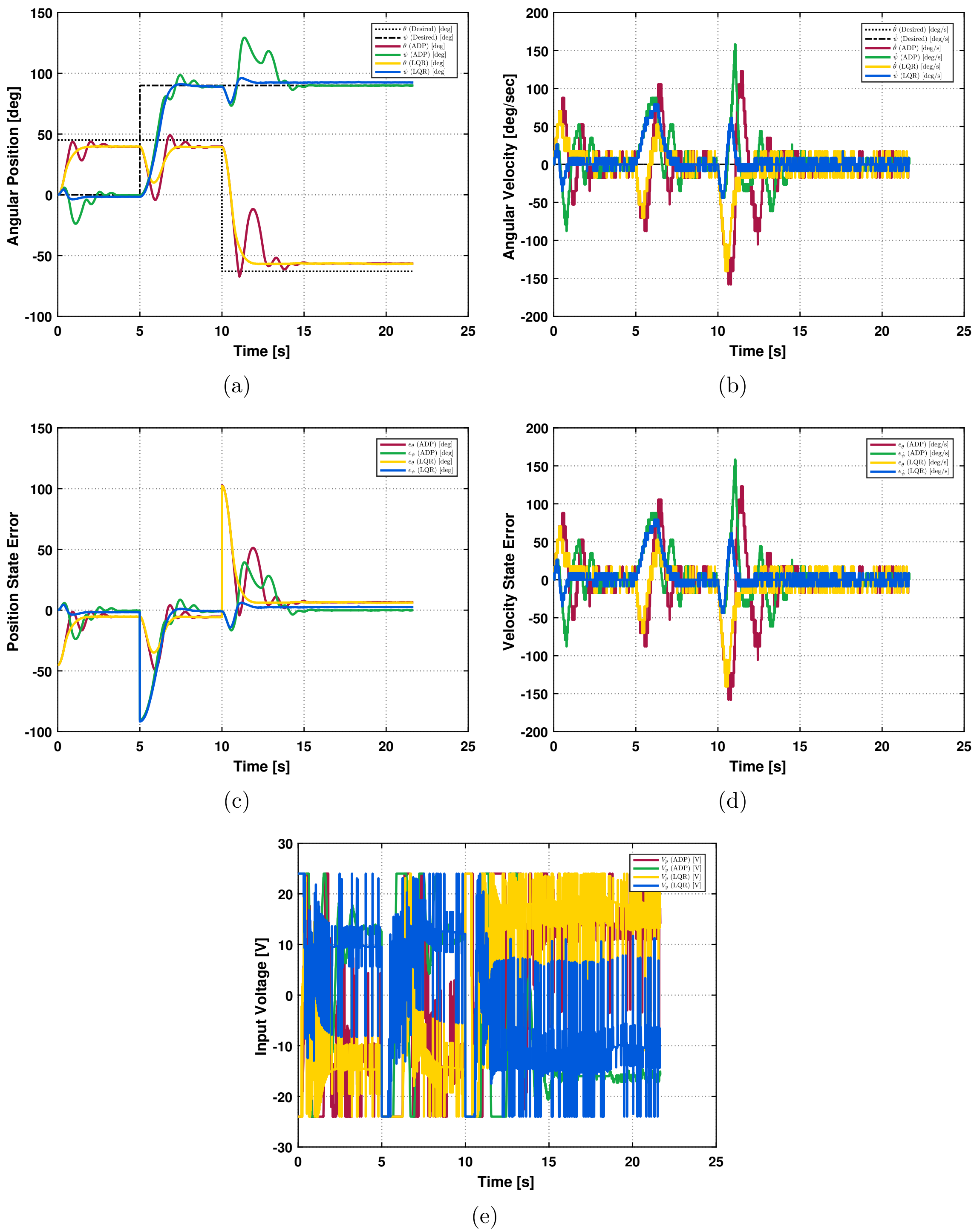
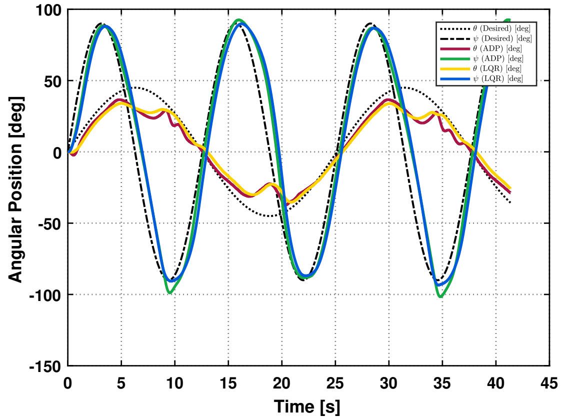
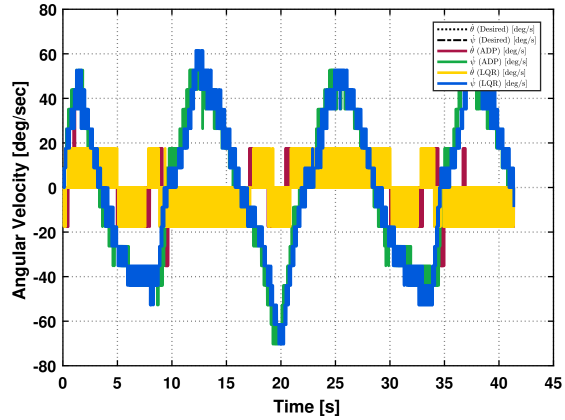


Figure 6.10: Step desired pitch and yaw with inputs saturated to 24 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

would minimize the energy consumed. We just cannot say for sure that this occurs with the experiments we have ran here.



(a)



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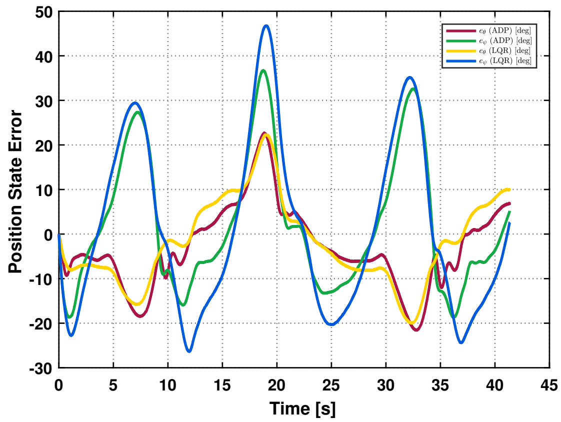
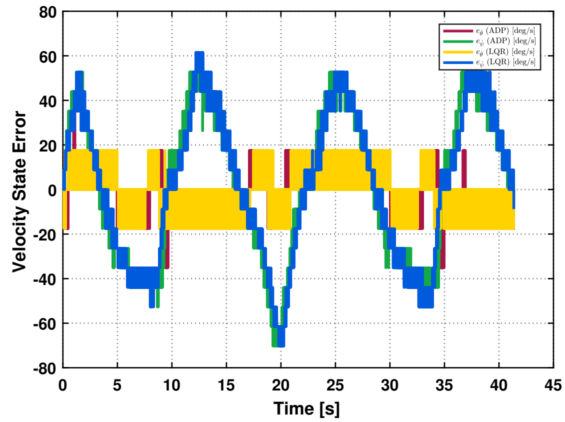


(c)



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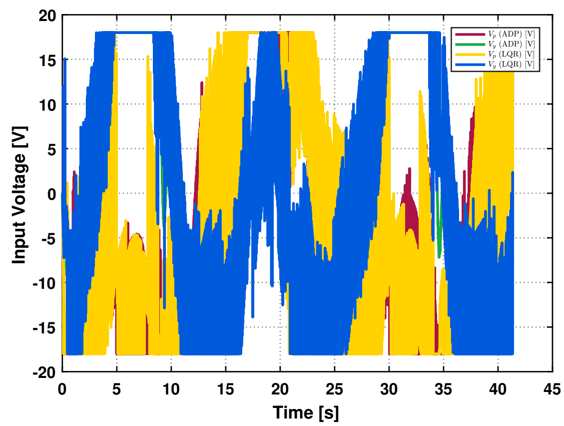


(e)

Figure 6.11: Sinusoidal desired pitch and yaw with inputs saturated to 18 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

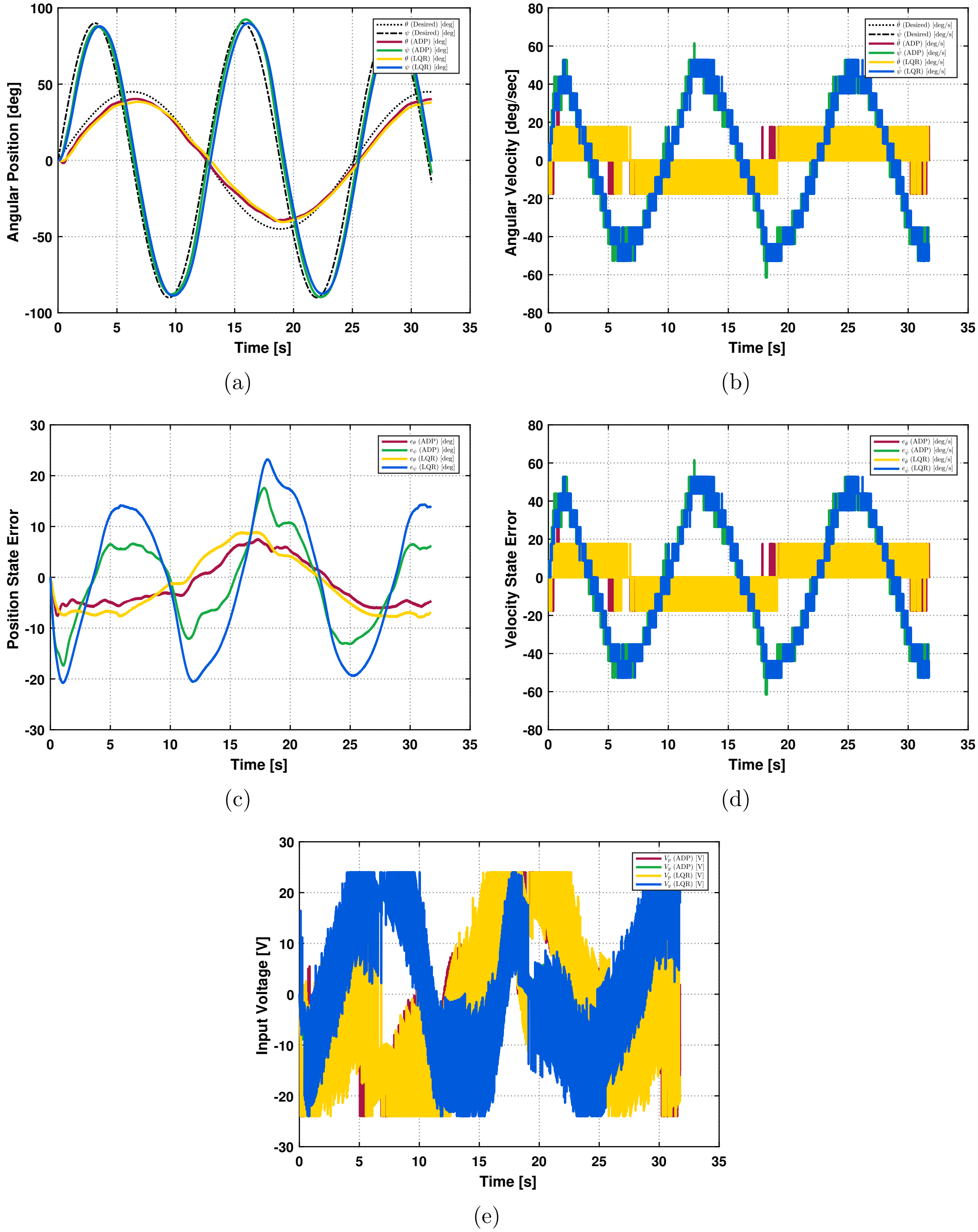


Figure 6.12: Sinusoidal desired pitch and yaw with inputs saturated to 24 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

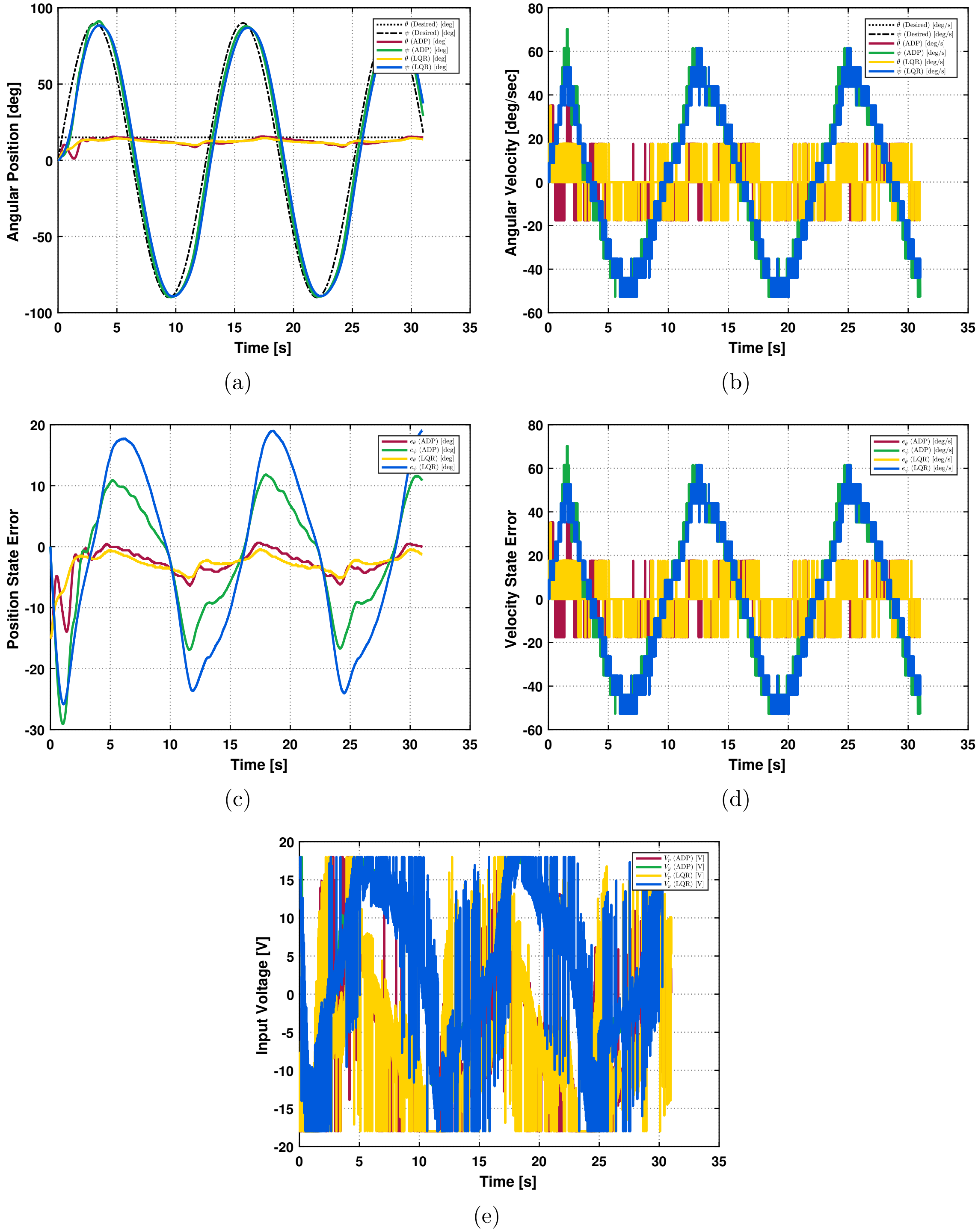


Figure 6.13: Constant desired pitch and sinusoidal desired yaw with inputs saturated to 18 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

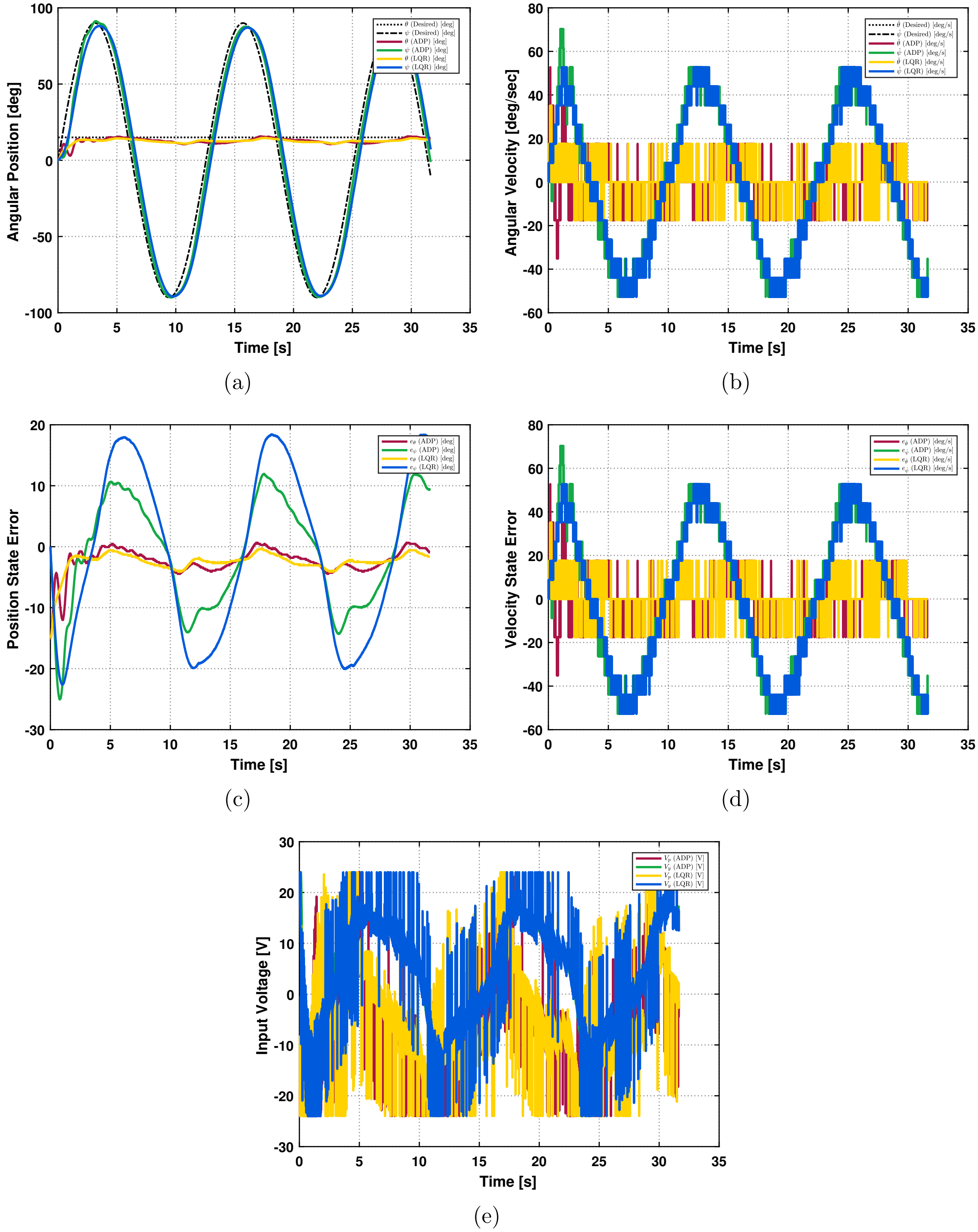
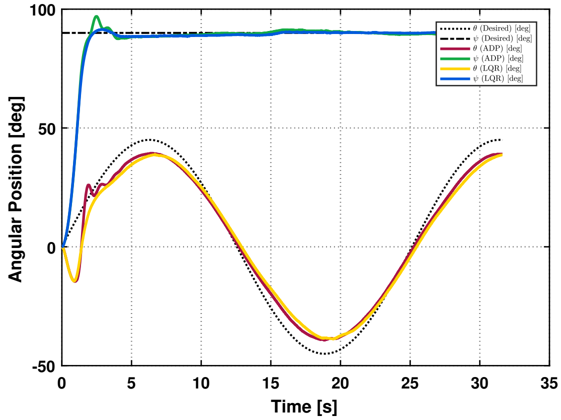
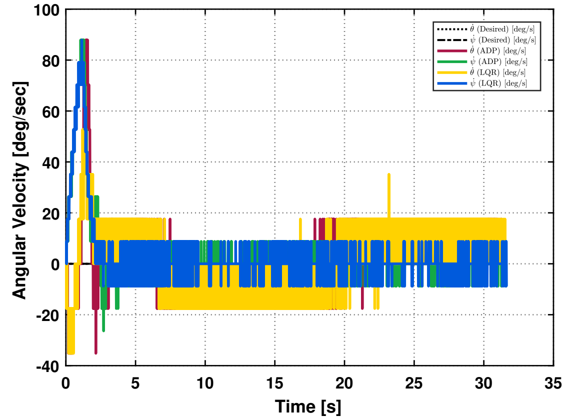


Figure 6.14: Constant desired pitch and sinusoidal desired yaw with inputs saturated to 24 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.



(a)



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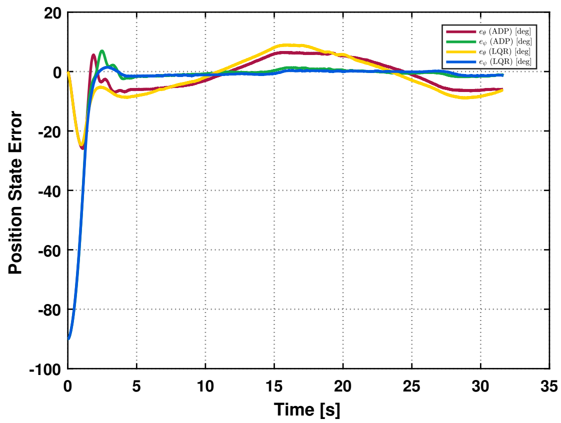

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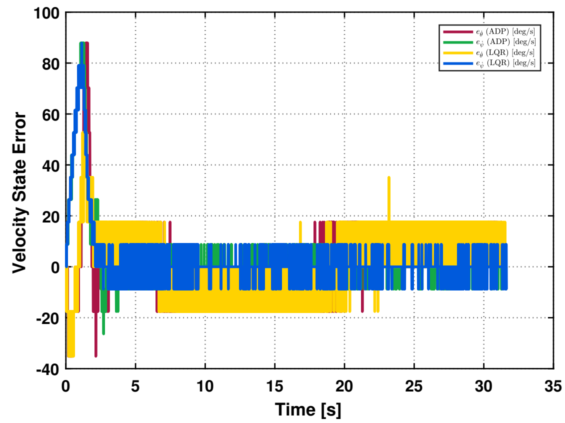

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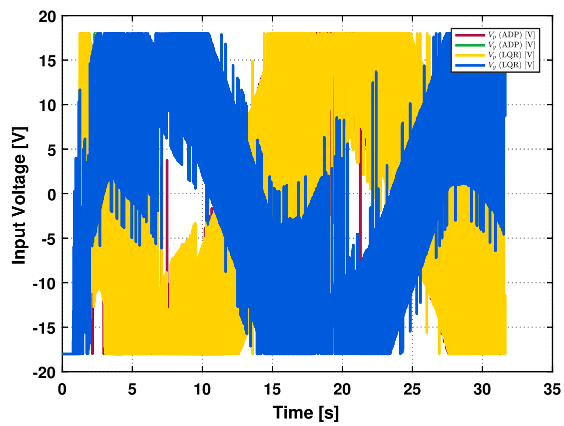
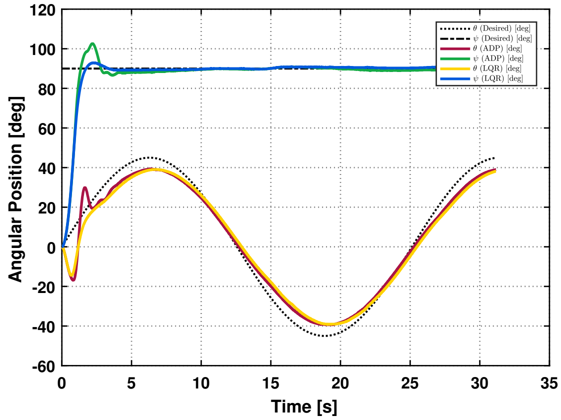
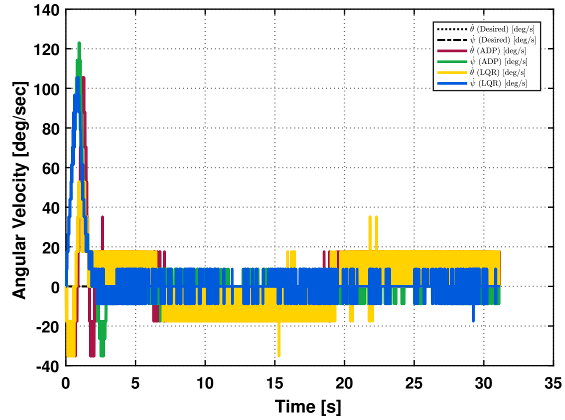


(e)

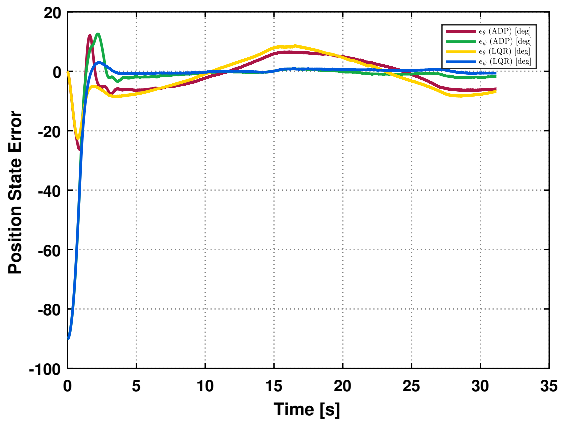
Figure 6.15: Sinusoidal desired pitch and constant desired yaw with inputs saturated to 18 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.



(a)



(b)



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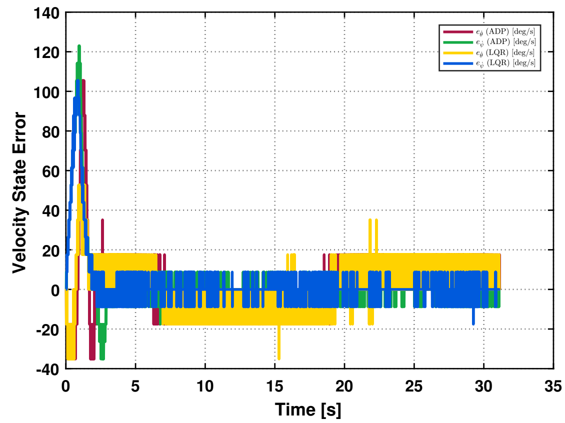

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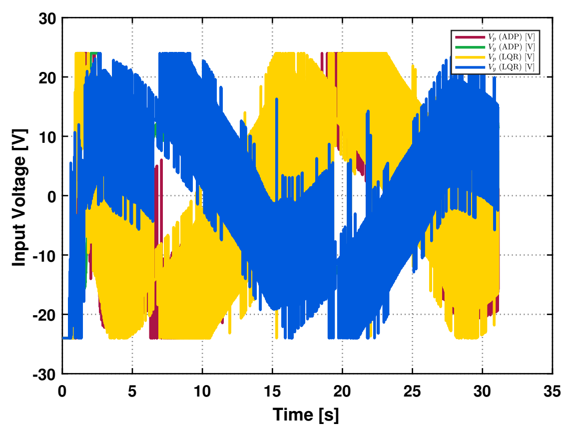
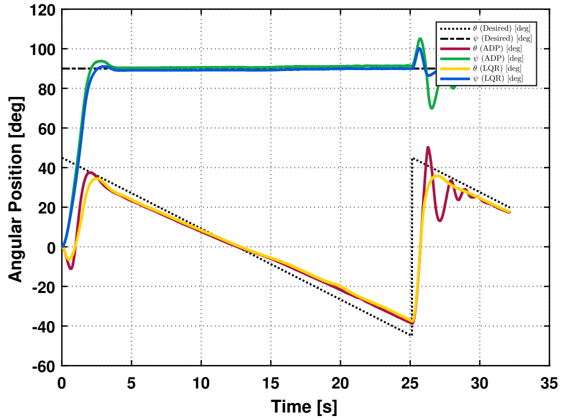
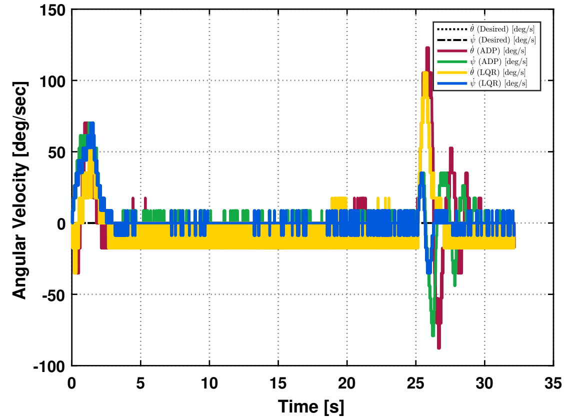


(e)

Figure 6.16: Sinusoidal desired pitch and constant desired yaw with inputs saturated to 24 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.



(a)



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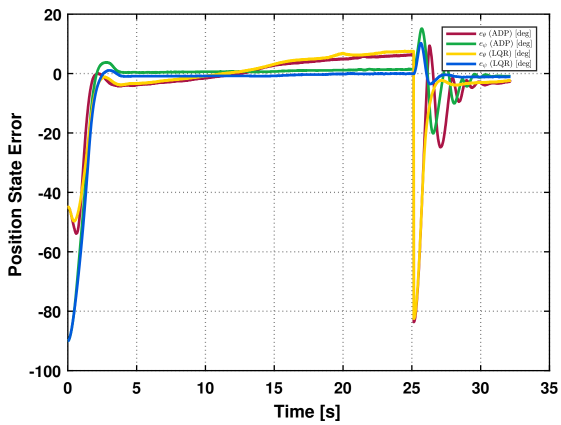

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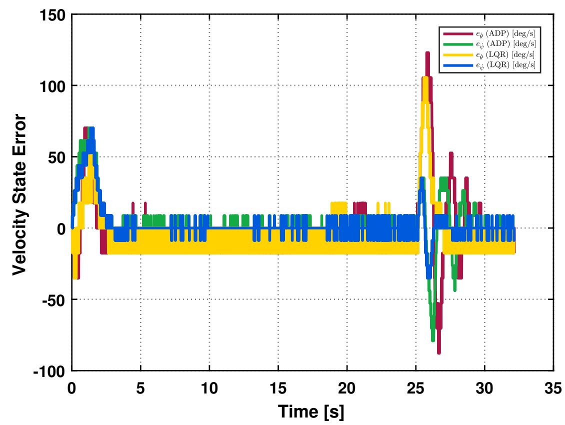

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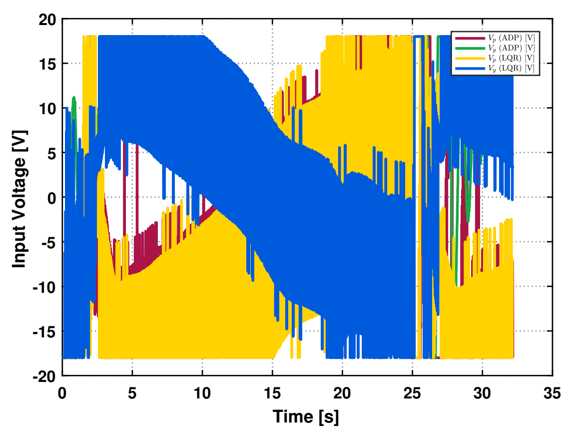
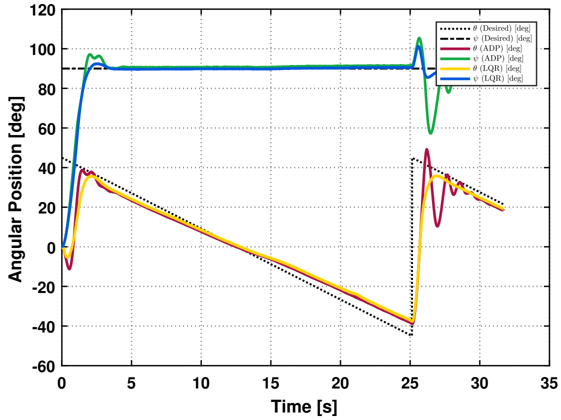
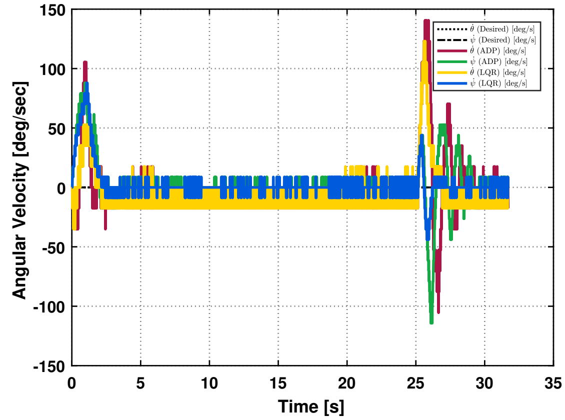


(e)

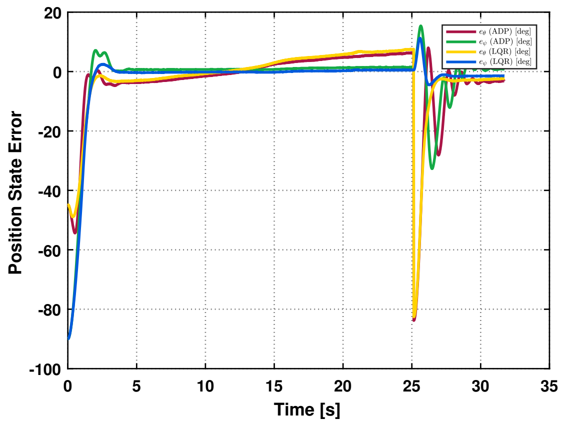
Figure 6.17: Sawtooth desired pitch and constant desired yaw with inputs saturated to 18 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.



(a)



(b)



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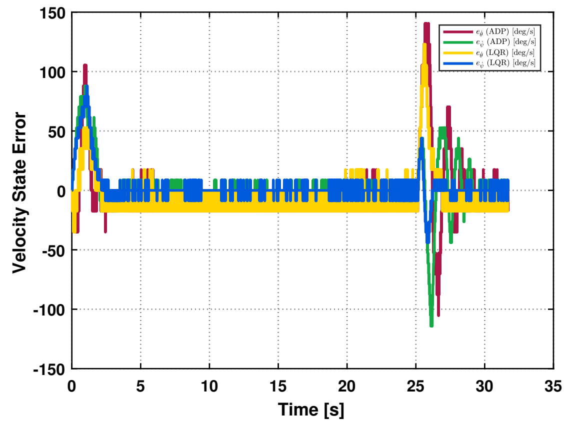

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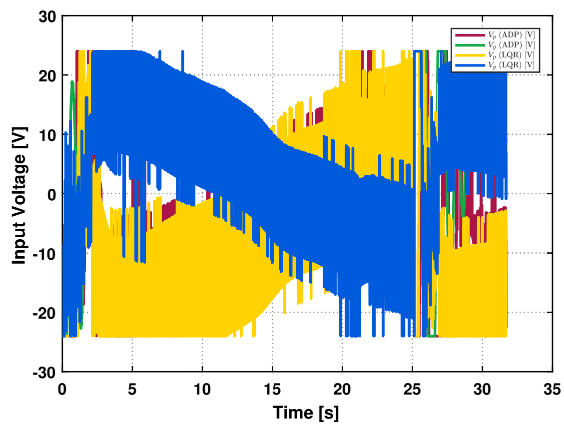


(e)

Figure 6.18: Sawtooth desired pitch and constant desired yaw with inputs saturated to 24 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

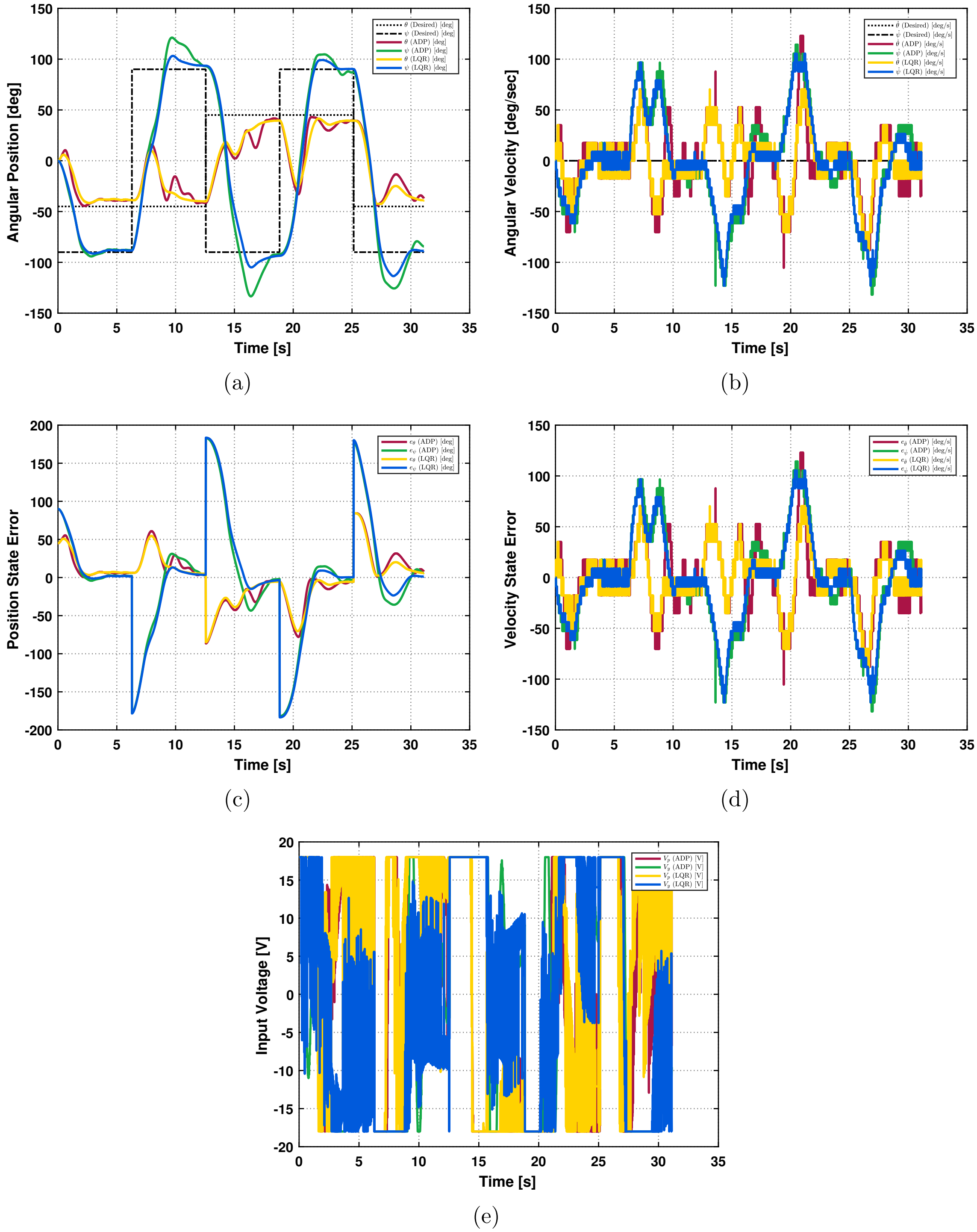
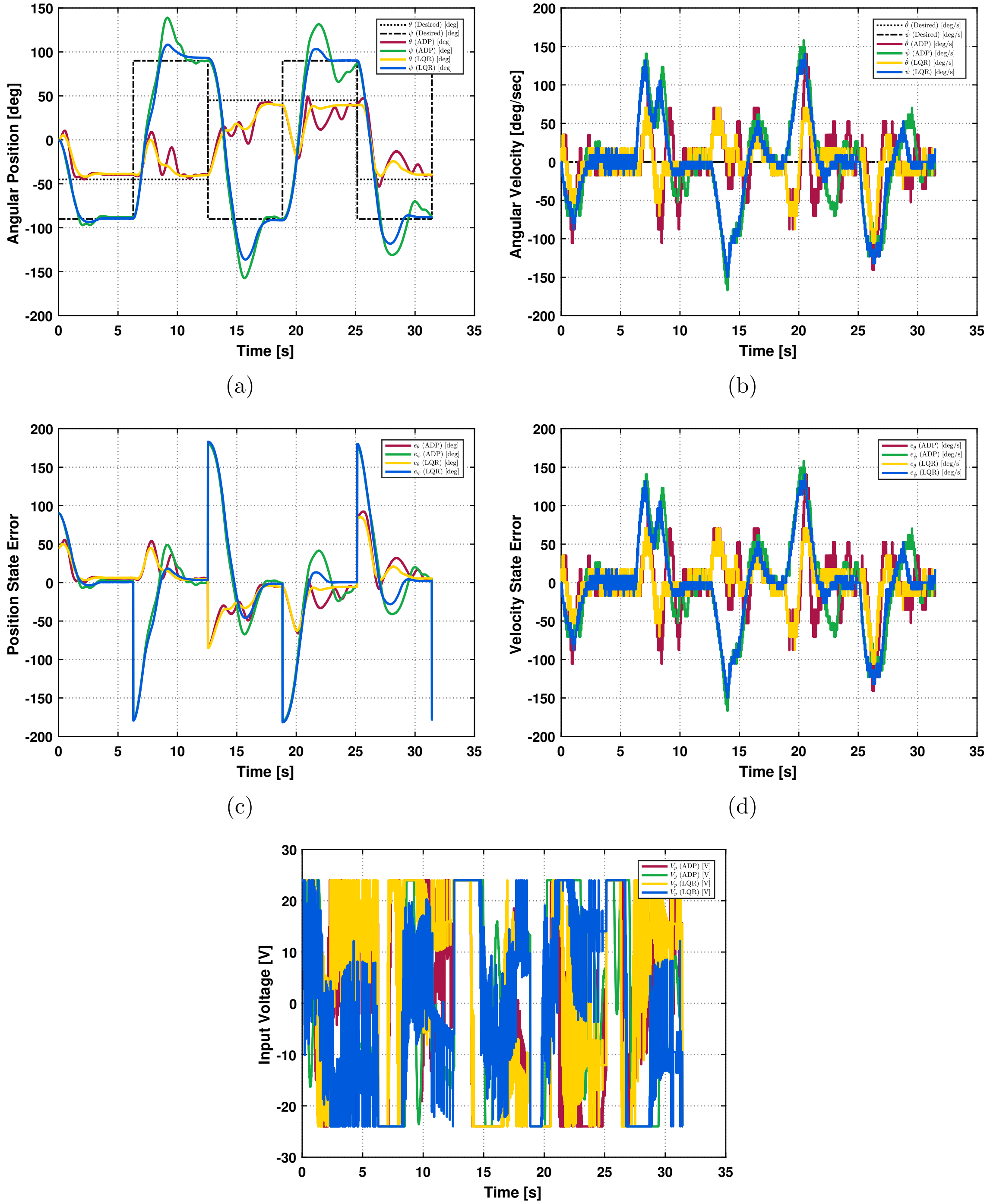


Figure 6.19: Square desired pitch and yaw with inputs saturated to 18 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.



(a) (b) (c) (d) (e)

Figure 6.20: Square desired pitch and yaw with inputs saturated to 24 Volts. (a) Angular position. (b) Angular velocity. (c) Angular position error. (d) Angular velocity error. (e) Voltage inputs.

Table 6.2: Experimental Simulink angular velocity RMSE measurements.

Figure	$\dot{\theta}$ ADP [deg /s]	$\dot{\theta}$ LQR [deg /s]	$\dot{\psi}$ ADP [deg /s]	$\dot{\psi}$ LQR [deg /s]
6.7	15.338	22.159	11.205	20.706
6.8	19.229	26.309	11.923	23.233
6.9	34.616	20.821	24.156	15.431
6.10	38.509	28.835	23.992	17.961
6.11	11.556	33.329	10.588	32.736
6.12	10.892	32.345	10.616	31.863
6.13	7.162	32.494	5.905	31.616
6.14	6.929	32.827	5.726	31.761
6.15	13.835	13.742	11.990	13.107
6.16	15.412	16.558	12.239	14.713
6.17	22.027	16.116	15.976	12.396
6.18	24.220	19.828	16.577	14.104
6.19	32.972	48.646	26.456	44.624
6.20	39.173	58.155	26.308	52.175

Table 6.3: Experimental Simulink power RMS measurements.

Figure	Pitch ADP [W]	Pitch LQR [W]	Yaw ADP [W]	Yaw LQR [W]
6.7	8.076	7.102	7.418	6.332
6.8	13.532	11.845	12.628	9.272
6.9	12.140	13.725	11.994	11.271
6.10	19.356	16.368	21.413	12.878
6.11	8.250	10.956	6.247	8.544
6.12	10.275	14.123	7.019	11.655
6.13	5.874	7.358	5.819	6.309
6.14	7.272	9.266	7.241	6.737
6.15	9.305	12.979	5.532	9.620
6.16	11.851	15.868	8.287	10.886
6.17	8.811	9.215	7.004	7.809
6.18	13.406	12.252	12.312	9.589
6.19	11.874	12.993	11.634	10.315
6.20	19.969	19.9	20.693	16.344

6.3 QFLEX 2 Embedded/Raspberry Pi

Quanser provides the QFLEX 2 USB panel with the Quanser AERO making the Quanser AERO simple and easy to use. The problem with the QFLEX 2 USB panel is that a Simulink model must be ran with the QUARC software in order to actually control the Quanser AERO. This can become an issue when multiple licenses for QUARC are not an option or an embedded device is desired as the controller. To overcome these setbacks, Quanser has developed the QFLEX 2 Embedded panel. This panel's datasheet can be found in Appendix F.3.

The QFLEX 2 Embedded panel is different from the QFLEX 2 USB panel in the fact that it uses SPI communication from an embedded device to the Quanser AERO. The SPI protocol will be discussed in more detail later in this section. The universality of the SPI protocol allows most embedded systems to be used for control of the Quanser AERO. In our case, the embedded system is the Raspberry Pi 3. The challenge with the Raspberry Pi is that we have to convert the Simulink model used with the QFLEX 2 USB panel to one compatible with a Raspberry Pi and the QFLEX 2 Embedded panel. These challenges were overcame, but much research and innovation was needed on our part which we will discuss in the following sections.

6.3.1 Raspberry Pi 3

The Raspberry Pi 3 was given to us as a design constraint for the hardware implementation. Dr. Miah chose the Raspberry Pi 3 because it is readily available, and it has the functionality for Wi-Fi. Better embedded systems could have been considered, but we implemented ADP within the design constraints of the Raspberry Pi 3. More information on the Raspberry Pi 3 can be found at https://www.raspberrypi.org/products/raspberry-pi-3-model-b/.

6.3.2 Required Software

Associated with the Raspberry Pi is the difficulty in generating code for ADP. We basically had two options on how to implement ADP on the Raspberry Pi using the QFLEX 2 Embedded panel. The first option was to write code from scratch that was compatible with the Raspberry Pi and the SPI protocol. This could either be C, C++, Python, etc. The difficulty with this approach would be converting all of the MATLAB code into another language less "math-friendly." This could be considered a project all in its own.

Luckily for us, we found a solution that would make the task of generating the code much easier. We considered using Simulink to generate the C-code which we could then use on the Raspberry Pi. We would then have to manipulate the C-code such that the SPI protocol could be observed. This would, again, be an extreme task.

After some research, we found the solution. We found two support packages that help with code generation for the Raspberry Pi family. The Raspberry Pi Support Package for MATLAB (https://www.mathworks.com/hardware-support/raspberry-pi-matlab.html) and the Raspberry Pi Support Package for Simulink (https://www.mathworks.com/hardware-support/raspberry-pi-simulink.html) make communication between MATLAB/Simulink and a Raspberry Pi possible. The Raspberry Support Package for MATLAB is more intended for running code using MATLAB on the Raspberry Pi. On the other hand, the Raspberry Support Package for Simulink is intended to generate code to be ran solely on the Raspberry Pi.

These packages can be obtained by going to one's MATLAB. In the Home tab, click on Add-Ons. You can either use Manage Add-Ons or Get Add-Ons and search for Raspberry Pi. These support packages are free, but they do require some setup. The setup includes configuring an SD card for the Raspberry Pi to be used. The SD card is configured with a Linux version specified by MATLAB. To complete the setup just follow the steps provided in the prompts; It is rather simple. Once the setup is complete, you can create a Simulink model and run the generated code on a Raspberry Pi. A tutorial of this can be found in Appendix D.1.

It should also be noted that we were using MATLAB 2016b for the extent of this project. Newer versions of MATLAB correspond to newer versions of the support packages which may have additional features.

6.3.3 SPI Communication

In order to utilize the QFLEX 2 Embedded panel, transmission between an embedded system and the Quanser AERO must be achieved via SPI communication. SPI stands for serial peripheral interface. In a nutshell, SPI is the process of sending data between two devices one bit at a time. These bits can be combined to form data needed by each device. An overview of the SPI communication process can be seen in Figure 6.21.

There are four main signals to consider in SPI communication: SS (slave-select), clock, MOSI (master-in slave-out), and MISO (master-in slave-out). One of the devices is the considered the master, and the other device is considered the slave. The master generates a clock signal. This clock signal is used to synchronize communication between the master and slave devices. Essentially, one bit is passed for each period of the clock cycle. Different variations of SPI specify when a bit is starting to send and when a bit is supposed to be stable, but these specifications will not be discussed in detail in this report. The master also controls the value of the slave-select signal. This signal is active-low to specify to the slave that transmission is required. The last two signals are MOSI and MISO. MOSI is the signal generated by the master to be sent to the slave. For our case, we are only concerned with the sending the data for the motor voltages and the base colors. MISO, on the other hand, is the signal generated by the slave to be sent to the master. For our case, we are only concerned with receiving the data from the Quanser AERO corresponding to the pitch and yaw measurements.

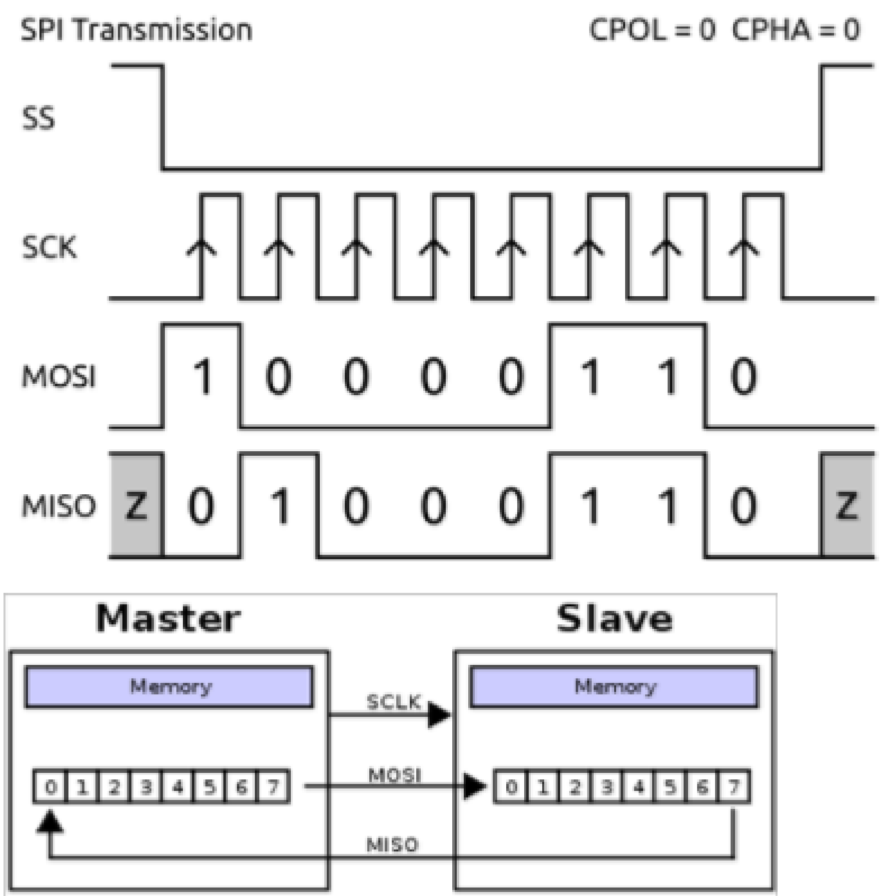


Figure 6.21: Overview of the SPI protocol. Image provided by https://www.mathworks.com/help/supportpkg/raspberrypi/ug/support-spi-communication.html.

There are actually multiple difficulties associated with SPI communication that Quanser failed to realize. To begin, SPI communication is intended for short information; that is low numbers of bits to be sent and received. We want to keep the information short, so we can update the information much faster. This is not the case with the Quanser AERO. Appendix F.3 specifies the data sent and received between the Quanser AERO and an embedded device. We can see that for each communication we are sending 51 bytes, or 408 bits. That is a lot of information to send via SPI communication. In our opinion, this is a major design flaw of the Quanser AERO. Specifically, we are only concerned with a few bytes, but we need to send everything for the system function properly.

The only way to make our system fast enough to send all of the bytes in a reasonable time is to increase the SPI clock frequency generated by the master. This is where you can run into hardware limitations. In the next section, we will discuss that Simulink can be used to generate signals on pins of the Raspberry Pi, but this can cause some difficulties. When we generated a clock signal on the Raspberry Pi 3, we were not getting any communication between the Quanser AERO and the Raspberry Pi. We set the clock period at 2 microseconds. With this period, we can send the 51 bytes in between each data collection in ADP. This seemed reasonable, but there was no communication.

After much thought, we decided to connect an oscilloscope to the Raspberry Pi pin that was supposed to generate the clock signal. There was no clock signal. Why is that? We decided to try larger clock periods. When we reach a clock period of 100 microseconds, we actually begin to see a clock signal that is accurate. It appears that the Raspberry Pi is incapable of switching its GPIO pins at very fast rates. This seemed very odd given that the built in SPI clock of the Raspberry Pi can run in the mHz range. We, however, could not access this built in SPI clock, so we had to generate our own clock signal which was not very fast. So, we have a 10 kHz clock signal. That means we will update the motor voltages every 0.051 seconds. This is not the value of τ we anticipated in the ADP. With this type of τ, the gain of the state-feedback system is updated roughly every 1 second which is at all not ideal. This is, however, what we are dealt because of hardware limitations. This could either make or break the implementation of ADP.

There is also one more issue we found with Quanser's implementation of SPI communication. The standard is to send the least-significant bit first in the communication. By trial-and-error and some research, we determined that Quanser sends the most-significant bit first. This design is not at all conventional. This can make troubleshooting very difficult if this is not known.

6.3.4 SPI Communication in Simulink

We already knew the capabilities of Simulink in generating C-code for the QFLEX 2 USB panel. We also found a Simulink support package for the Raspberry Pi as discussed earlier. Our thought was can we combine these two aspects in Simulink to generate C-code to be used on the Raspberry Pi? The answer is yes we can. To do this, we need to take our ADP Simulink model and convert it to SPI communication that can be used on the Quanser AERO. The model used for this whole process can be seen in Figure 6.22.

There are two basic subsystem blocks in Figure 6.22. The subsystem block on the left is ADP. This subsystem block has the exact same structure used in the previous models. The structure can be seen in Figure 6.23. We kept the same structure, but we made ADP its own entity. Remember, we needed an initialization function to determine the gain for the first T seconds. We still use this approach, but we also add commands to set the timing of the SPI communication which will be discussed next. We also secure a connection to the Raspberry Pi through MATLAB and enable the GPIO pins of the Raspberry Pi. This initialization code can be seen in Appendix D.2.

The other subsystem block seen in Figure 6.22 is the SPI communication block which is zoomed in on in Figure 6.24. In Figure 6.24, we can see that the SPI communication block is only triggered on the rising-edge of a clock signal generated by a GPIO pin. We already discussed the hardware limitations of this pin earlier. The inputs to the subsystem are the calculated motor voltages and the color values for the base which were found in ADP. The outputs of the subsystem are the current pitch and yaw values.

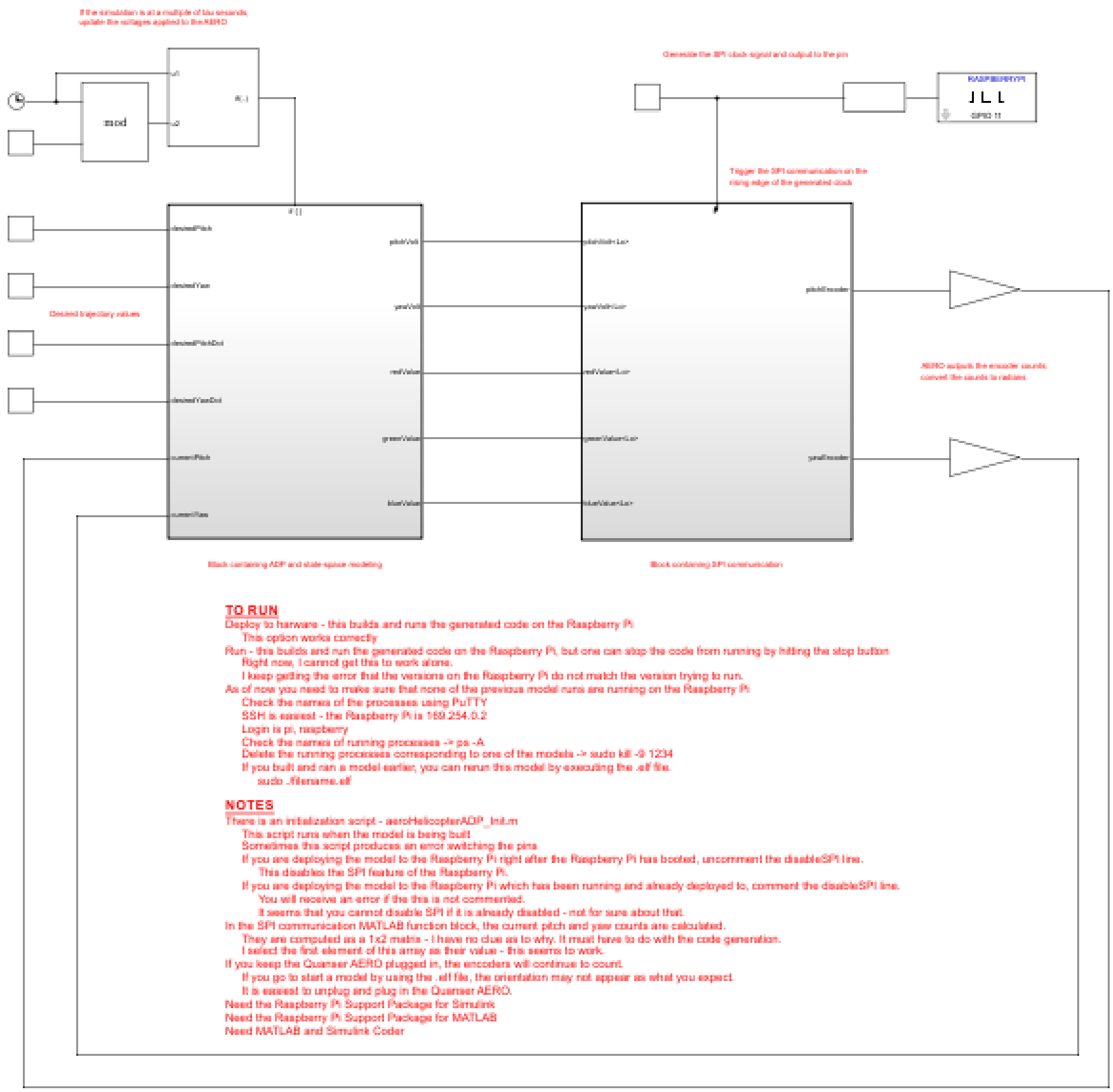


Figure 6.22: Simulink model used to generate the code for the Raspberry Pi 3.

Inside the SPI communication subsystem block, we can see what is really happening in Figure 6.25. Figure 6.25 shows how the information to be sent and received in the SPI communication is handled. Determining what bit to send is handled by a MATLAB function which can be found in Appendix D.3. The MATLAB function follows the transmission information provided in the QFLEX 2 Embedded datasheet found in Appendix F.3. The difficulty with this is we need to keep track of what bit and byte we are currently sending and receiving. This is why we have Data Store blocks in Figure 6.25. These Data Store blocks act as global variables for the MATLAB function. The MATLAB function in Appendix D.3 takes the information about the bit and byte we are currently sending along with the voltage and color information and sets the GPIO pins of the Raspberry Pi. Please refer to Appendix D.3 for the MATLAB code and to Appendix F.3 for how we thought

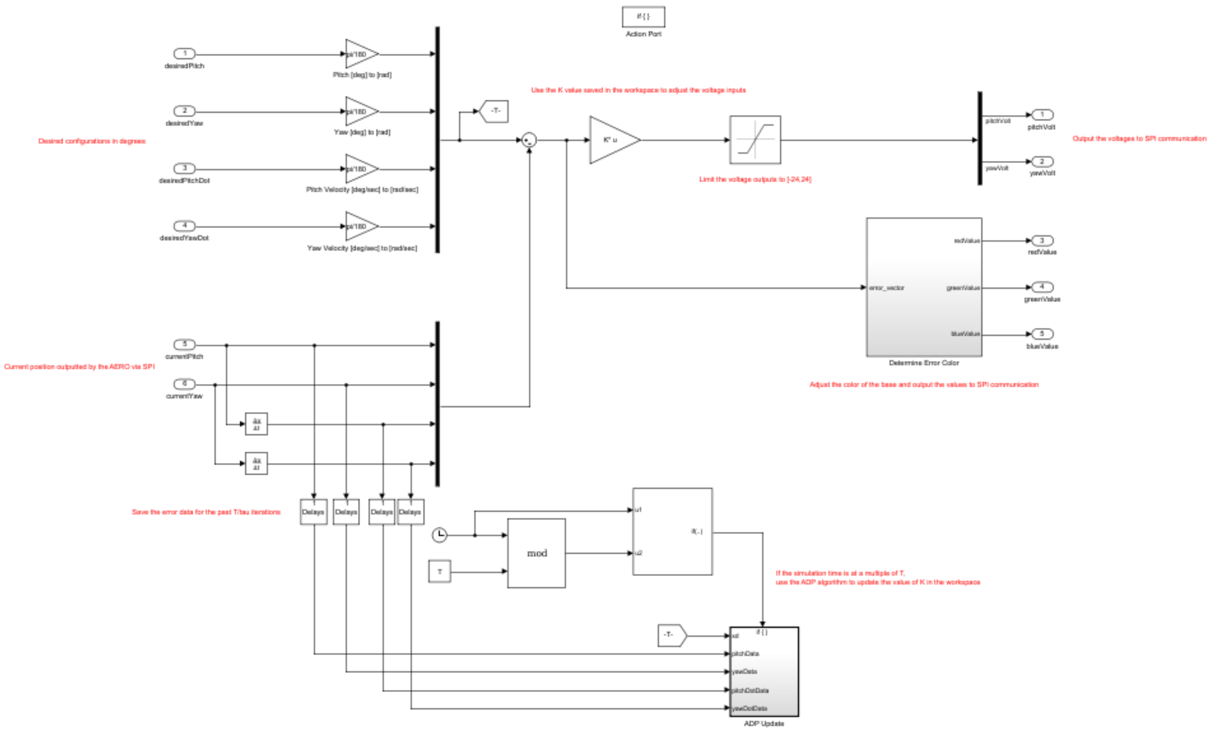


Figure 6.23: Simulink subsystem used for the ADP algorithm.

out the MATLAB code.

6.3.5 Results

As for the results with the Raspberry Pi, it is quite difficult to get a numerical analysis. We uses Simulink to generate the C-code onto the Raspberry Pi. With that method, there is no way to record data that we know of. This will require further research. The only result we can get is physically looking at the motions to determine if the Quanser AERO is in the correct position.

When we generated the C-code using Simulink, we kept getting one persistent error. We had a size mismatch because the code generation made the current pitch and yaw values 1×2 matrices. We have no idea as to why these values are matrices; they should only be single values. There are two options as to why this happened. The first option is the C-code generation is just wrong. The second option is the way we wrote the MATLAB function. We could have wrote the MATLAB function in such a way that the C-code generator interpreted it as a matrix instead of a single value. This might have worked and been undetectable in MATLAB, but the C-code generator experienced problems. Either way, we do not know the cause of the error which will require further research.

To overcome the error and proceed as scheduled, we decided to take the first element of the matrix we were supposedly generating. We know this is not the best approach, but at

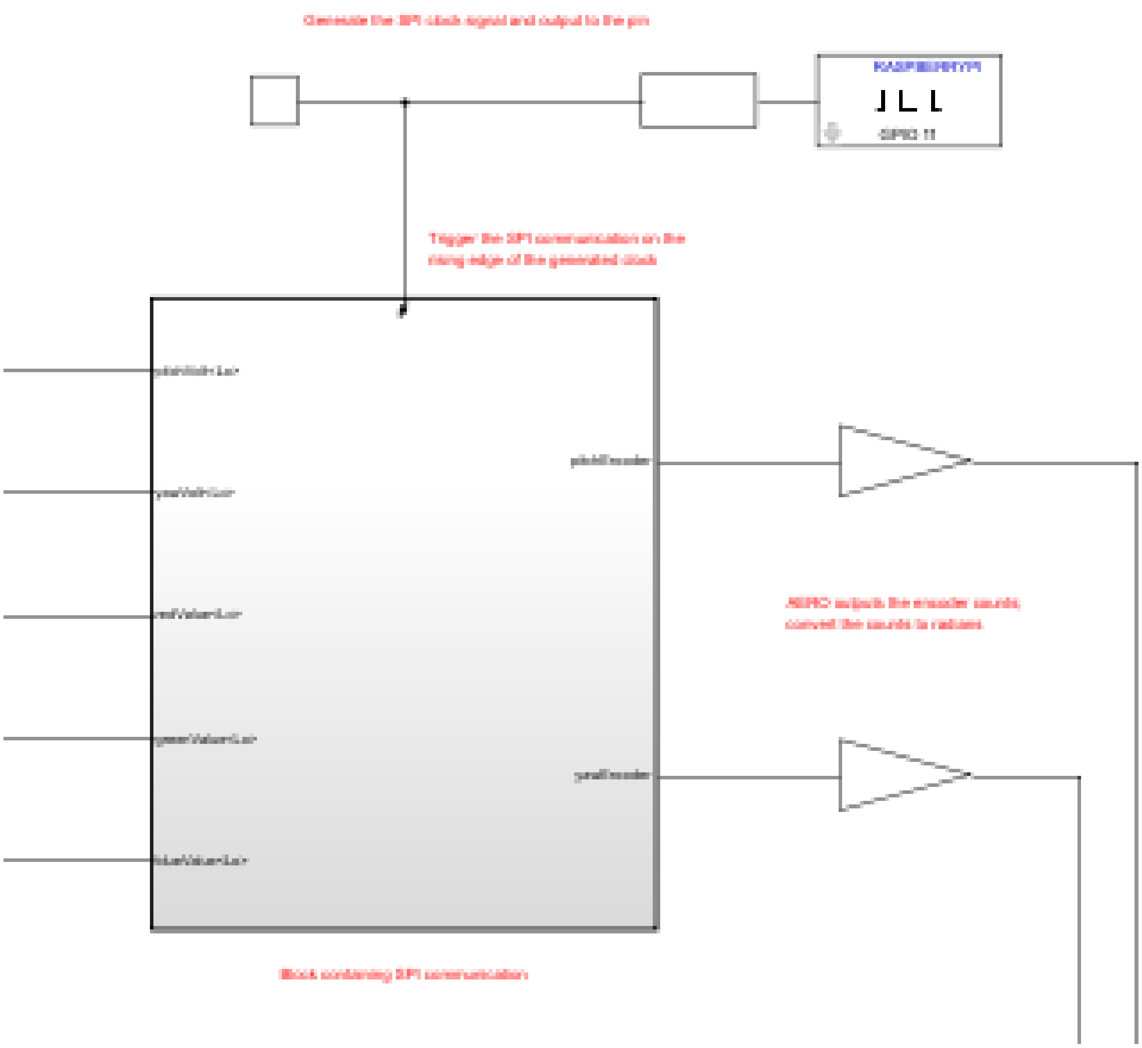


Figure 6.24: Simulink subsystem used specifically by the Raspberry Pi 3.

this point, we were behind schedule, and we needed some results. With that fix, Simulink was able to successfully generate the code on the Raspberry Pi. A full tutorial of how to execute the code is given in Appendix D.1. When we executed the code, we could see the results. It appears that the Quanser AERO goes to the position specified in the code. Because of our error and the hardware limitations discussed, the position is not perfect. The Quanser AERO appears to oscillate around the desired position. We believe, because of the timing, that the state-feedback gain is not being updated fast enough. The Quanser AERO overshoots the desired position, so it is forced to go back on the next update. There is really nothing else we can do given the hardware and algorithm constraints.

Even though ADP works but not very well in this case, we did show a proof of concept. The proof of concept is that we are able to control the Quanser AERO using a Raspberry Pi

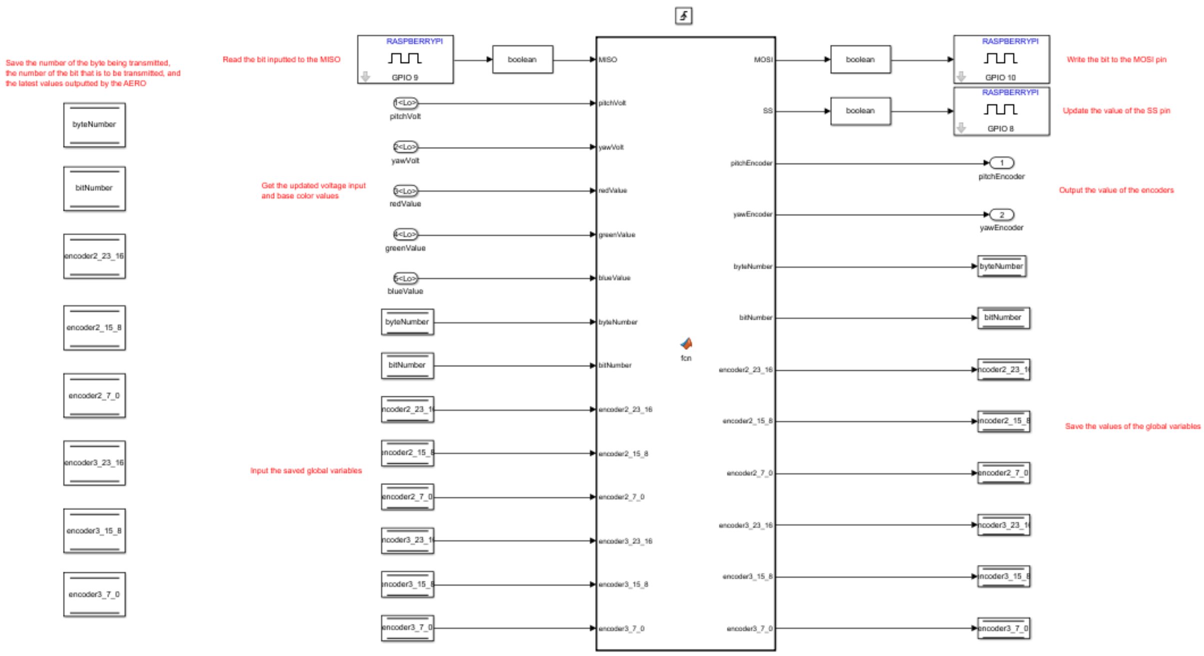


Figure 6.25: Simulink subsystem used to establish communication between the Raspberry Pi 3 and the Quanser AERO.

running ADP. This means ADP should be portable to other physical systems and embedded systems. The one drawback is that the desired position of the Quanser AERO has to be preprogrammed on the Raspberry Pi.

6.4 Raspberry Pi/Android

Our original objective of the project was to control the Quanser AERO via a Raspberry Pi and smart phone. Due to the difficulties with the C-code generation for the Raspberry Pi, we were unsure we would be able to complete the smart phone objective.

It turns out that by using Simulink, we actually made the process a lot easier. Simulink is actually able to create smart phone applications that can communicate with a Raspberry Pi. We literally had everything ready except for the application on the smart phone.

But, what is the point of the smart phone application? The development of a smart phone application can show another proof of concept. What if your system to be controlled is remote or your are actually flying an unmanned aerial vehicle? You cannot preprogram the embedded system like we did in the previous section; you need some sort of communication such as a smart phone.

6.4.1 Required Software

Because we are using the same concepts that we used for generating the C-code for the Raspberry Pi, we do not need much additional software. We do, however, need something to develop the smart phone application. Just as there was a Raspberry Pi Support Package for Simulink, there is an Android Support Package for Simulink (https://www.mathworks.com/hardware-support/android-programming-simulink.html). A helpful reference is located at https://www.mathworks.com/videos/control-raspberry-pi-from-your-android-device.html. This package can be obtained by going to one's MATLAB. In the Home tab, click on Add-Ons. You can either use Manage Add-Ons or Get Add-Ons and search for Android. This support packages is free, but it does require some setup. The setup includes setting your smart phone to allow you to develop applications. Follow the prompts in the setup, and the configuration is simple.

Because we will be using Wi-Fi as our communication medium, we also need a network. We need a local network connection for both the smart phone and the Raspberry Pi. We need a local network because it is much easier than the encrypted school network.

We also need to know the IP addresses of the smart phone and the Raspberry Pi once they are connected to the network. This task can be made simpler by downloading a device manager for your smart phone. We downloaded the IP Tools application from the Google Play Store. To manage the IP addresses of the devices on the network, we set the IP addresses as static using the router settings.

6.4.2 Simulink Model for Android Applications

Once we have all of the software set up for the Android smart phone application, we can begin updating the Simulink models. The Simulink model for the Raspberry Pi does not need to be edited significantly. We only need to change how we receive the desired pitch and yaw and also what we do with the current pitch and yaw. The updated Simulink model can be seen in Figure 6.26. This is the exact same Simulink model that was previously implemented. The only difference is how we are receiving the desired pitch and yaw.

The Simulink model in Figure 6.26 makes use of the UDP Send and Receive blocks. The UDP Receive block is specific for the Raspberry Pi. The only information needed in this block is the local port of the wireless network. This port number is specified in the UDP Send block. On the outputs of the current pitch and yaw, we see the UDP Send block. We are sending the UDP packet to an Android device, but we are doing it from a Raspberry Pi, so we need to use the Raspberry Pi UDP Send block. In the UDP Send block, we specify the local port number of the wireless network we want to connect through. We also need to specify the IP address of the receiver of the UDP packet. In our case, this will be the Android smart phone's IP address.

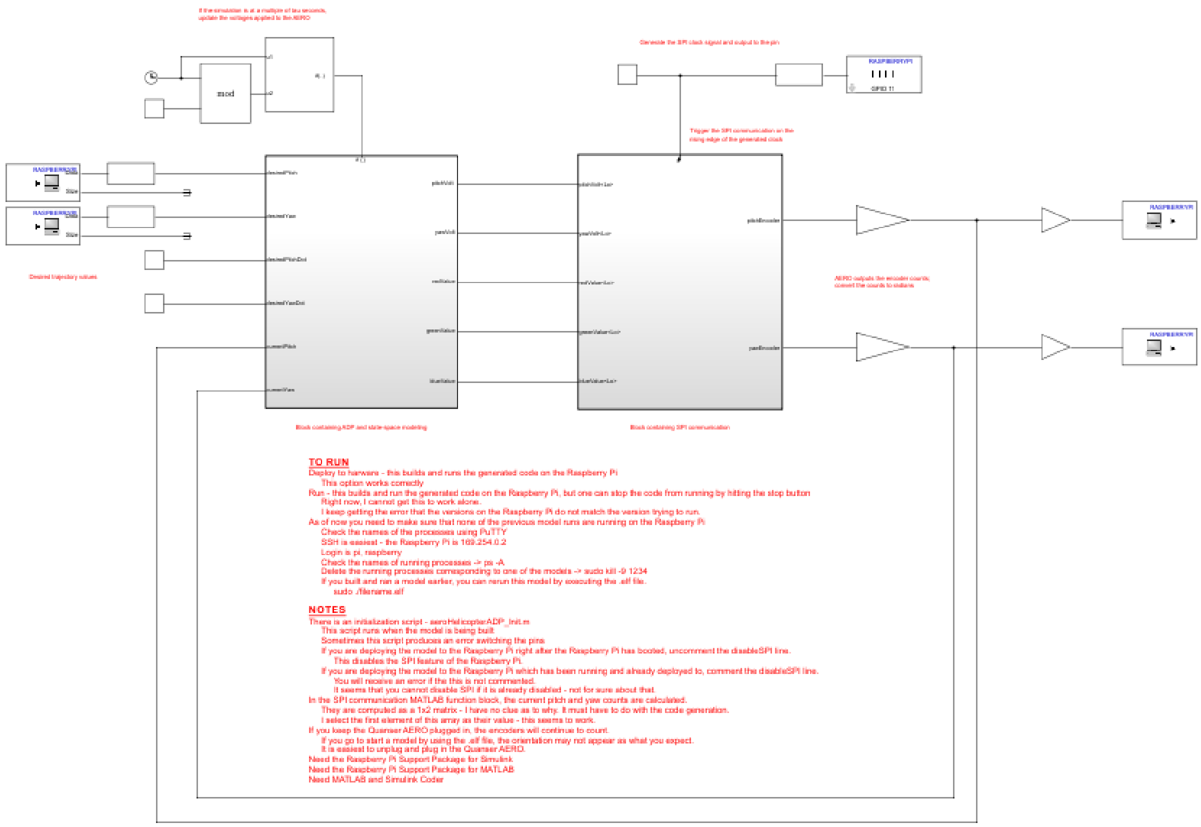


Figure 6.26: Simulink model used to communicate with the Android smart phone.

On the other end of the Wi-Fi communication is the Android smart phone. As discussed earlier, there is an Android Support Package for Simulink. With this package, we can create Simulink models that run on Android devices. The Simulink model can essentially be an application on the smart phone. Just as we did with the Raspberry Pi, we create the model for the Android smart phone, and then we generate the code on the physical device. A tutorial of how to accomplish this is given in Appendix E.1.

In order to generate the application, we need to create the Simulink model. This Simulink model can be seen in Figure 6.27. The Android Support Package for Simulink library has the necessary blocks to create the application. In our case, we used the Android Slider block and the Android Display block. These blocks basically imply what they mean, but they do it in the Android application. The Android Slider block is used to set the desired pitch and yaw. This value is then sent from the Android UDP Send block. The Android UDP Send block specifies the port number to be used on the wireless network. The block also specifies the IP address to where to send the UDP packet. In our case, this will be the Raspberry Pi's IP address. In this Simulink model, we also utilize the Android UDP Receive block. This block receives the UDP packet of the actual pitch and yaw measurements from the Raspberry Pi model discussed earlier. This value can then be displayed on the Android application with the use of the Android Display block. This model can be generated on the Android smart phone and then ready to use with the Raspberry Pi.

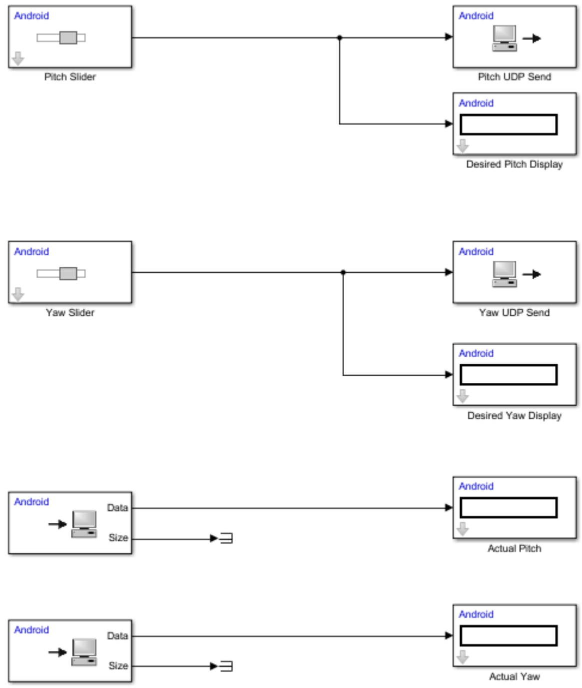


Figure 6.27: Simulink model used to create the Android smart phone application.

6.4.3 Results

The results for the smart phone application are more a matter of seeing the application work correctly. The model builds correctly for the Raspberry Pi 3. The model also builds correctly for the Android smart phone, but the build process takes a while longer. It turns out that you need to be connected to a network with Internet for the build process to complete. We are unsure why this needs to be true, but it does. A tutorial of how to run the application and get everything set up can be found in Appendix E.1.

When we run the Android smart phone application, we actually see communication to the Quanser AERO. We see the desired pitch and yaw changing as the sliders are adjusted. We also see the actual pitch and yaw readings on the application as well. Overall, this accomplishes our objective.

Chapter 7

Conclusion and Future Work

The work presented in this project successfully followed electrical engineering methodologies. We were given a system and method of control for that system; our job was to make it work. Following electrical engineering methodologies, we started with the system. We derived a highly assumed and linearized model. With the system model in hand, we analyzed ADP modifying it so it would work correctly with the new model. Once we completed the mathematical analysis and modeling, we could begin simulations to see if ADP actually worked for our system.

Simulations were conducted in both MATLAB and V-REP, but linear system models limited the simulation results. For our linear system, simulations showed proper trajectory tracking. V-REP simulations took much longer to complete due to no available Quanser AERO platforms. With successful simulations, we could begin implementation.

For the implementation process, we started simple. We created ADP in the form a Simulink model which we could then apply directly to the Quanser AERO via the QFLEX 2 USB panel. With successful control of the Quanser AERO, we modified the Simulink model slightly for interfacing with the QFLEX 2 Embedded panel. We utilized Simulink support packages to expedite the process. Using Simulink, we also were able to generate C-code that functioned correctly on the Raspberry Pi and Android smart phone.

7.1 Conclusion

This project encompassed a large amount of work for only being a year long, but several conclusions can be drawn from that work. It is possible to use ADP on the Quanser AERO, but there are some drawbacks. The Quanser AERO behaves, for the most part, as a linear system; the coupling is not too significant. ADP, thus, could not be tested accurately for this system. It would be just as beneficial to use a conventional linear control technique such as LQR. The results shown in this report show similar results for both methods. In our opinion, it would be more beneficial to use LQR for this system to limit control complexity and computational requirements. There really is no advantage to using ADP for this particular system.

Even though ADP shows no advantage for this particular system, simulations and implementation were successfully completed for the Quanser AERO. We were able to simulate the Quanser AERO in MATLAB using the system model. The MATLAB simulations were limited in the fact that it is difficult to simulate the nonlinear coupling. V-REP also provided accurate simulation results for LQR, but the complexity of ADP made real-time simulation almost impossible.

For implementation, we conclude that it is possible to apply ADP directly to a Quanser AERO using Simulink or a Raspberry Pi. Implementing the Raspberry Pi is a feat due to the fact that Quanser provides very limited documentation on the Raspberry Pi interfacing. We were able to successfully implement ADP on the Raspberry Pi with SPI communication to the Quanser QFLEX 2 Embedded panel. ADP tracks the desired trajectory reasonably well but limitations noted suggest why perfect tracking is not achieved. Extending our Simulink models and capabilities, we were also able to successfully implement an Android smart phone application that communicates with the Raspberry Pi.

Overall, we conclude that it is possible to modify, simulate, and implement ADP on the Quanser AERO.

7.2 Future Directions

Even though we accomplished many objectives in this project, there is still room for improvement and further research. Further research can be broken down into the following categories: system, ADP, and implementation. A better system can be used to test ADP. The Quanser AERO, for the most part, was a linear system. We were not able to see the true capabilities of ADP with such a linear system.

ADP itself can also be researched further. Because our project encompassed so many objective, we were unable to fully validate ADP. The use of the actor neural network can be researched such that the system can be truly learned by the algorithm. We needed an approximate system model in order to utilize ADP. We were also unable to research the optimal sampling times for ADP. We were given these sampling times at the beginning of the project, but are they truly optimal? Lastly, ADP is very slow because of the recursive nature of the algorithm. This was especially detrimental in the V-REP simulations. If ADP could be made faster or streamlined, simulations and implementation code could run much faster and more efficient.

Implementation can also be researched further. We were constrained to the Raspberry Pi, but can other, simpler embedded systems use ADP. With the Raspberry Pi, we were limited in the switching frequency of the GPIO pins. Further research could determine if there are workarounds to this or if we are truly limited. Above all, further research is needed in determining the error we received from the C-code generator in Simulink. We outputted a matrix instead of a single value, but we do not know the cause of this error. Was it code or compiler error? Or both? We were also limited in our ability to switch the desired trajectory once the C-code was generated for the Raspberry Pi. Further Linux research can be performed to determine alternate methods to change this once the code has been generated.

Our research may have come to an end for this project, but there is plenty more to discover.

APPENDICES

Appendix A

MATLAB

A.1 ADP Simulation (MATLAB)

```
% Andrew Fandel
% Use exact the linearized model to find error data points
% Use the error data points directly in the neural network instead of
% randomizing the error
close all; clear; clc;

testName = 'heli_sineWave';
heli = 0;
halfq = 1;

[ A, B ] = getAeroAB(heli);
% 2-DOF QUANSER HELICOPTER PARAMETERS
% Maximum applied voltage for the rotor motors
maxVolt = 18;
% Number of state variables - theta, psi, dtheta, dpsi
n = 4;
theta = 1;    % Pitch
psi = 2;      % Yaw
thetaDot = 3; % Pitch angular velocity
psiDot = 4;   % Yaw angular velocity
% Number of inputs - Vp, Vy
m = 2;

% SIMULATION TIMING
% Initial and final simulation times [s]
t0 = 0; tf = 25;
% Sampling time [s]
tau = 0.01;
% Larger sampling time for updating the inputs [s]
T = 0.2;
% Number of time steps
tsteps = floor((tf-t0)/tau);
% Discrete time vecotor of sampling time (tau)
dt = tau*(0:tsteps);
```

```
% Number of equations for actor-critic neural network
% Number of training samples per T
nbar = floor(T/tau);

% COST FUNCTION
% Q and R matrices used in the cost function
%Q_Mat_ADP = diag([270 100 1 1]);
%R_Mat_ADP = 0.0001*diag([1 1]);
% Use the matrices used in Quanser documentation except add ones for the
% velocities -> need Q to be positive definite
Q_Mat_ADP = diag([200 75 1 1]);
R_Mat_ADP = 0.005*diag([1 1]);
% Modified LQR cost matrices
Q_Mat_LQR = diag([200 75 1 1]);
R_Mat_LQR = 0.005*diag([1 1]);
nonLinearAdjustment = 0;
% Name of .mat file to save the data


% INITIAL STATES
xInit = [deg2rad(-87) deg2rad(-45) (pi/180)*10 (pi/180)*7]';
%xInit = [0 0 0 0]';

% MATRIX INITIALIZATION
% Actual states - state variable x
xADP = zeros(n,tsteps+1);
xLQR = zeros(n,tsteps+1);
% Initialize x with initial conditions
xADP(:,1) = xInit;
xLQR(:,1) = xInit;
% Desired states
% Uncomment for desired states of all zero
%xd = zeros(n,tsteps+1);
% Uncomment for desired states that switch constant values
%xd = [deg2rad(37)*ones(1,tsteps+1); deg2rad(80)*ones(1,tsteps+1); zeros(1,tsteps+1); zeros(1,tsteps+1)];
%xd = [deg2rad(10)*ones(1,1000), deg2rad(60)*ones(1,tsteps+1-1000); deg2rad(80)*ones(1,1500), deg2rad(-120)*ones(1,tsteps+1-1500); zeros(1,tsteps+1); zeros(1,tsteps+1)];
% Uncomment for desired states that are sine and cosine waves
 pitchSine = deg2rad(60)*sin(0.5*dt);
 yawCosine = deg2rad(90)*cos(0.5*dt);
%  pitchSquare = deg2rad(57)*square(0.5*dt);
%  yawSquare = deg2rad(-36)*square(0.25*dt);
 xd = [pitchSine; yawCosine; zeros(1,tsteps+1); zeros(1,tsteps+1)];
% xd = [pitchSquare; yawSquare; zeros(1,tsteps+1); zeros(1,tsteps+1)];
% State errors for entire simulation
errorADP = zeros(n,tsteps+1);
errorLQR = zeros(n,tsteps+1);
% Initialize the error
errorADP(:,1) = xd(:,1) - xADP(:,1);
errorLQR(:,1) = xd(:,1) - xLQR(:,1);
% Actual inputs
uADP = zeros(m,tsteps+1);
```

```
uLQR = zeros(m, tsteps+1);

% CALCULATE THE CONTROLLER GAIN MATRIX USING LQR
fprintf('WORKING ON LQR GAIN MATRIX...\n');
kLQR = lqr(A,B,Q_Mat_LQR,R_Mat_LQR);

% FIND THE CRITIC NEURAL NETWORK WEIGHTS BEFORE APPLYING ANY INPUTS
% Calculate the critic weights
wcInit = quanserAEROCriticTuningInitial(A,B,((2*pi).*rand(4,nbar)-pi),tau,
    R_Mat_ADP,Q_Mat_ADP,xd(:,1));
% Use the weights to determine the P matrix
P_Mat = [wcInit(5) wcInit(6) wcInit(7) wcInit(8);
         wcInit(6) wcInit(9) wcInit(10) wcInit(11);
         wcInit(7) wcInit(10) wcInit(12) wcInit(13);
         wcInit(8) wcInit(11) wcInit(13) wcInit(14)];
% Use P to determine the state-feedback gain
kADP = 0.5*(R_Mat_ADP^-1)*B'*P_Mat;

% RUN THE SIMULATION
% Start at the next time tau after time zero
for k = 2:tsteps+1
    % Current time
    t = (k-1)*tau; % generate value discrete time index
    if (mod(t,5) == 0)
        % Print the current time
        fprintf('Current time, t = %g [s]\n',t);
    end

    % EXACT 2-DOF HELICOPTER MODEL
    % Take previous values to find the derivative of the exact model
    xdotNonLinearADP = A*xADP(:,k-1) + B*uADP(:,k-1);
    xdotNonLinearLQR = A*xLQR(:,k-1) + B*uLQR(:,k-1);
    % Update the states of the exact model
    % Use Euler integration to estimate the current states of the nonlinear
    % model
    xADP(:,k) = xADP(:,k-1) + xdotNonLinearADP*tau + nonLinearAdjustment
    .*(2.*rand(4,1)-1);
    xLQR(:,k) = xLQR(:,k-1) + xdotNonLinearLQR*tau + nonLinearAdjustment
    .*(2.*rand(4,1)-1);
    % ADD SOME DISTURBANCE TO THE STATES
    % This forces the states to some other position
    %if ((t > 15) && (t < 17))
        %xADP(:,k) = [rad2deg(-45) 0 0 0]';
        %xLQR(:,k) = [rad2deg(-45) 0 0 0]';
    %end
    % Force the pitch angle to be in the range [-pi/2,pi/2] because of
    % physical constraints
    xADP(theta,k) = angleLimiterPitch(xADP(theta,k));
    xLQR(theta,k) = angleLimiterPitch(xLQR(theta,k));
    % Force the yaw angle to be in the range [-pi,pi] because the yaw is
    % free to do a complete circle
    xADP(psi,k) = angleLimiterYaw(xADP(psi,k));
    xLQR(psi,k) = angleLimiterYaw(xLQR(psi,k));

```

```
    % FIND THE ERROR BETWEEN THE EXACT STATES AND THE DESIRED STATES
    errorADP(:,k) = xd(:,k) - xADP(:,k);
    errorLQR(:,k) = xd(:,k) - xLQR(:,k);

    % UPDATE THE CRITIC WEIGHTS EVERY T TIME
    % Determine if the time is a multiple of T
    if (mod(t,T) == 0)
        % Update the critic weights
        wc = quanserAEROCriticTuning(A, B, errorADP(:,(k-nbar):k),tau,
    R_Mat_ADP,Q_Mat_ADP,xd(:,k),wcInit);
        % Use the weights to determine the P matrix
        P_Mat = [wc(5) wc(6) wc(7) wc(8);
                 wc(6) wc(9) wc(10) wc(11);
                 wc(7) wc(10) wc(12) wc(13);
                 wc(8) wc(11) wc(13) wc(14)];
        % Use P to determine the state-feedback gain
        kADP = 0.5*(R_Mat_ADP^-1)*B'*P_Mat;
    end
    % Update the inputs using ADP
    uNewADP = kADP*errorADP(:,k);
    % Limit the voltages
    uNewADP(1) = sign(uNewADP(1))*min(abs(uNewADP(1)),maxVolt);
    uNewADP(2) = sign(uNewADP(2))*min(abs(uNewADP(2)),maxVolt);
    % Update the input matrices
    uADP(1,k) = uNewADP(1);
    uADP(2,k) = uNewADP(2);
    % Update the inputs using LQR
    uNewLQR = kLQR*errorLQR(:,k);
    % Limit the voltages
    uNewLQR(1) = sign(uNewLQR(1))*min(abs(uNewLQR(1)),maxVolt);
    uNewLQR(2) = sign(uNewLQR(2))*min(abs(uNewLQR(2)),maxVolt);
    % Update the input matrices
    uLQR(1,k) = uNewLQR(1);
    uLQR(2,k) = uNewLQR(2);
end

% SAVE OUTPUTS FOR PLOTTING
% Save the time, error, input, states and desired states vectors
save([testName,'.mat'],'dt','errorADP','uADP','xADP','xd','tau','R_Mat_ADP',
    'Q_Mat_ADP','errorLQR','uLQR','xLQR','R_Mat_LQR','Q_Mat_LQR');
% Notify that the simulation is complete
disp('SIMULATION COMPLETE');

% ANGLE LIMITER FOR YAW FUNCTION
function angle = angleLimiterYaw(angle)
% This function limits the angle between -pi and pi
    angle = mod(angle, 2*pi);

    i=find(angle>pi);
    angle(i)=angle(i)-2*pi;

    i=find(angle<-pi);
    angle(i)=angle(i)+2*pi;
end
```

```

% ANGLE LIMITER FOR PITCH FUNCTION
function angle = angleLimiterPitch(angle)
% This function limits the physical constraint of the pitch measurement to
% 90 degrees
    if (angle < 0)
        angle = max(angle,-pi/2);
    end

    if (angle > 0)
        angle = min(angle,pi/2);
    end
end

% CRITIC WEIGHT TUNING NEURAL NETWORK
% This function is specific to the helicopter because of the number of
% weights and error model
function weights = quanserAEROCriticTuningInitial(A,B,e_vec,tau,R_Mat,Q_Mat,
    xd)
    % REFERENCE DR. MIAH'S PAPER FOR EQUATION NUMBERS

    % System dimensions specific for our model
    [n,~] = size(B);

    % ERROR MODEL OF THE HELICOPTER
    % fbar and gbar -- EQ 8
    fbar = @(e) A*e;
    gbar = -B;
    hbar = -A*xd;

    % DISCRETE-TIME ERROR MODEL FOR TIME TAU
    % f and g -- EQ 10
    f = @(e) fbar(e)*tau + e;
    g = gbar*tau;
    h = hbar*tau;

    % COST FUNCTION PARAMETERS
    % State penalizing function in the continuous cost function
    Qbar = @(e) e'*Q_Mat*e;
    % Control penalizing matrix in the continuous cost function
    Rbar = R_Mat;
    % The discrete-time cost function will have terms:
    % Right after EQ 11 in paper
    % State penalizing function in the discretized cost function
    Q = @(e) Qbar(e)*tau;
    % Control penalizing matrix in the discretized cost function
    R = Rbar*tau;

    % NEURAL NETWORK FUNCTIONS
    % Critic neural network activation functions
    rho = @(e) [e(1); e(2); e(3); e(4);...
                e(1)^2; e(1)*e(2); e(1)*e(3); e(1)*e(4);...
                e(2)^2; e(2)*e(3); e(2)*e(4); e(3)^2; e(3)*e(4); e(4)^2];
    % Partial derivative of rho with respect to e
```

```
    drhode =@(e) [1, 0, 0, 0;
                  0, 1, 0, 0;
                  0, 0, 1, 0;
                  0, 0, 0, 1;
                  2*e(1), 0, 0, 0;
                  e(2), e(1), 0, 0;
                  e(3), 0, e(1), 0;
                  e(4), 0, 0, e(1);
                  0, 2*e(2), 0, 0;
                  0, e(3), e(2), 0;
                  0, e(4), 0, e(2);
                  0, 0, 2*e(3), 0;
                  0, 0, e(4), e(3);
                  0, 0, 0, 2*e(4)];

    % TOLERANCES
    % Convergence tolerance for control policy
    EpsilonPolicy = 0.1;
    % Convergence tolerance for critic neural network
    EpsilonWcritic = 0.1;

    % TRAINING PARAMETERS
    % Number of outer loop iterations
    outerLoopMax = 700;
    % Number of inner loop iterations
    innerLoopMax = 100;
    % Number of equations needed for training, number of sub-intervals
    % Number of training samples
    [~,nbar] = size(e_vec);

    % WEIGHT INITIALIZATION
    % Initialize the weights of the critic neural network to zero
    WcLast = zeros(length(rho(e_vec(:,1))),1);

    % LEAST-SQUARES COMPUTATION INITIALIZATION
    % Matrices required for computing least squares weights of the critic
    % neural networks -- EQ 20
    V = zeros(nbar,1);
    Lambda = zeros(nbar,length(rho(e_vec(:,1))));

    % Matrix to hold the derivative of the error model during policy
    % updating
    e_k_plus_1 = zeros(n,nbar);

    % Product of the least squares matrices must be invertible
    % Logic flag indicating if the critic weights are unsolvable
    % The weights are unsolvable because the least squares matrices have no
    % solution -- not invertible
    diverged = 0;

    % OUTER LOOP
    for i = 1:(outerLoopMax-1)
    % Determine if the least squares matrices are invertible
    if diverged == 0
```

```
        % For each of the data collection (discrete time index)
        for k = 1:nbar
            % Initialize the optimal inputs to zero
            uNew = [0; 0];
            % INNER LOOP
            for j = 1:(innerLoopMax-1)
                % Get the updated input value
                uLast = uNew;
                % Update the error model
                e_k_plus_1(:,k) = f(e_vec(:,k)) + g*uLast + h;
                % Compute the new optimal inputs
                uNew  = -0.5*(R^(-1))*g'*drhode(e_k_plus_1(:,k))'*WcLast;

                % Check convergence of the optimal inputs
                if norm(uNew - uLast) < EpsilonPolicy
                    break;
                end
            end

            % Update the values for the least-squares computation
            V(k,:) = Q(e_vec(:,k)) + uNew'*R*uNew + WcLast'*rho(e_k_plus_1(:,k));
            Lambda(k,:) = rho(e_vec(:,k))';
        end
    end

    % Verify the least square solution exists for the critic's weights
    % If the error data is consistent or there is no error, this will not
    % hold, so set the weights to zero
    if det(Lambda'*Lambda) == 0
        fprintf('AWESOME...YOU HAVE NO ERROR...I''M GOING TO SET THE WEIGHTS TO ZERO\n');
        weights = zeros(length(rho(e_vec(:,1))),1);
        break;
    end;

    % Calculate least squares solution of critic's weights -- EQ 20
    WcNew = (Lambda'*Lambda)^(-1)*Lambda'*V;
    % Make sure the weights did not diverge
    % If the weights are diverging, just set them to a large number
    if isnan(WcNew)
        fprintf('OOPS...DIVERGING WEIGHTS...I''M GOING TO USE LARGE WEIGHTS\n');
        weights = 1000*ones(length(rho(e_vec(:,1))),1);
        break;
    end;

    % Check for convergence of the critic weights
    if norm(WcNew - WcLast) < EpsilonWcritic
        weights = WcNew;
        fprintf('GREAT...THE WEIGHTS CONVERGED\n');
        break;
    end
    % If the weights did not converge, repeat the loop
```

```
    WcLast = WcNew;

    % If we reached the last iteration of the loop, just use the last
    % weights found
    if (i == (outerLoopMax-1))
        fprintf('OOPS...YOU REACHED THE END OF THE OUTER LOOP...I''M GOING
    TO USE THE LAST VALUE\n');
        weights = WcNew;
    end
    end
end

% CRITIC WEIGHT TUNING NEURAL NETWORK
% This function is specific to the helicopter because of the number of
% weights and error model
function weights = quanserAEROCriticTuning(A, B, e_vec,tau,R_Mat,Q_Mat,xd,
    wcInit)
    % REFERENCE DR. MIAH'S PAPER FOR EQUATION NUMBERS

    % System dimensions specific for our model
    [n,~] = size(B);

    % ERROR MODEL OF THE HELICOPTER
    % fbar and gbar -- EQ 8
    fbar = @(e) A*e;
    gbar = -B;
    hbar = -A*xd;

    % DISCRETE-TIME ERROR MODEL FOR TIME TAU
    % f and g -- EQ 10
    f = @(e) fbar(e)*tau + e;
    g = gbar*tau;
    h = hbar*tau;

    % COST FUNCTION PARAMETERS
    % State penalizing function in the continuous cost function
    Qbar = @(e) e'*Q_Mat*e;
    % Control penalizing matrix in the continuous cost function
    Rbar = R_Mat;
    % The discrete-time cost function will have terms:
    % Right after EQ 11 in paper
    % State penalizing function in the discretized cost function
    Q = @(e) Qbar(e)*tau;
    % Control penalizing matrix in the discretized cost function
    R = Rbar*tau;

    % NEURAL NETWORK FUNCTIONS
    % Critic neural network activation functions
    rho = @(e) [e(1); e(2); e(3); e(4);...
                e(1)^2; e(1)*e(2); e(1)*e(3); e(1)*e(4);...
                e(2)^2; e(2)*e(3); e(2)*e(4); e(3)^2; e(3)*e(4); e(4)^2];
    % Partial derivative of rho with respect to e
    drhode =@(e) [1, 0, 0, 0;
                  0, 1, 0, 0;
```

```
                    0, 0, 1, 0;
                    0, 0, 0, 1;
                    2*e(1), 0, 0, 0;
                    e(2), e(1), 0, 0;
                    e(3), 0, e(1), 0;
                    e(4), 0, 0, e(1);
                    0, 2*e(2), 0, 0;
                    0, e(3), e(2), 0;
                    0, e(4), 0, e(2);
                    0, 0, 2*e(3), 0;
                    0, 0, e(4), e(3);
                    0, 0, 0, 2*e(4)];

    % TOLERANCES
    % Convergence tolerance for control policy
    EpsilonPolicy = 0.1;
    % Convergence tolerance for critic neural network
    EpsilonWcritic = 0.1;

    % TRAINING PARAMETERS
    % Number of outer loop iterations
    outerLoopMax = 700;
    % Number of inner loop iterations
    innerLoopMax = 100;
    % Number of equations needed for training, number of sub-intervals
    % Number of training samples
    [~,nbar] = size(e_vec);

    % WEIGHT INITIALIZATION
    % Initialize the weights of the critic neural network to zero
    WcLast = zeros(length(rho(e_vec(:,1))),1);

    % LEAST-SQUARES COMPUTATION INITIALIZATION
    % Matrices required for computing least squares weights of the critic
    % neural networks -- EQ 20
    V = zeros(nbar,1);
    Lambda = zeros(nbar,length(rho(e_vec(:,1))));

    % Matrix to hold the derivative of the error model during policy
    % updating
    e_k_plus_1 = zeros(n,nbar);

    % Product of the least squares matrices must be invertible
    % Logic flag indicating if the critic weights are unsolvable
    % The weights are unsolvable because the least squares matrices have no
    % solution -- not invertible
    diverged = 0;

    % OUTER LOOP
    for i = 1:(outerLoopMax-1)
    % Determine if the least squares matrices are invertible
    if diverged == 0
        % For each of the data collection (discrete time index)
        for k = 1:nbar
```

```
            % Initialize the optimal inputs to zero
            uNew = [0; 0];
            % INNER LOOP
            for j = 1:(innerLoopMax-1)
                % Get the updated input value
                uLast = uNew;
                % Update the error model
                e_k_plus_1(:,k) = f(e_vec(:,k)) + g*uLast + h;
                % Compute the new optimal inputs
                uNew  = -0.5*(R^(-1))*g'*drhode(e_k_plus_1(:,k))'*WcLast;

                % Check convergence of the optimal inputs
                if norm(uNew - uLast) < EpsilonPolicy
                    break;
                end
            end

            % Update the values for the least-squares computation
            V(k,:) = Q(e_vec(:,k)) + uNew'*R*uNew + WcLast'*rho(e_k_plus_1(:,k));
            Lambda(k,:) = rho(e_vec(:,k))';
        end
    end

    % Verify the least square solution exists for the critic's weights
    % If the error data is consistent or there is no error, this will not
    % hold, so set the weights to what they were initially before the
    % simulation
    if det(Lambda'*Lambda) == 0
        fprintf('AWESOME...YOU HAVE NO ERROR...I''M GOING TO USE THE ORIGINAL WEIGHTS\n');
        weights = wcInit;
        break;
    end;

    % Calulcate least squares solution of critic's weights -- EQ 20
    WcNew = (Lambda'*Lambda)^(-1)*Lambda'*V;
    % Make sure the weights did not diverge
    % If the weights diverged, set the weights to what they were initially
    % before the simulation
    if isnan(WcNew)
        fprintf('OOPS...DIVERGING WEIGHTS...I''M GOING TO USE THE ORIGINAL WEIGHTS\n');
        weights = wcInit;
        break;
    end;

    % Check for convergence of the critic weights
    if norm(WcNew - WcLast) < EpsilonWcritic
        fprintf('GREAT...THE WEIGHTS CONVERGED\n');
        weights = WcNew;
        break;
    end
    % If the weights did not converge, do another iteration of the loop
```

```
    WcLast = WcNew;

    % If we reach the last iteration of the loop, just use the last weights
    if (i == (outerLoopMax-1))
        fprintf('OOPS...YOU REACHED THE END OF THE OUTER LOOP...I''M GOING TO USE THE LAST VALUE\n');
        weights = WcNew;
    end
    end
end

function [A, B] = getAeroAB(choice)
% This function builds the A and B matrix for either configuration of the Quanser Aero
% choice 0 selects the helicopter mode
% choice 1 selects the halfquad   mode

% Quanser Aero Parameters
% Moment of Inertia of helicopter body (kg-m^2)
L_body = 6.5*0.0254; % length of horizontal body (metal tube)
m_body = 0.094; % mass of horizontal body (metal tube)
J_body = m_body * L_body^2 / 12; % horizontal cylinder rotating about CM

% Moment of Inertia of yoke fork that rotates about yaw axis (kg-m^2)
m_yoke = 0.526; % mass of entire yoke assembly (kg)
% h_yoke = 9*0.0254; % height of yoke assembly (m)
r_fork = 0.04/2; % radius of each fork (approximated as cylinder)
J_yoke = 0.5*m_yoke*r_fork^2;

% Moment of Inertia from motor + guard assembly about pivot (kg-m^2)
m_prop = 0.43; % mass of dc motor + shield + propeller shield
% m_motor = 0.203; % mass of dc motor
r_prop = 6.25*0.0254; % distance from CM to center of pitch axis
J_prop = m_prop * r_prop^2; % using parallel axis theorem

% Equivalent Moment of Inertia about Pitch and Yaw Axis (kg-m^2)
Jp = J_body + 2*J_prop; % pitch: body and 2 props
Jy = J_body + 2*J_prop + J_yoke; % yaw: body, 2 props, and yoke

% Thrust-torque constant (N-m/V) [found experimentally]
Kpp = 0.0011; % (pre-production unit: 0.0015)
Kyy = 0.0022; % (pre-production unit: 0.0040)
Kpy = 0.0021; % thrust acting on pitch from yaw (pre-production unit: 0.0020)
Kyp = -0.0027; % thrust acting on yaw from pitch (pre-production unit: -0.0017)

% Stiffness (N-m/rad)[found experimentally]
Ksp = 0.037463;

% Viscous damping (N-m-s/rad) [found experimentally]
Dp = 0.0071116; % pitch axis (pre-production unit: Dp = 0.0226)
Dy = 0.0220; % yaw axis (pre-production unit: Dy = 0.0211)

```

```
if choice == 0
    disp ('helicopter mode');
    A = [0 0 1 0;
         0 0 0 1;
        -Ksp/Jp 0 -Dp/Jp 0;
         0 0 0 -Dy/Jy];
    B = [0 0;
         0 0;
         Kpp/Jp Kpy/Jp;
         Kyp/Jy Kyy/Jy];

    disp('A - Matrix');
    disp(A);
    disp('B - Matrix');
    disp(B);
else
    disp ('halfquad mode');
    A = [0 0 1 0;
         0 0 0 1;
        -Ksp/Jp 0 -Dp/Jp 0;
         0 0 0 -Dy/Jy];
    B = [0 0;
         0 0;
         -Kpp/Jy Kpp/Jy;
         -Kyy/Jy -Kyy/Jy];
    disp('A - Matrix');
    disp(A);
    disp('B - Matrix');
    disp(B);
end

end
```

Appendix B

Running V-REP

B.1 Running a Simulation

B.1.1 File Setup

Setup the API Files

First, unpack the vrepAeroADP file to a spot on the computer. To use the following three MATLAB files, the version of them must be selected to reflect the 32/64 bit-architecture of the host system.

- remApi.m
- remoteApiProto.m
- remoteApi.dll (optional - MATLAB will compile)

Make sure your MATLAB uses the same bit-architecture as the remoteApi library: 64bit MATLAB with 32bit remoteApi library will not work, and vice-versa! The above files are located in V-REP's installation directory at:

```
../programming/remoteApiBindings/matlab
```

The Vortex Physics Engine

The Vortex Physics Engine is a unified real-time simulation platform that V-REP uses to calculate the dynamics of the scene. To obtain Vortex, an account at CM-Labs.com must be made, and a free license acquired.

1. CM Labs Account and Vortex License
2. Vortex Download

B.1.2 Running the Simulation

To run a V-REP scene, controlled by MATLAB:

1. VREP: Select the Vortex physics engine.
2. VREP: Select $dt = 50$ ms.
3. VREP: Click the clock icon, to the right of the play button. Make sure the output console reads: Toggled to non real-time simulation mode.
4. V-REP: Press the Play button in the V-REP scene.
5. MATLAB: Press the Play Button/Run icon under the **Editor** tab.

B.2 Skeletal Framework

B.2.1 MATLAB Framework

```

% 2-DOF QUANSER HELICOPTER PARAMETERS
% Maximum applied voltage for the rotor motors
maxVolt = 18;
% Number of state variables - theta, psi, dtheta, dpsi
n = 4;
theta = 1;    % Pitch
psi = 2;      % Yaw
thetaDot = 3; % Pitch angular velocity
psiDot = 4;   % Yaw angular velocity
% Number of inputs - Vp, Vy
m = 2;

% SIMULATION TIMING
% Initial and final simulation times [s]
t0 = 0; tf = 30;
% Sampling time [s]
tau = 0.05;
% Number of time steps
tsteps = floor((tf-t0)/tau);
% Discrete time vecotor of sampling time (tau)
dt = tau*(0:tsteps);

state = zeros(n,tsteps+1);
u = zeros(m,tsteps+1);

%% connection routine

ip = '127.0.0.1';
port = 19999;

```

```
% create a handle to remote API
vrep = remApi('remoteApi'); % using the prototype file (remoteApiProto.m)
vrep.simxFinish(-1); % just in case, close all opened connections
clientID = vrep.simxStart(ip,port,true,true,5000,5);

%% simulation
if (clientID>-1) % if clientID exists
    disp('Connected to remote API server');

    %Set up the handles
    [rtn, aero] = vrep.simxGetObjectHandle(clientID,...
                                            'Aero',...
                                             vrep.simx_opmode_blocking)
    [rtn, yJ] = vrep.simxGetObjectHandle(clientID,...
                                        'yawJoint',...
                                         vrep.simx_opmode_blocking)
    [rtn, pJ] = vrep.simxGetObjectHandle(clientID,...
                                        'pitchJoint',...
                                        vrep.simx_opmode_blocking)

    % setup joint position streaming
    vrep.simxGetJointPosition(clientID,...
                                yJ,...
                                vrep.simx_opmode_streaming);
    vrep.simxGetJointPosition(clientID,...
                                pJ,...
                                vrep.simx_opmode_streaming);

    % add a delay to let the streaming operations initialize
    pause(1);

    [ret, state(1,1)] = vrep.simxGetJointPosition(clientID,...
                                                    pJ,...
                                                    vrep.simx_opmode_buffer);

    [ret, state(2,1)] = vrep.simxGetJointPosition(clientID,...
                                                    yJ,...
                                                    vrep.simx_opmode_buffer);


    state(3,1) = 0;
    state(4,1) = 0;

    % start a timer
    tic;
    % RUN THE SIMULATION
    % Start at the next time tau after time zero
    for k = 2:tsteps+1
        % Current time
        t = (k-1)*tau; % generate value discrete time index

        % Step 1 - get/store the state of the quadcopter from VREP
        [ret, state(1,k)] = vrep.simxGetJointPosition(clientID,...
                                                        pJ,...
```

```
                                                                vrep.simx_opmode_buffer
    );

        [ret, state(2,k)] = vrep.simxGetJointPosition(clientID ,...
                                                                yJ ,...
                                                                vrep.simx_opmode_buffer
    );



    % algorithm goes here


        u(1,k) = 5; % motor 0
        u(2,k) = 5.01; % motor 1

        % Step 3 - Update Inputs
        V0V1=[u(1,k) u(2,k)];
        packedData=vrep.simxPackFloats(V0V1);
        vrep.simxSetStringSignal(clientID ,...
                                'Vin_matlab' ,...
                                packedData ,...
                                vrep.simx_opmode_oneshot);

        pause(tau);

        t = t + tau;


    end
    toc; % end the timer

    ret = 1;
    while(ret ~= 0)
        [ret]=vrep.simxClearStringSignal(clientID ,...
                                        'Vin_matlab' ,...
                                        vrep.simx_opmode_blocking);
    end

else
    disp('Failed connecting to remote API server');
end % if (clientID>-1)

disp('Sim ended');
```

B.2.2 LUA Framework

```
function sysCall_init()
  -- port to talk to MATLAB
    simRemoteApi.start(19999)

    -- Assign handles to Propeller Scripts
    propellerScripts={-1,-1}
    propellerScripts[1] = sim.getScriptHandle('propeller')
```

```
    propellerScripts[2] = sim.getScriptHandle('propeller#0')

end

function sysCall_actuation()
     -- Receive velocities from server
    local packedData = sim.getStringSignal('Vin_matlab')

    if packedData then
         -- Clear the signal
        sim.clearStringSignal('Vin_matlab')
         -- unpack 2 floats, save in array "a={V0, V1}"
        local a = sim.unpackFloatTable(packedData,0,2,0)

        for i=1,2,1 do
            -- set script variable Vin associated
            -- with the propellers
      sim.setScriptSimulationParameter(propellerScripts[i],'Vin',a[i])
        end
    end
end

function sysCall_sensing()

end

function sysCall_cleanup()

end
```

B.3 Calculate State-Space

```
function [ A, B ] = getAeroAB( choice )
%% this function builds the A and B matrix for either configuration of the
    Quanser Aero
% choice 0 selects the helicopter mode
% choice 1 selects the halfquad   mode

    %% Quanser Aero Parameters
    % Moment of Inertia of helicopter body (kg-m^2)
    L_body = 6.5*0.0254; % length of horizontal body (metal tube)
    m_body = 0.094; % mass of horizontal body (metal tube)
    J_body = m_body * L_body^2 / 12; % horizontal cylinder rotating about CM

    % Moment of Inertia of yoke fork that rotates about yaw axis (kg-m^2)
    m_yoke = 0.526; % mass of entire yoke assembly (kg)
    % h_yoke = 9*0.0254; % height of yoke assembly (m)
    r_fork = 0.04/2; % radius of each fork (approximated as cylinder)
    J_yoke = 0.5*m_yoke*r_fork^2;

    % Moment of Inertia from motor + guard assembly about pivot (kg-m^2)
    m_prop = 0.43; % mass of dc motor + shield + propeller shield
    % m_motor = 0.203; % mass of dc motor
```

```
    r_prop = 6.25*0.0254; % distance from CM to center of pitch axis
    J_prop = m_prop * r_prop^2; % using parallel axis theorem

   % Equivalent Moment of Inertia about Pitch and Yaw Axis (kg-m^2)
   Jp = J_body + 2*J_prop; % pitch: body and 2 props
   Jy = J_body + 2*J_prop + J_yoke; % yaw: body, 2 props, and yoke

   % Thrust-torque constant (N-m/V) [found experimentally]
   Kpp = 0.0011; % (pre-production unit: 0.0015)
   Kyy = 0.0022; % (pre-production unit: 0.0040)
   Kpy = 0.0021; % thrust acting on pitch from yaw (pre-production unit:
   0.0020)
   Kyp = -0.0027; % thrust acting on yaw from pitch (pre-production unit:
   -0.0017)

   % Stiffness (N-m/rad)[found experimentally]
   Ksp = 0.037463;

   % Viscous damping (N-m-s/rad) [found experimentally]
   Dp = 0.0226; % pitch axis (pre-production unit: Dp = 0.0226)
   Dy = 0.0220; % yaw axis (pre-production unit: Dy = 0.0211)

    if choice == 0
        disp ('helicopter mode');
        A = [0 0 1 0;
             0 0 0 1;
            -Ksp/Jp 0 -Dp/Jp 0;
             0 0 0 -Dy/Jy];
        B = [0 0;
             0 0;
             Kpp/Jp Kpy/Jp;
             Kyp/Jy Kyy/Jy];

        disp('A - Matrix');
        disp(A);
        disp('B - Matrix');
        disp(B);
    else
        disp ('halfquad mode');
        A = [0 0 1 0;
             0 0 0 1;
            -Ksp/Jp 0 -Dp/Jp 0;
             0 0 0 -Dy/Jy];
        B = [0 0;
             0 0;
             -Kpp/Jy Kpp/Jy;
             -Kyy/Jy -Kyy/Jy];
        disp('A - Matrix');
        disp(A);
        disp('B - Matrix');
        disp(B);
    end

end
```

```
function [ A, B ] = getVREPAeroAB( choice )
%% this function builds the A and B matrix for either configuration of the
    Quanser Aero
% choice 0 selects the helicopter mode
% choice 1 selects the halfquad   mode

    %% VREP parameters
    % Moment of Inertia of pitcharm
%     M_fuselage = 3.375e-1;
%     M_sideblock = 4.275e-2;
    M_fuselage = 0.85;
    M_sideblock = 0.4;
    M_motor     = 0.225;

    x_fuselage = 3.0000e-2;
    y_fuselage = 3.75000e-1;
    z_fuselage = 3.0000e-2;

    x_sideblock = 4.7500e-2;
    y_sideblock = 3.0000e-2;
    z_sideblock = 3.0000e-2;
    r_sideblock = 3.8750e-2;

    r_motor = 1.5875e-1;

    % MoI of yoke
    side_m    = 0.18;
    cyl_mass = 0.05;
    cross_m   = .1;

    x_side    = 5.0000e-2;
    y_side    = 5.0000e-2;
    z_side    = 2.0000e-1;
    r_side    = 8.7500e-2;

    x_cross   = 1.2500e-1;
    y_cross   = 5.0000e-2;
    z_cross   = 5.0000e-2;

    dia_cyl   = 5.0000e-2;
    h_cyl     = 2.0000e-2;

    J_body = getrectPrismI(y_fuselage,z_fuselage,M_fuselage)
           + getrectPrismI(y_sideblock,z_sideblock,M_sideblock)
           + getrectPrismI(y_sideblock,z_sideblock,M_sideblock);

    % Moment of Inertia of yoke fork that rotates about yaw axis (kg-m^2)
    J_support_on_axis = getrectPrismI(x_side, y_side, side_m);
    J_support = parAxisTheorem(J_support_on_axis,side_m,r_side);
    J_cross = getrectPrismI(x_cross, y_cross, cross_m);
    J_cyl = 0.5*h_cyl*(dia_cyl/2).^2;

    J_yoke = 2*J_support + J_cross + J_cyl;

```

```
    % Moment of Inertia from motor + guard assembly about pivot (kg-m^2)
    J_prop = M_motor * r_motor^2; % using parallel axis theorem

    % Equivalent Moment of Inertia about Pitch and Yaw Axis (kg-m^2)
    Jp = J_body + 2*J_prop % pitch: body and 2 props
    Jy = J_body + 2*J_prop + J_yoke % yaw: body, 2 props, and yoke

    % Thrust-torque constant (N-m/V) [found experimentally]
    Kpp = 0.03; % (pre-production unit: 0.0015)
    Kyy = 0.0022; % (pre-production unit: 0.0040)
    Kpy = 0.0021; % thrust acting on pitch from yaw (pre-production unit: 0.0020)
    Kyp = -0.0027; % thrust acting on yaw from pitch (pre-production unit: -0.0017)

    % Stiffness (N-m/rad)[found experimentally]
     Ksp = 0.037463;

    % Viscous damping (N-m-s/rad) [found experimentally]
      Dp = 0.0071116; % pitch axis (pre-production unit: Dp = 0.0226)
      Dy = 0.0220; % yaw axis (pre-production unit: Dy = 0.0211)
%     Ksp = 0;
%     Dp = 0;
%     Dy = 0;

    if choice == 0
        disp ('helicopter mode');
        A = [0 0 1 0;
             0 0 0 1;
            -Ksp/Jp 0 -Dp/Jp 0;
             0 0 0 -Dy/Jy];
        B = [0 0;
             0 0;
             Kpp/Jp Kpy/Jp;
             Kyp/Jy Kyy/Jy];

        disp('A - Matrix');
        disp(A);
        disp('B - Matrix');
        disp(B);
    else
        disp ('halfquad mode');
        A = [0 0 1 0;
             0 0 0 1;
            -Ksp/Jp 0 -Dp/Jp 0;
             0 0 0 -Dy/Jy];
        B = [0 0;
             0 0;
             -Kpp/Jy Kpp/Jy;
             -Kyy/Jy Kyy/Jy];
        disp('A - Matrix');
        disp(A);
        disp('B - Matrix');
        disp(B);
```

```
    end

end

function [I] = getrectPrismI(a,b,m)
    I = m*(a.^2 + b.^2)/12;
end

function [pax] = parAxisTheorem(I,m,r)
    pax = I + m*r*r;
end
```

B.4 Joint Control

B.4.1 LUA 2-DOF Helicopter

```
-- Initialization ==================================================================
if (sim_call_type==sim.syscb_init) then
    simRemoteApi.start(19999)

    -- Get Handles for Propeller Objects and simulation timestep
  pJ = sim.getObjectHandle('pitchJoint');
  yJ = sim.getObjectHandle('yawJoint');
    prop0 = sim.getObjectHandle('motor0');
    prop1 = sim.getObjectHandle('motor1');
    timestep = sim.getSimulationTimeStep()
    print('timestep is : ', timestep)
    -- Get parameter values to work with
    Jy = sim.getScriptSimulationParameter(sim.handle_self,'Jy')
    print('Jy is : ', Jy)
    Jp = sim.getScriptSimulationParameter(sim.handle_self,'Jp')
    print('Jp is : ', Jp)
    Dp = sim.getScriptSimulationParameter(sim.handle_self,'Dp')
    print('Dp is : ', Dp)
    Dy = sim.getScriptSimulationParameter(sim.handle_self,'Dy')
    print('Dy is : ', Dy)
    Kpp = sim.getScriptSimulationParameter(sim.handle_self,'Kpp')
    print('Kpp is : ', Kpp)
    Kpy = sim.getScriptSimulationParameter(sim.handle_self,'Kpy')
    print('Kpy is : ', Kpy)
    Kyp = sim.getScriptSimulationParameter(sim.handle_self,'Kyp')
    print('Kyp is : ', Kyp)
    Kyy = sim.getScriptSimulationParameter(sim.handle_self,'Kyy')
    print('Kyy is : ', Kyy)
  Ksp = sim.getScriptSimulationParameter(sim.handle_self,'Ksp')
    print('Ksp is : ', Ksp)

    -- Store current measured angular velocity to calculate angular acceleration
  prev_pJ_pose = sim.getJointPosition(pJ)
  prev_yJ_pose = sim.getJointPosition(yJ)
```

```
    sim.setJointTargetVelocity(pJ, 0)
    sim.setJointTargetVelocity(yJ, 0)
    sim.setJointTargetVelocity(prop0,0)
    sim.setJointTargetVelocity(prop1,0)
    pitchTarVel = 0
    yawTarVel = 0

   graphHandle=sim.getObjectHandle("Graph")

    PitchDesired = sim.getIntegerSignal("PitchDesired")
    YawDesired = sim.getIntegerSignal("YawDesired")
    lastPAcc = 0
    lastYAcc = 0

    sim.setJointTargetVelocity(prop0,1*0.8)
    sim.setJointTargetVelocity(prop1,-1*0.8)

end

-- Looping code ==================================================================================

if (sim_call_type==sim.syscb_actuation) then


        local t = sim.getSimulationTime()

     -- Receive velocities from server
    local packedData=sim.getStringSignal('MATLAB_SIG')
    if packedData then
        sim.clearStringSignal('MATLAB_SIG') -- Clear the signal
        local V=sim.unpackFloatTable(packedData,0,4,0)

    pJ_pose = sim.getJointPosition(pJ)
    yJ_pose = sim.getJointPosition(yJ)
        pJ_vel  = (pJ_pose - prev_pJ_pose)/timestep
        yJ_vel  = (yJ_pose - prev_yJ_pose)/timestep

        pAcc = -(Ksp/Jp)*math.sin(pJ_pose) + -(Dy/Jp)*pJ_vel + (Kpp/Jp)*V[1]
    + (Kpy/Jp)*V[2]
        yAcc = -(Dp/Jy)*yJ_vel + (Kyp/Jy)*V[1] + (Kyy/Jy)*V[2]


        pitchTarVel = pitchTarVel + ((pAcc+lastPAcc)/2)*timestep
        yawTarVel = yawTarVel + ((yAcc+lastYAcc)/2)*timestep

        sim.setJointTargetVelocity(pJ, pitchTarVel)
        sim.setJointTargetVelocity(yJ, yawTarVel)

        prev_pJ_pose = pJ_pose
        prev_yJ_pose = yJ_pose
        lastPAcc = pAcc
        lastYAcc = yAcc
```

```
        sim.setGraphUserData(graphHandle,'PitchDesired',V[3])
        sim.setGraphUserData(graphHandle,'YawDesired',V[4])

        sim.setJointTargetVelocity(prop0,V[1]*0.8)
        sim.setJointTargetVelocity(prop1,-V[2]*0.8)

    end

end

function sysCall_sensing()
    -- put your sensing code here
end

if (sim_call_type==sim.syscb_cleanup) then

end
```

B.4.2 LUA 2-DOF Half-Quadcopter

```
-- Initialization
   ==============================================================
if (sim_call_type==sim.syscb_init) then
    simRemoteApi.start(19999)

  pJ = sim.getObjectHandle('pitchJoint');
  yJ = sim.getObjectHandle('yawJoint');
    prop0 = sim.getObjectHandle('motor0');
    prop1 = sim.getObjectHandle('motor1');
    timestep = sim.getSimulationTimeStep()
    print('timestep is : ', timestep)
    -- Get parameter values to work with
    Jy = sim.getScriptSimulationParameter(sim.handle_self,'Jy')
    print('Jy is : ', Jy)
    Jp = sim.getScriptSimulationParameter(sim.handle_self,'Jp')
    print('Jp is : ', Jp)
    Dp = sim.getScriptSimulationParameter(sim.handle_self,'Dp')
    print('Dp is : ', Dp)
    Dy = sim.getScriptSimulationParameter(sim.handle_self,'Dy')
    print('Dy is : ', Dy)
    Kpp = sim.getScriptSimulationParameter(sim.handle_self,'Kpp')
    print('Kpp is : ', Kpp)
    Kyy = sim.getScriptSimulationParameter(sim.handle_self,'Kyy')
    print('Kyy is : ', Kyy)
  Ksp = sim.getScriptSimulationParameter(sim.handle_self,'Ksp')
    print('Ksp is : ', Ksp)

    -- Store current measured angular velocity to calculate angular
   acceleration
  prev_pJ_pose = sim.getJointPosition(pJ)
  prev_yJ_pose = sim.getJointPosition(yJ)

    sim.setJointTargetVelocity(pJ, 0)
    sim.setJointTargetVelocity(yJ, 0)
```

```
    sim.setJointTargetVelocity(prop0,0)
    sim.setJointTargetVelocity(prop1,0)
    pitchTarVel = 0
    yawTarVel = 0

    graphHandle=sim.getObjectHandle("Graph")

    PitchDesired = sim.getIntegerSignal("PitchDesired")
    lastPAcc = 0
    lastYAcc = 0

    sim.setJointTargetVelocity(prop0,1*0.8)
    sim.setJointTargetVelocity(prop1,-1*0.8)

end

-- Looping code
   ==============================================================================

if (sim_call_type==sim.syscb_actuation) then

        local t = sim.getSimulationTime()
     -- Receive velocities from server
    local packedData=sim.getStringSignal('MATLAB_SIG')
    if packedData then
        sim.clearStringSignal('MATLAB_SIG') -- Clear the signal
        local V=sim.unpackFloatTable(packedData,0,4,0)

        -- Measure pitch and yaw
    pJ_pose = sim.getJointPosition(pJ)
    yJ_pose = sim.getJointPosition(yJ)

        -- Calculate Velocites
        pJ_vel  = (pJ_pose - prev_pJ_pose)/timestep
        yJ_vel  = (yJ_pose - prev_yJ_pose)/timestep

        pAcc = -(Ksp/Jy)*math.sin(pJ_pose) + -(Dy/Jy)*pJ_vel + -(Kpp/Jy)*V
   [1] + (Kpp/Jy)*V[2]
        yAcc = -(Dp/Jp)*yJ_vel + -(Kyy/Jy)*V[1] + -(Kyy/Jy)*V[2]


        pitchTarVel = pitchTarVel + ((pAcc+lastPAcc)/2)*timestep
        yawTarVel = yawTarVel + ((yAcc+lastYAcc)/2)*timestep

        sim.setJointTargetVelocity(pJ, pitchTarVel)
        sim.setJointTargetVelocity(yJ, yawTarVel)

        prev_pJ_pose = pJ_pose
        prev_yJ_pose = yJ_pose
        lastPAcc = pAcc
        lastYAcc = yAcc
        sim.setGraphUserData(graphHandle,'PitchDesired',V[3])
        sim.setGraphUserData(graphHandle,'YawDesired',V[4])

```

```
        sim.setJointTargetVelocity(prop0,V[1]*0.8)
        sim.setJointTargetVelocity(prop1,-V[2]*0.8)

    end

end

function sysCall_sensing()
end

if (sim_call_type==sim.syscb_cleanup) then

end
```

B.4.3 LQR Half-Quadcopter

```
% Use exact model of the 2-DOF helicopter and the linearized model to find
% error data points
% Use the error data points directly in the neural network instead of
% randomizing the error
close all; clear; clc;
% Name of .mat file to save the data
testName = 'half_joint_sinesine';
[ A, B ] = getAeroAB( 1 )


% 2-DOF QUANSER HELICOPTER PARAMETERS
% Maximum applied voltage for the rotor motors
maxVolt = 18;
% Number of state variables - theta, psi, dtheta, dpsi
n = 4;
theta = 1;    % Pitch
psi = 2;      % Yaw
thetaDot = 3; % Pitch angular velocity
psiDot = 4;   % Yaw angular velocity
% Number of inputs - Vp, Vy
m = 2;

% SIMULATION TIMING
% Initial and final simulation times [s]
t0 = 0; tf = 30;
% Sampling time [s]
tau = 0.05;
% Larger sampling time for updating the inputs [s]
T = 0.2;
% Number of time steps
tsteps = floor((tf-t0)/tau);
% Discrete time vecotor of sampling time (tau)
dt = tau*(0:tsteps);

% COST FUNCTION
% Q and R matrices used in the cost function
% Use the matrices used in Quanser documentation except add ones for the
% velocities -> need Q to be positive definite
```

```
% Modified LQR cost matrices
Q_Mat_LQR = diag([250 30 1 1]);
R_Mat_LQR = 0.005*diag([1 1]);
nonLinearAdjustment = 0.01;


% INITIAL STATES
% xInit = [deg2rad(0) deg2rad(0) (0*pi/180)*10 (0*pi/180)*7]';
%xInit = [0 0 0 0]';

% MATRIX INITIALIZATION
% Actual states - state variable x
%xADP = zeros(n,tsteps+1);
xLQR = zeros(n,tsteps+1);
pose = zeros(n,tsteps+1);
% xLQR(:,1) = xInit;
% State errors for entire simulation
errorLQR = zeros(n,tsteps+1);
% Actual inputs
uLQR = zeros(m,tsteps+1);

% Desired states
% Uncomment for desired states of all zero
%xd = zeros(n,tsteps+1);
% Uncomment for desired states that switch constant values
%xd = [deg2rad(37)*ones(1,tsteps+1); deg2rad(80)*ones(1,tsteps+1); zeros(1,tsteps+1); zeros(1,tsteps+1)];
%xd = [deg2rad(10)*ones(1,1000), deg2rad(60)*ones(1,tsteps+1-1000); deg2rad(80)*ones(1,1500), deg2rad(-120)*ones(1,tsteps+1-1500); zeros(1,tsteps+1); zeros(1,tsteps+1)];
% Uncomment for desired states that are sine and cosine waves
 pitchSine = deg2rad(30)*sin(0.5*dt);
 yawCosine = deg2rad(60)*cos(0.5*dt);
%  pitchSquare = deg2rad(25)*square(0.5*dt);
%  yawSquare = deg2rad(-36)*square(0.25*dt);
% xd = [pitchSine; yawCosine; zeros(1,tsteps+1); zeros(1,tsteps+1)];
% xd = [pitchSquare; yawSquare; zeros(1,tsteps+1); zeros(1,tsteps+1)];



%% CALCULATE THE CONTROLLER GAIN MATRIX USING LQR
fprintf('WORKING ON LQR GAIN MATRIX...\n');
kLQR = lqr(A,B,Q_Mat_LQR,R_Mat_LQR);

pause(1);
%% connection routine

ip = '127.0.0.1';
port = 19999;

% create a handle to remote API
vrep = remApi('remoteApi'); % using the prototype file (remoteApiProto.m)
vrep.simxFinish(-1); % just in case, close all opened connections
clientID = vrep.simxStart(ip,port,true,true,5000,5);
```

```

%% simulation
if (clientID>-1) % if clientID exists
    disp('Connected to remote API server');

%Set up the handles
[rtn, aero] = vrep.simxGetObjectHandle(clientID,...
                                        'Aero',...
                                         vrep.simx_opmode_blocking)
[rtn, yJ] = vrep.simxGetObjectHandle(clientID,...
                                    'yawJoint',...
                                     vrep.simx_opmode_blocking)
[rtn, pJ] = vrep.simxGetObjectHandle(clientID,...
                                    'pitchJoint',...
                                    vrep.simx_opmode_blocking)

% setup joint position streaming
vrep.simxGetJointPosition(clientID,...
                           yJ,...
                           vrep.simx_opmode_streaming);
vrep.simxGetJointPosition(clientID,...
                           pJ,...
                           vrep.simx_opmode_streaming);

% add a delay to let the streaming operations initialize
pause(1);

[ret, pose(1,1)] = vrep.simxGetJointPosition(clientID,...
                                                pJ,...
                                                vrep.simx_opmode_buffer);

[ret, pose(2,1)] = vrep.simxGetJointPosition(clientID,...
                                                yJ,...
                                                vrep.simx_opmode_buffer);


pose(3,1) = 0;
pose(4,1) = 0;
xLQR(:,1) = pose(:,1);
% Initialize the error
errorLQR(:,1) = xd(:,1) - xLQR(:,1);


tic;
% RUN THE SIMULATION
% Start at the next time tau after time zero
for k = 2:tsteps+1
    % Current time
    t = (k-1)*tau; % generate value discrete time index

    % Step 1 - get/store the state of the quadcopter from VREP
    [ret, pose(1,k)] = vrep.simxGetJointPosition(clientID,...
                                                    pJ,...
                                                    vrep.simx_opmode_buffer)
```

```
    ;

     [ret, pose(2,k)] = vrep.simxGetJointPosition(clientID,...
                                                  yJ,...
                                                  vrep.simx_opmode_buffer)
    ;

     pose(3,k) =  (pose(1,k) -  pose(1,k-1))\tau;
     pose(4,k) = (pose(2,k) -  pose(2,k-1))\tau;


    xLQR(:,k) = pose(:,k);


    % EXACT 2-DOF MODEL
    % Take previous values to find the derivative of the exact model
    xdotNonLinearLQR = A*xLQR(:,k-1) + B*uLQR(:,k-1);
    % Update the states of the exact model
    % Use Euler integration to estimate the current states of the nonlinear
    % model
     xLQR(:,k) = xLQR(:,k-1) + xdotNonLinearLQR*tau; % + nonLinearAdjustment
    .*(2.*rand(4,1)-1);

%      xLQR(theta,k) = angleLimiterPitch(xLQR(theta,k));
    % Force the yaw angle to be in the range [-pi,pi]
     xLQR(psi,k) = angleLimiterYaw(xLQR(psi,k));

    % FIND THE ERROR BETWEEN THE EXACT STATES AND THE DESIRED STATES
    errorLQR(:,k) = xd(:,k) - xLQR(:,k);

    % Update the inputs using LQR
    uNewLQR = kLQR*errorLQR(:,k);
    % Limit the voltages
    uNewLQR(1) = sign(uNewLQR(1))*min(abs(uNewLQR(1)),maxVolt);
    uNewLQR(2) = sign(uNewLQR(2))*min(abs(uNewLQR(2)),maxVolt);
    % Update the input matrices
    uLQR(1,k) = uNewLQR(1);
    uLQR(2,k) = uNewLQR(2);

    % Step 3 - Update Inputs
    VREP_PACKET=[uNewLQR(1) uNewLQR(2) rad2deg(xd(1,k)) rad2deg(xd(2,k))];
 %   motorSpeeds=[20 0];
     packedData=vrep.simxPackFloats(VREP_PACKET);
     vrep.simxSetStringSignal(clientID,...
                              'MATLAB_SIG',...
                              packedData,...
                              vrep.simx_opmode_oneshot);

   pause(tau);

   t = t + tau;


end
```

```
toc;

%ret = 1;
while(ret ~= 0)
    [ret]=vrep.simxClearStringSignal(clientID,...
                                    'MATLAB_SIG',...
                                    vrep.simx_opmode_blocking);
end

else
    disp('Failed connecting to remote API server');
end % if (clientID>-1)

disp('Sim ended');

figure
plot(dt,rad2deg(pose(1,:))); hold on;
plot(dt,rad2deg(xd(1,:))); hold off;
title('pitch')

figure
plot(dt,rad2deg(pose(2,:))); hold on;
plot(dt,rad2deg(xd(2,:))); hold off;
title('yaw')

figure
plot(dt,uLQR(1,:)); hold on;
plot(dt,uLQR(2,:)); hold off;
title('Inputs')

% SAVE OUTPUTS FOR PLOTTING
% Save the time, error, input, states and desired states vectors
save([testName,'.mat'],'dt', 'xd','tau','errorLQR','uLQR','xLQR','R_Mat_LQR'
    ,'Q_Mat_LQR');
% Notify that the simulation is complete
disp('SIMULATION COMPLETE');

% ANGLE LIMITER FOR YAW FUNCTION
function angle = angleLimiterYaw(angle)
% This function limits the angle between -pi and pi
    angle = mod(angle, 2*pi);

    i=find(angle>pi);
    angle(i)=angle(i)-2*pi;

    i=find(angle<-pi);
    angle(i)=angle(i)+2*pi;
end
```

B.4.4 LQR 2-DOF Helicopter

```
 % Use exact model of the 2-DOF helicopter and the linearized model to find
% error data points
% Use the error data points directly in the neural network instead of
```

```
% randomizing the error
close all; clear; clc;

% Name of .mat file to save the data
testName = 'heli_joint_sqsq';

% LINEAR MODEL MATRIX PARAMETERS (2DoF Helicopter)
[ A, B ] = getAeroAB( 0 );

% 2-DOF QUANSER HELICOPTER PARAMETERS
% Maximum applied voltage for the rotor motors
maxVolt = 18;
% Number of state variables - theta, psi, dtheta, dpsi
n = 4;
theta = 1;    % Pitch
psi = 2;      % Yaw
thetaDot = 3; % Pitch angular velocity
psiDot = 4;   % Yaw angular velocity
% Number of inputs - Vp, Vy
m = 2;

% SIMULATION TIMING
% Initial and final simulation times [s]
t0 = 0; tf = 30;
% Sampling time [s]
tau = 0.05;
% Larger sampling time for updating the inputs [s]
T = 0.2;
% Number of time steps
tsteps = floor((tf-t0)/tau);
% Discrete time vecotor of sampling time (tau)
dt = tau*(0:tsteps);

% COST FUNCTION
% Q and R matrices used in the cost function
% Use the matrices used in Quanser documentation except add ones for the
% velocities -> need Q to be positive definite
% Modified LQR cost matrices
Q_Mat_LQR = diag([250 30 1 1]);
R_Mat_LQR = 0.005*diag([1 1]);
nonLinearAdjustment = 0.01;

% INITIAL STATES
% xInit = [deg2rad(0) deg2rad(0) (0*pi/180)*10 (0*pi/180)*7]';
%xInit = [0 0 0 0]';

% MATRIX INITIALIZATION
% Actual states - state variable x
xLQR = zeros(n,tsteps+1);
pose = zeros(n,tsteps+1);

% State errors for entire simulation
errorLQR = zeros(n,tsteps+1);
% Actual inputs
```

```
uLQR = zeros(m, tsteps+1);

% Desired states
% Uncomment for desired states of all zero
%xd = zeros(n,tsteps+1);
% Uncomment for desired states that switch constant values
%xd = [deg2rad(37)*ones(1,tsteps+1); deg2rad(80)*ones(1,tsteps+1); zeros(1,
    tsteps+1); zeros(1,tsteps+1)];
%xd = [deg2rad(10)*ones(1,1000), deg2rad(60)*ones(1,tsteps+1-1000); deg2rad
    (80)*ones(1,1500), deg2rad(-120)*ones(1,tsteps+1-1500); zeros(1,tsteps+1)
    ; zeros(1,tsteps+1)];
% Uncomment for desired states that are sine and cosine waves
% pitchSine = deg2rad(30)*sin(0.5*dt);
% yawCosine = deg2rad(60)*cos(0.5*dt);
  pitchSquare = deg2rad(25)*square(0.5*dt);
  yawSquare = deg2rad(-36)*square(0.25*dt);
 %xd = [pitchSine; yawCosine; zeros(1,tsteps+1); zeros(1,tsteps+1)];
 xd = [pitchSquare; yawSquare; zeros(1,tsteps+1); zeros(1,tsteps+1)];


%% CALCULATE THE CONTROLLER GAIN MATRIX USING LQR
fprintf('WORKING ON LQR GAIN MATRIX...\n');
kLQR = lqr(A,B,Q_Mat_LQR,R_Mat_LQR);

%% connection routine
ip = '127.0.0.1';
port = 19999;

% create a handle to remote API
vrep = remApi('remoteApi'); % using the prototype file (remoteApiProto.m)
vrep.simxFinish(-1); % just in case, close all opened connections
clientID = vrep.simxStart(ip,port,true,true,5000,5);

%% simulation
if (clientID>-1) % if clientID exists
    disp('Connected to remote API server');

%Set up the handles
[rtn, aero] = vrep.simxGetObjectHandle(clientID,...
                                        'Aero',...
                                         vrep.simx_opmode_blocking)
[rtn, yJ] = vrep.simxGetObjectHandle(clientID,...
                                     'yawJoint',...
                                      vrep.simx_opmode_blocking)
[rtn, pJ] = vrep.simxGetObjectHandle(clientID,...
                                     'pitchJoint',...
                                     vrep.simx_opmode_blocking)

% setup joint position streaming
vrep.simxGetJointPosition(clientID,...
                          yJ,...
                          vrep.simx_opmode_streaming);
vrep.simxGetJointPosition(clientID,...
                          pJ,...
```

```
                              vrep.simx_opmode_streaming);

% add a delay to let the streaming operations initialize
pause(1);

[ret, pose(1,1)] = vrep.simxGetJointPosition(clientID,...
                                               pJ,...
                                               vrep.simx_opmode_buffer);

[ret, pose(2,1)] = vrep.simxGetJointPosition(clientID,...
                                               yJ,...
                                               vrep.simx_opmode_buffer);


pose(3,1) = 0;
pose(4,1) = 0;
xLQR(:,1) = pose(:,1);
% Initialize the error
errorLQR(:,1) = xd(:,1) - xLQR(:,1);


tic;
% RUN THE SIMULATION
% Start at the next time tau after time zero
for k = 2:tsteps+1
    % Current time
    t = (k-1)*tau; % generate value discrete time index

    % Step 1 - get/store the state of the quadcopter from VREP
    [ret, pose(1,k)] = vrep.simxGetJointPosition(clientID,...
                                                   pJ,...
                                                   vrep.simx_opmode_buffer)
    ;

    [ret, pose(2,k)] = vrep.simxGetJointPosition(clientID,...
                                                   yJ,...
                                                   vrep.simx_opmode_buffer)
    ;

    pose(3,k) =  (pose(1,k) -  pose(1,k-1))\tau;
    pose(4,k) = (pose(2,k) -  pose(2,k-1))\tau;


    xLQR(:,k) = pose(:,k);


    % EXACT 2-DOF MODEL
    % Take previous values to find the derivative of the exact model
    xdotNonLinearLQR = A*xLQR(:,k-1) + B*uLQR(:,k-1);
    % Update the states of the exact model
    % Use Euler integration to estimate the current states of the nonlinear
    % model
     xLQR(:,k) = xLQR(:,k-1) + xdotNonLinearLQR*tau; % + nonLinearAdjustment
    .*(2.*rand(4,1)-1);
```

```

%       xLQR(theta,k) = angleLimiterPitch(xLQR(theta,k));
    % Force the yaw angle to be in the range [-pi,pi]
     xLQR(psi,k) = angleLimiterYaw(xLQR(psi,k));

    % FIND THE ERROR BETWEEN THE EXACT STATES AND THE DESIRED STATES
    errorLQR(:,k) = xd(:,k) - xLQR(:,k);

    % Update the inputs using LQR
    uNewLQR = kLQR*errorLQR(:,k);
    % Limit the voltages
    uNewLQR(1) = sign(uNewLQR(1))*min(abs(uNewLQR(1)),maxVolt);
    uNewLQR(2) = sign(uNewLQR(2))*min(abs(uNewLQR(2)),maxVolt);
    % Update the input matrices
    uLQR(1,k) = uNewLQR(1);
    uLQR(2,k) = uNewLQR(2);

    % Step 3 - Update Inputs
    Vin=[uNewLQR(1) uNewLQR(2) rad2deg(xd(1,k)) rad2deg(xd(2,k))];
 %   motorSpeeds=[20 0];
    packedData=vrep.simxPackFloats(Vin);
    vrep.simxSetStringSignal(clientID,...
                             'MATLAB_SIG',...
                             packedData,...
                             vrep.simx_opmode_oneshot);

   pause(tau);

   t = t + tau;


end
toc;

%ret = 1;
while(ret ~= 0)
    [ret]=vrep.simxClearStringSignal(clientID,...
                                     'MATLAB_SIG',...
                                     vrep.simx_opmode_blocking);
end

else
    disp('Failed connecting to remote API server');
end % if (clientID>-1)

disp('Sim ended');

figure
plot(dt,rad2deg(pose(1,:))); hold on;
plot(dt,rad2deg(xd(1,:))); hold off;
title('pitch')

figure
plot(dt,rad2deg(pose(2,:))); hold on;
```

```
plot(dt,rad2deg(xd(2,:))); hold off;
title('yaw')

figure
plot(dt,uLQR(1,:)); hold on;
plot(dt,uLQR(2,:)); hold off;
title('Inputs')

% SAVE OUTPUTS FOR PLOTTING
% Save the time, error, input, states and desired states vectors
save([testName,'.mat'],'dt', 'xd','tau','errorLQR','uLQR','xLQR','R_Mat_LQR'
    ,'Q_Mat_LQR');
% Notify that the simulation is complete
disp('SIMULATION COMPLETE');

% ANGLE LIMITER FOR YAW FUNCTION
function angle = angleLimiterYaw(angle)
% This function limits the angle between -pi and pi
    angle = mod(angle, 2*pi);

    i=find(angle>pi);
    angle(i)=angle(i)-2*pi;

    i=find(angle<-pi);
    angle(i)=angle(i)+2*pi;
end
```

B.4.5 ADP Half-Quadcopter

```
% Use exact model of the 2-DOF helicopter and the linearized model to find
% error data points
% Use the error data points directly in the neural network instead of
% randomizing the error
close all; clear; clc;
testName = 'half_joint_adp_sinesine';

% LINEAR MODEL MATRIX PARAMETERS
[ A, B ] = getAeroAB( 1 )
% 2-DOF QUANSER HELICOPTER PARAMETERS
% Maximum applied voltage for the rotor motors
maxVolt = 18;
% Number of state variables - theta, psi, dtheta, dpsi
n = 4;
theta = 1;    % Pitch
psi = 2;      % Yaw
thetaDot = 3; % Pitch angular velocity
psiDot = 4;   % Yaw angular velocity
% Number of inputs - Vp, Vy
m = 2;

% SIMULATION TIMING
% Initial and final simulation times [s]
t0 = 0; tf = 25;
% Sampling time [s]
```

```
tau = 0.05;
% Larger sampling time for updating the inputs [s]
T = 0.75;
% Number of time steps
tsteps = floor((tf-t0)/tau);
% Discrete time vecotor of sampling time (tau)
dt = tau*(0:tsteps);
% Number of equations for actor-critic neural network
% Number of training samples per T
nbar = floor(T/tau);

% COST FUNCTION
% Q and R matrices used in the cost function
%Q_Mat_ADP = diag([270 100 1 1]);
%R_Mat_ADP = 0.0001*diag([1 1]);
% Use the matrices used in Quanser documentation except add ones for the
% velocities -> need Q to be positive definite
Q_Mat_ADP = diag([200 75 1 1]);
R_Mat_ADP = 0.005*diag([1 1]);
% Modified LQR cost matrices
Q_Mat_LQR = diag([200 75 1 1]);
R_Mat_LQR = 0.005*diag([1 1]);
nonLinearAdjustment = 0.03;
% Name of .mat file to save the data




% MATRIX INITIALIZATION
% Actual states - state variable x
xADP = zeros(n,tsteps+1);
% State errors for entire simulation
errorADP = zeros(n,tsteps+1);
% Actual inputs
uADP = zeros(m,tsteps+1);

% Desired states
% Uncomment for desired states of all zero
%xd = zeros(n,tsteps+1);
% Uncomment for desired states that switch constant values
%xd = [deg2rad(37)*ones(1,tsteps+1); deg2rad(80)*ones(1,tsteps+1); zeros(1,
    tsteps+1); zeros(1,tsteps+1)];
%xd = [deg2rad(10)*ones(1,1000), deg2rad(60)*ones(1,tsteps+1-1000); deg2rad
    (80)*ones(1,1500), deg2rad(-120)*ones(1,tsteps+1-1500); zeros(1,tsteps+1)
    ; zeros(1,tsteps+1)];
% Uncomment for desired states that are sine and cosine waves
pitchSine = deg2rad(30)*sin(0.5*dt);
yawCosine = deg2rad(90)*cos(0.5*dt);
% pitchSquare = deg2rad(57)*square(0.5*dt);
% yawSquare = deg2rad(-36)*square(0.25*dt);
xd = [pitchSine; yawCosine; zeros(1,tsteps+1); zeros(1,tsteps+1)];
%xd = [pitchSquare; yawSquare; zeros(1,tsteps+1); zeros(1,tsteps+1)];


```

```

% FIND THE CRITIC NEURAL NETWORK WEIGHTS BEFORE APPLYING ANY INPUTS
% Calculate the critic weights
wcInit = quanserAEROCriticTuningInitial(A, B, ((2*pi).*rand(4,nbar)-pi),tau,
    R_Mat_ADP,Q_Mat_ADP,xd(:,1));
% Use the weights to determine the P matrix
P_Mat = [wcInit(5) wcInit(6) wcInit(7) wcInit(8);
         wcInit(6) wcInit(9) wcInit(10) wcInit(11);
         wcInit(7) wcInit(10) wcInit(12) wcInit(13);
         wcInit(8) wcInit(11) wcInit(13) wcInit(14)];
% Use P to determine the state-feedback gain
kADP = 0.5*(R_Mat_ADP^-1)*B'*P_Mat;





%% connection routine

ip = '127.0.0.1';
port = 19999;

% create a handle to remote API
vrep = remApi('remoteApi'); % using the prototype file (remoteApiProto.m)
vrep.simxFinish(-1); % just in case, close all opened connections
clientID = vrep.simxStart(ip,port,true,true,5000,5);

%% simulation
if (clientID>-1) % if clientID exists
    disp('Connected to remote API server');

%Set up the handles
[rtn, aero] = vrep.simxGetObjectHandle(clientID,...
                                        'Aero',...
                                         vrep.simx_opmode_blocking)
[rtn, yJ] = vrep.simxGetObjectHandle(clientID,...
                                     'yawJoint',...
                                      vrep.simx_opmode_blocking)
[rtn, pJ] = vrep.simxGetObjectHandle(clientID,...
                                     'pitchJoint',...
                                     vrep.simx_opmode_blocking)

% setup joint position streaming
vrep.simxGetJointPosition(clientID,...
                          yJ,...
                          vrep.simx_opmode_streaming);
vrep.simxGetJointPosition(clientID,...
                          pJ,...
                          vrep.simx_opmode_streaming);

% add a delay to let the streaming operations initialize
pause(1);

[ret, pose(1,1)] = vrep.simxGetJointPosition(clientID,...
```

```
                                                                    pJ,...
                                                                    vrep.simx_opmode_buffer);

[ret, pose(2,1)] = vrep.simxGetJointPosition(clientID,...
                                                                    yJ,...
                                                                    vrep.simx_opmode_buffer);


pose(3,1) = 0;
pose(4,1) = 0;
xADP(:,1) = pose(:,1);
% Initialize the error
errorADP(:,1) = xd(:,1) - xADP(:,1);



% RUN THE SIMULATION
% Start at the next time tau after time zero


% RUN THE SIMULATION
% Start at the next time tau after time zero
tic;
for k = 2:tsteps+1
    % Current time
    t = (k-1)*tau; % generate value discrete time index

    % Step 1 - get/store the state of the quadcopter from VREP
    [ret, pose(1,k)] = vrep.simxGetJointPosition(clientID,...
                                                                    pJ,...
                                                                    vrep.simx_opmode_buffer)
    ;

    [ret, pose(2,k)] = vrep.simxGetJointPosition(clientID,...
                                                                    yJ,...
                                                                    vrep.simx_opmode_buffer)
    ;

    pose(3,k) =  (pose(1,k) -  pose(1,k-1))/tau;
    pose(4,k) = (pose(2,k) -  pose(2,k-1))/tau;


    xADP(:,k) = pose(:,k);
    % EXACT 2-DOF HELICOPTER MODEL
    % Take previous values to find the derivative of the exact model
    xdotNonLinearADP = A*xADP(:,k-1) + B*uADP(:,k-1);
    % Update the states of the exact model
    % Use Euler integration to estimate the current states of the nonlinear
    % model
    xADP(:,k) = xADP(:,k-1) + xdotNonLinearADP*tau + nonLinearAdjustment
    .*(2.*rand(4,1)-1);
    % ADD SOME DISTURBANCE TO THE STATES
    % This forces the states to some other position
    %if ((t > 15) && (t < 17))
```

```
        %xADP(:,k) = [rad2deg(-45) 0 0 0]';
        %xLQR(:,k) = [rad2deg(-45) 0 0 0]';
    %end
    % Force the pitch angle to be in the range [-pi/2,pi/2] because of
    % physical constraints
    xADP(theta,k) = angleLimiterPitch(xADP(theta,k));
    % Force the yaw angle to be in the range [-pi,pi] because the yaw is
    % free to do a complete circle
    xADP(psi,k) = angleLimiterYaw(xADP(psi,k));

    % FIND THE ERROR BETWEEN THE EXACT STATES AND THE DESIRED STATES
    errorADP(:,k) = xd(:,k) - xADP(:,k);

    % UPDATE THE CRITIC WEIGHTS EVERY T TIME
    % Determine if the time is a multiple of T
    if (mod(t,T) == 0)
        % Update the critic weights
        wc = quanserAEROCriticTuning(A, B, errorADP(:,(k-nbar):k),tau,
    R_Mat_ADP,Q_Mat_ADP,xd(:,k),wcInit);
        % Use the weights to determine the P matrix
        P_Mat = [wc(5) wc(6) wc(7) wc(8);
                 wc(6) wc(9) wc(10) wc(11);
                 wc(7) wc(10) wc(12) wc(13);
                 wc(8) wc(11) wc(13) wc(14)];
        % Use P to determine the state-feedback gain
        kADP = 0.5*(R_Mat_ADP^-1)*B'*P_Mat;
    end
    % Update the inputs using ADP
    uNewADP = kADP*errorADP(:,k);
    % Limit the voltages
    uNewADP(1) = sign(uNewADP(1))*min(abs(uNewADP(1)),maxVolt);
    uNewADP(2) = sign(uNewADP(2))*min(abs(uNewADP(2)),maxVolt);
    % Update the input matrices
    uADP(1,k) = uNewADP(1);
    uADP(2,k) = uNewADP(2);

    Vin=[uNewADP(1) uNewADP(2) rad2deg(xd(1,k)) rad2deg(xd(2,k))];
    packedData=vrep.simxPackFloats(Vin);
    vrep.simxSetStringSignal(clientID,...
                             'MATLAB_SIG',...
                             packedData,...
                             vrep.simx_opmode_oneshot);

   pause(tau);

   t = t + tau;


end
toc;

%ret = 1;
while(ret ~= 0)
    [ret]=vrep.simxClearStringSignal(clientID,...
```

```
                                            'MATLAB_SIG' ,...
                                            vrep.simx_opmode_blocking);
end

else
    disp('Failed connecting to remote API server');
end % if (clientID>-1)

disp('Sim ended');


figure
plot(dt,rad2deg(pose(1,:))); hold on;
plot(dt,rad2deg(xd(1,:))); hold off;
title('pitch')

figure
plot(dt,rad2deg(pose(2,:))); hold on;
plot(dt,rad2deg(xd(2,:))); hold off;
title('yaw')

figure
plot(dt,uADP(1,:)); hold on;
plot(dt,uADP(2,:)); hold off;
title('Inputs')

% SAVE OUTPUTS FOR PLOTTING
% Save the time, error, input, states and desired states vectors
save([testName,'.mat'],'dt','errorADP','uADP','xADP','xd','tau','R_Mat_ADP',
    'Q_Mat_ADP');
% Notify that the simulation is complete
disp('SIMULATION COMPLETE');

% ANGLE LIMITER FOR YAW FUNCTION
function angle = angleLimiterYaw(angle)
% This function limits the angle between -pi and pi
    angle = mod(angle, 2*pi);

    i=find(angle>pi);
    angle(i)=angle(i)-2*pi;

    i=find(angle<-pi);
    angle(i)=angle(i)+2*pi;
end

% ANGLE LIMITER FOR PITCH FUNCTION
function angle = angleLimiterPitch(angle)
% This function limits the physical constraint of the pitch measurement to
% 90 degrees
    if (angle < 0)
        angle = max(angle,-pi/2);
    end

    if (angle > 0)
```

```
            angle = min(angle,pi/2);
        end
end

% CRITIC WEIGHT TUNING NEURAL NETWORK
% This function is specific to the helicopter because of the number of
% weights and error model
function weights = quanserAEROCriticTuningInitial(A, B, e_vec,tau,R_Mat,
    Q_Mat,xd)
    % REFERENCE DR. MIAH'S PAPER FOR EQUATION NUMBERS

    % SYSTEM PARAMETERS SPECIFIC TO THE 2-DOF QUANSER AERO
%     A = [0 0 1 0; 0 0 0 1; -1.7442 0 -0.3307 0; 0 0 0 -0.9283];
%     B = [0 0; 0 0; 0.0512 0.0977; -0.1139 0.0928];
    % System dimensions specific for our model
    [n,~] = size(B);

    % ERROR MODEL OF THE HELICOPTER
    % fbar and gbar -- EQ 8
    fbar = @(e) A*e;
    gbar = -B;
    hbar = -A*xd;

    % DISCRETE-TIME ERROR MODEL FOR TIME TAU
    % f and g -- EQ 10
    f = @(e) fbar(e)*tau + e;
    g = gbar*tau;
    h = hbar*tau;

    % COST FUNCTION PARAMETERS
    % State penalizing function in the continuous cost function
    Qbar = @(e) e'*Q_Mat*e;
    % Control penalizing matrix in the continuous cost function
    Rbar = R_Mat;
    % The discrete-time cost function will have terms:
    % Right after EQ 11 in paper
    % State penalizing function in the discretized cost function
    Q = @(e) Qbar(e)*tau;
    % Control penalizing matrix in the discretized cost function
    R = Rbar*tau;

    % NEURAL NETWORK FUNCTIONS
    % Critic neural network activation functions
    rho = @(e) [e(1); e(2); e(3); e(4);...
                e(1)^2; e(1)*e(2); e(1)*e(3); e(1)*e(4);...
                e(2)^2; e(2)*e(3); e(2)*e(4); e(3)^2; e(3)*e(4); e(4)^2];
    % Partial derivative of rho with respect to e
    drhode =@(e) [1, 0, 0, 0;
                  0, 1, 0, 0;
                  0, 0, 1, 0;
                  0, 0, 0, 1;
                  2*e(1), 0, 0, 0;
                  e(2), e(1), 0, 0;
                  e(3), 0, e(1), 0;
```

```
                    e(4), 0, 0, e(1);
                    0, 2*e(2), 0, 0;
                    0, e(3), e(2), 0;
                    0, e(4), 0, e(2);
                    0, 0, 2*e(3), 0;
                    0, 0, e(4), e(3);
                    0, 0, 0, 2*e(4)];

    % TOLERANCES
    % Convergence tolerance for control policy
    EpsilonPolicy = 0.1;
    % Convergence tolerance for critic neural network
    EpsilonWcritic = 0.1;

    % TRAINING PARAMETERS
    % Number of outer loop iterations
    outerLoopMax = 700;
    % Number of inner loop iterations
    innerLoopMax = 100;
    % Number of equations needed for training, number of sub-intervals
    % Number of training samples
    [~,nbar] = size(e_vec);

    % WEIGHT INITIALIZATION
    % Initialize the weights of the critic neural network to zero
    WcLast = zeros(length(rho(e_vec(:,1))),1);

    % LEAST-SQUARES COMPUTATION INITIALIZATION
    % Matrices required for computing least squares weights of the critic
    % neural networks -- EQ 20
    V = zeros(nbar,1);
    Lambda = zeros(nbar,length(rho(e_vec(:,1))));

    % Matrix to hold the derivative of the error model during policy
    % updating
    e_k_plus_1 = zeros(n,nbar);

    % Product of the least squares matrices must be invertible
    % Logic flag indicating if the critic weights are unsolvable
    % The weights are unsolvable because the least squares matrices have no
    % solution -- not invertible
    diverged = 0;

    % OUTER LOOP
    for i = 1:(outerLoopMax-1)
    % Determine if the least squares matrices are invertible
    if diverged == 0
        % For each of the data collection (discrete time index)
        for k = 1:nbar
            % Initialize the optimal inputs to zero
            uNew = [0; 0];
            % INNER LOOP
            for j = 1:(innerLoopMax-1)
                % Get the updated input value
```

```
                uLast = uNew;
                % Update the error model
                e_k_plus_1(:,k) = f(e_vec(:,k)) + g*uLast + h;
                % Compute the new optimal inputs
                uNew  = -0.5*(R^(-1))*g'*drhode(e_k_plus_1(:,k))'*WcLast;

                % Check convergence of the optimal inputs
                if norm(uNew - uLast) < EpsilonPolicy
                    break;
                end
            end

            % Update the values for the least-squares computation
            V(k,:) = Q(e_vec(:,k)) + uNew'*R*uNew + WcLast'*rho(e_k_plus_1(:,k));
            Lambda(k,:) = rho(e_vec(:,k))';
        end
    end

    % Verify the least square solution exists for the critic's weights
    % If the error data is consistent or there is no error, this will not
    % hold, so set the weights to zero
    if det(Lambda'*Lambda) == 0
        fprintf('AWESOME...YOU HAVE NO ERROR...I''M GOING TO SET THE WEIGHTS TO ZERO\n');
        weights = zeros(length(rho(e_vec(:,1))),1);
        break;
    end;

    % Calculate least squares solution of critic's weights -- EQ 20
    WcNew = (Lambda'*Lambda)^(-1)*Lambda'*V;
    % Make sure the weights did not diverge
    % If the weights are diverging, just set them to a large number
    if isnan(WcNew)
        fprintf('OOPS...DIVERGING WEIGHTS...I''M GOING TO USE LARGE WEIGHTS\n');
        weights = 1000*ones(length(rho(e_vec(:,1))),1);
        break;
    end;

    % Check for convergence of the critic weights
    if norm(WcNew - WcLast) < EpsilonWcritic
        weights = WcNew;
        fprintf('GREAT...THE WEIGHTS CONVERGED\n');
        break;
    end
    % If the weights did not converge, repeat the loop
    WcLast = WcNew;

    % If we reached the last iteration of the loop, just use the last
    % weights found
    if (i == (outerLoopMax-1))
        fprintf('OOPS...YOU REACHED THE END OF THE OUTER LOOP...I''M GOING TO USE THE LAST VALUE\n');
```

```
        weights = WcNew;
    end
    end
end

% CRITIC WEIGHT TUNING NEURAL NETWORK
% This function is specific to the helicopter because of the number of
% weights and error model
function weights = quanserAEROCriticTuning(A, B, e_vec,tau,R_Mat,Q_Mat,xd,
    wcInit)
    % REFERENCE DR. MIAH'S PAPER FOR EQUATION NUMBERS

    % SYSTEM PARAMETERS SPECIFIC TO THE 2-DOF QUANSER AERO
%     A = [0 0 1 0; 0 0 0 1; -1.7442 0 -0.3307 0; 0 0 0 -0.9283];
%     B = [0 0; 0 0; 0.0512 0.0977; -0.1139 0.0928];
    % System dimensions specific for our model
    [n,~] = size(B);

    % ERROR MODEL OF THE HELICOPTER
    % fbar and gbar -- EQ 8
    fbar = @(e) A*e;
    gbar = -B;
    hbar = -A*xd;

    % DISCRETE-TIME ERROR MODEL FOR TIME TAU
    % f and g -- EQ 10
    f = @(e) fbar(e)*tau + e;
    g = gbar*tau;
    h = hbar*tau;

    % COST FUNCTION PARAMETERS
    % State penalizing function in the continuous cost function
    Qbar = @(e) e'*Q_Mat*e;
    % Control penalizing matrix in the continuous cost function
    Rbar = R_Mat;
    % The discrete-time cost function will have terms:
    % Right after EQ 11 in paper
    % State penalizing function in the discretized cost function
    Q = @(e) Qbar(e)*tau;
    % Control penalizing matrix in the discretized cost function
    R = Rbar*tau;

    % NEURAL NETWORK FUNCTIONS
    % Critic neural network activation functions
    rho = @(e) [e(1); e(2); e(3); e(4);...
                e(1)^2; e(1)*e(2); e(1)*e(3); e(1)*e(4);...
                e(2)^2; e(2)*e(3); e(2)*e(4); e(3)^2; e(3)*e(4); e(4)^2];
    % Partial derivative of rho with respect to e
    drhode =@(e) [1, 0, 0, 0;
                  0, 1, 0, 0;
                  0, 0, 1, 0;
                  0, 0, 0, 1;
                  2*e(1), 0, 0, 0;
                  e(2), e(1), 0, 0;
```

```
                    e(3), 0, e(1), 0;
                    e(4), 0, 0, e(1);
                    0, 2*e(2), 0, 0;
                    0, e(3), e(2), 0;
                    0, e(4), 0, e(2);
                    0, 0, 2*e(3), 0;
                    0, 0, e(4), e(3);
                    0, 0, 0, 2*e(4)];

    % TOLERANCES
    % Convergence tolerance for control policy
    EpsilonPolicy = 0.1;
    % Convergence tolerance for critic neural network
    EpsilonWcritic = 0.1;

    % TRAINING PARAMETERS
    % Number of outer loop iterations
    outerLoopMax = 100;
    % Number of inner loop iterations
    innerLoopMax = 100;
    % Number of equations needed for training, number of sub-intervals
    % Number of training samples
    [~,nbar] = size(e_vec);

    % WEIGHT INITIALIZATION
    % Initialize the weights of the critic neural network to zero
    WcLast = zeros(length(rho(e_vec(:,1))),1);

    % LEAST-SQUARES COMPUTATION INITIALIZATION
    % Matrices required for computing least squares weights of the critic
    % neural networks -- EQ 20
    V = zeros(nbar,1);
    Lambda = zeros(nbar,length(rho(e_vec(:,1))));

    % Matrix to hold the derivative of the error model during policy
    % updating
    e_k_plus_1 = zeros(n,nbar);

    % Product of the least squares matrices must be invertible
    % Logic flag indicating if the critic weights are unsolvable
    % The weights are unsolvable because the least squares matrices have no
    % solution -- not invertible
    diverged = 0;

    % OUTER LOOP
    for i = 1:(outerLoopMax-1)
    % Determine if the least squares matrices are invertible
    if diverged == 0
        % For each of the data collection (discrete time index)
        for k = 1:nbar
            % Initialize the optimal inputs to zero
            uNew = [0; 0];
            % INNER LOOP
            for j = 1:(innerLoopMax-1)
```

```
                % Get the updated input value
                uLast = uNew;
                % Update the error model
                e_k_plus_1(:,k) = f(e_vec(:,k)) + g*uLast + h;
                % Compute the new optimal inputs
                uNew  = -0.5*(R^(-1))*g'*drhode(e_k_plus_1(:,k))'*WcLast;

                % Check convergence of the optimal inputs
                if norm(uNew - uLast) < EpsilonPolicy
                    break;
                end
            end

            % Update the values for the least-squares computation
            V(k,:) = Q(e_vec(:,k)) + uNew'*R*uNew + WcLast'*rho(e_k_plus_1
    (:,k));
            Lambda(k,:) = rho(e_vec(:,k))';
        end
    end

    % Verify the least square solution exists for the critic's weights
    % If the error data is consistent or there is no error, this will not
    % hold, so set the weights to what they were initially before the
    % simulation
    if det(Lambda'*Lambda) == 0
        %fprintf('AWESOME...YOU HAVE NO ERROR...I''M GOING TO USE THE
    ORIGINAL WEIGHTS\n');
        weights = wcInit;
        break;
    end;

    % Calulcate least squares solution of critic's weights -- EQ 20
    WcNew = (Lambda'*Lambda)^(-1)*Lambda'*V;
    % Make sure the weights did not diverge
    % If the weights diverged, set the weights to what they were initially
    % before the simulation
    if isnan(WcNew)
        %fprintf('OOPS...DIVERGING WEIGHTS...I''M GOING TO USE THE ORIGINAL
    WEIGHTS\n');
        weights = wcInit;
        break;
    end;

    % Check for convergence of the critic weights
    if norm(WcNew - WcLast) < EpsilonWcritic
        %fprintf('GREAT...THE WEIGHTS CONVERGED\n');
        weights = WcNew;
        break;
    end
    % If the weights did not converge, do another iteration of the loop
    WcLast = WcNew;

    % If we reach the last iteration of the loop, just use the last weights
    if (i == (outerLoopMax-1))
```

```
        %fprintf('OOPS...YOU REACHED THE END OF THE OUTER LOOP...I''M GOING
    TO USE THE LAST VALUE\n');
        weights = WcNew;
    end
    end
end
```

B.4.6 ADP 2-DOF Helicopter

```
% Use exact model of the 2-DOF helicopter and the linearized model to find
% error data points
% Use the error data points directly in the neural network instead of
% randomizing the error
close all; clear; clc;
testName = 'heli_joint_adp_sinesine';

% LINEAR MODEL MATRIX PARAMETERS
[ A, B ] = getAeroAB( 0 )
% 2-DOF QUANSER HELICOPTER PARAMETERS
% Maximum applied voltage for the rotor motors
maxVolt = 18;
% Number of state variables - theta, psi, dtheta, dpsi
n = 4;
theta = 1;    % Pitch
psi = 2;      % Yaw
thetaDot = 3; % Pitch angular velocity
psiDot = 4;   % Yaw angular velocity
% Number of inputs - Vp, Vy
m = 2;

% SIMULATION TIMING
% Initial and final simulation times [s]
t0 = 0; tf = 25;
% Sampling time [s]
tau = 0.05;
% Larger sampling time for updating the inputs [s]
T = 0.75;
% Number of time steps
tsteps = floor((tf-t0)/tau);
% Discrete time vecotor of sampling time (tau)
dt = tau*(0:tsteps);
% Number of equations for actor-critic neural network
% Number of training samples per T
nbar = floor(T/tau);

% COST FUNCTION
% Q and R matrices used in the cost function
%Q_Mat_ADP = diag([270 100 1 1]);
%R_Mat_ADP = 0.0001*diag([1 1]);
% Use the matrices used in Quanser documentation except add ones for the
% velocities -> need Q to be positive definite
Q_Mat_ADP = diag([200 75 1 1]);
R_Mat_ADP = 0.005*diag([1 1]);
% Modified LQR cost matrices
```

```
Q_Mat_LQR = diag([200 75 1 1]);
R_Mat_LQR = 0.005*diag([1 1]);
nonLinearAdjustment = 0.03;
% Name of .mat file to save the data




% MATRIX INITIALIZATION
% Actual states - state variable x
xADP = zeros(n,tsteps+1);
% State errors for entire simulation
errorADP = zeros(n,tsteps+1);
% Actual inputs
uADP = zeros(m,tsteps+1);

% Desired states
% Uncomment for desired states of all zero
%xd = zeros(n,tsteps+1);
% Uncomment for desired states that switch constant values
%xd = [deg2rad(37)*ones(1,tsteps+1); deg2rad(80)*ones(1,tsteps+1); zeros(1,
    tsteps+1); zeros(1,tsteps+1)];
%xd = [deg2rad(10)*ones(1,1000), deg2rad(60)*ones(1,tsteps+1-1000); deg2rad
    (80)*ones(1,1500), deg2rad(-120)*ones(1,tsteps+1-1500); zeros(1,tsteps+1)
    ; zeros(1,tsteps+1)];
% Uncomment for desired states that are sine and cosine waves
pitchSine = deg2rad(60)*sin(0.5*dt);
yawCosine = deg2rad(90)*cos(0.5*dt);
% pitchSquare = deg2rad(57)*square(0.5*dt);
% yawSquare = deg2rad(-36)*square(0.25*dt);
xd = [pitchSine; yawCosine; zeros(1,tsteps+1); zeros(1,tsteps+1)];
%xd = [pitchSquare; yawSquare; zeros(1,tsteps+1); zeros(1,tsteps+1)];



% FIND THE CRITIC NEURAL NETWORK WEIGHTS BEFORE APPLYING ANY INPUTS
% Calculate the critic weights
wcInit = quanserAEROCriticTuningInitial(A, B, ((2*pi).*rand(4,nbar)-pi),tau,
    R_Mat_ADP,Q_Mat_ADP,xd(:,1));
% Use the weights to determine the P matrix
P_Mat = [wcInit(5) wcInit(6) wcInit(7) wcInit(8);
         wcInit(6) wcInit(9) wcInit(10) wcInit(11);
         wcInit(7) wcInit(10) wcInit(12) wcInit(13);
         wcInit(8) wcInit(11) wcInit(13) wcInit(14)];
% Use P to determine the state-feedback gain
kADP = 0.5*(R_Mat_ADP^-1)*B'*P_Mat;





%% connection routine

ip = '127.0.0.1';
```

```
port = 19999;

% create a handle to remote API
vrep = remApi('remoteApi'); % using the prototype file (remoteApiProto.m)
vrep.simxFinish(-1); % just in case, close all opened connections
clientID = vrep.simxStart(ip,port,true,true,5000,5);

%% simulation
if (clientID>-1) % if clientID exists
    disp('Connected to remote API server');

%Set up the handles
[rtn, aero] = vrep.simxGetObjectHandle(clientID,...
                                        'Aero',...
                                         vrep.simx_opmode_blocking)
[rtn, yJ] = vrep.simxGetObjectHandle(clientID,...
                                    'yawJoint',...
                                     vrep.simx_opmode_blocking)
[rtn, pJ] = vrep.simxGetObjectHandle(clientID,...
                                    'pitchJoint',...
                                    vrep.simx_opmode_blocking)

% setup joint position streaming
vrep.simxGetJointPosition(clientID,...
                            yJ,...
                            vrep.simx_opmode_streaming);
vrep.simxGetJointPosition(clientID,...
                            pJ,...
                            vrep.simx_opmode_streaming);

% add a delay to let the streaming operations initialize
pause(1);

[ret, pose(1,1)] = vrep.simxGetJointPosition(clientID,...
                                                pJ,...
                                                vrep.simx_opmode_buffer);

[ret, pose(2,1)] = vrep.simxGetJointPosition(clientID,...
                                                yJ,...
                                                vrep.simx_opmode_buffer);


pose(3,1) = 0;
pose(4,1) = 0;
xADP(:,1) = pose(:,1);
% Initialize the error
errorADP(:,1) = xd(:,1) - xADP(:,1);



% RUN THE SIMULATION
% Start at the next time tau after time zero


```

```
% RUN THE SIMULATION
% Start at the next time tau after time zero
tic;
for k = 2:tsteps+1
    % Current time
    t = (k-1)*tau; % generate value discrete time index

    % Step 1 - get/store the state of the quadcopter from VREP
    [ret, pose(1,k)] = vrep.simxGetJointPosition(clientID,...
                                                   pJ,...
                                                   vrep.simx_opmode_buffer)
    ;

    [ret, pose(2,k)] = vrep.simxGetJointPosition(clientID,...
                                                   yJ,...
                                                   vrep.simx_opmode_buffer)
    ;

    pose(3,k) =  (pose(1,k) -  pose(1,k-1))\tau;
    pose(4,k) = (pose(2,k) -  pose(2,k-1))\tau;


    xADP(:,k) = pose(:,k);
    % EXACT 2-DOF HELICOPTER MODEL
    % Take previous values to find the derivative of the exact model
    xdotNonLinearADP = A*xADP(:,k-1) + B*uADP(:,k-1);
    % Update the states of the exact model
    % Use Euler integration to estimate the current states of the nonlinear
    % model
    xADP(:,k) = xADP(:,k-1) + xdotNonLinearADP*tau + nonLinearAdjustment
    .*(2.*rand(4,1)-1);
    % ADD SOME DISTURBANCE TO THE STATES
    % This forces the states to some other position
    %if ((t > 15) && (t < 17))
        %xADP(:,k) = [rad2deg(-45) 0 0 0]';
        %xLQR(:,k) = [rad2deg(-45) 0 0 0]';
    %end
    % Force the pitch angle to be in the range [-pi/2,pi/2] because of
    % physical constraints
    xADP(theta,k) = angleLimiterPitch(xADP(theta,k));
    % Force the yaw angle to be in the range [-pi,pi] because the yaw is
    % free to do a complete circle
    xADP(psi,k) = angleLimiterYaw(xADP(psi,k));

    % FIND THE ERROR BETWEEN THE EXACT STATES AND THE DESIRED STATES
    errorADP(:,k) = xd(:,k) - xADP(:,k);

    % UPDATE THE CRITIC WEIGHTS EVERY T TIME
    % Determine if the time is a multiple of T
    if (mod(t,T) == 0)
        % Update the critic weights
        wc = quanserAEROCriticTuning(A, B, errorADP(:,(k-nbar):k),tau,
    R_Mat_ADP,Q_Mat_ADP,xd(:,k),wcInit);
        % Use the weights to determine the P matrix
```

```
        P_Mat = [wc(5) wc(6) wc(7) wc(8);
                 wc(6) wc(9) wc(10) wc(11);
                 wc(7) wc(10) wc(12) wc(13);
                 wc(8) wc(11) wc(13) wc(14)];
        % Use P to determine the state-feedback gain
        kADP = 0.5*(R_Mat_ADP^-1)*B'*P_Mat;
    end
    % Update the inputs using ADP
    uNewADP = kADP*errorADP(:,k);
    % Limit the voltages
    uNewADP(1) = sign(uNewADP(1))*min(abs(uNewADP(1)),maxVolt);
    uNewADP(2) = sign(uNewADP(2))*min(abs(uNewADP(2)),maxVolt);
    % Update the input matrices
    uADP(1,k) = uNewADP(1);
    uADP(2,k) = uNewADP(2);

    Vin=[uNewADP(1) uNewADP(2) rad2deg(xd(1,k)) rad2deg(xd(2,k))];
    packedData=vrep.simxPackFloats(Vin);
    vrep.simxSetStringSignal(clientID,...
                             'MATLAB_SIG',...
                             packedData,...
                             vrep.simx_opmode_oneshot);

   pause(tau);

   t = t + tau;


end
toc;

%ret = 1;
while(ret ~= 0)
    [ret]=vrep.simxClearStringSignal(clientID,...
                                     'MATLAB_SIG',...
                                     vrep.simx_opmode_blocking);
end

else
    disp('Failed connecting to remote API server');
end % if (clientID>-1)

disp('Sim ended');


figure
plot(dt,rad2deg(pose(1,:))); hold on;
plot(dt,rad2deg(xd(1,:))); hold off;
title('pitch')

figure
plot(dt,rad2deg(pose(2,:))); hold on;
plot(dt,rad2deg(xd(2,:))); hold off;
title('yaw')
```

```
figure
plot(dt,uADP(1,:)); hold on;
plot(dt,uADP(2,:)); hold off;
title('Inputs')

% SAVE OUTPUTS FOR PLOTTING
% Save the time, error, input, states and desired states vectors
save([testName,'.mat'],'dt','errorADP','uADP','xADP','xd','tau','R_Mat_ADP',
    'Q_Mat_ADP');
% Notify that the simulation is complete
disp('SIMULATION COMPLETE');

% ANGLE LIMITER FOR YAW FUNCTION
function angle = angleLimiterYaw(angle)
% This function limits the angle between -pi and pi
    angle = mod(angle, 2*pi);

    i=find(angle>pi);
    angle(i)=angle(i)-2*pi;

    i=find(angle<-pi);
    angle(i)=angle(i)+2*pi;
end

% ANGLE LIMITER FOR PITCH FUNCTION
function angle = angleLimiterPitch(angle)
% This function limits the physical constraint of the pitch measurement to
% 90 degrees
    if (angle < 0)
        angle = max(angle,-pi/2);
    end

    if (angle > 0)
        angle = min(angle,pi/2);
    end
end

% CRITIC WEIGHT TUNING NEURAL NETWORK
% This function is specific to the helicopter because of the number of
% weights and error model
function weights = quanserAEROCriticTuningInitial(A, B, e_vec,tau,R_Mat,
    Q_Mat,xd)
    % REFERENCE DR. MIAH'S PAPER FOR EQUATION NUMBERS

    % SYSTEM PARAMETERS SPECIFIC TO THE 2-DOF QUANSER AERO
%     A = [0 0 1 0; 0 0 0 1; -1.7442 0 -0.3307 0; 0 0 0 -0.9283];
%     B = [0 0; 0 0; 0.0512 0.0977; -0.1139 0.0928];
    % System dimensions specific for our model
    [n,~] = size(B);

    % ERROR MODEL OF THE HELICOPTER
    % fbar and gbar -- EQ 8
    fbar = @(e) A*e;
```

```
    gbar = -B;
    hbar = -A*xd;

    % DISCRETE-TIME ERROR MODEL FOR TIME TAU
    % f and g -- EQ 10
    f = @(e) fbar(e)*tau + e;
    g = gbar*tau;
    h = hbar*tau;

    % COST FUNCTION PARAMETERS
    % State penalizing function in the continuous cost function
    Qbar = @(e) e'*Q_Mat*e;
    % Control penalizing matrix in the continuous cost function
    Rbar = R_Mat;
    % The discrete-time cost function will have terms:
    % Right after EQ 11 in paper
    % State penalizing function in the discretized cost function
    Q = @(e) Qbar(e)*tau;
    % Control penalizing matrix in the discretized cost function
    R = Rbar*tau;

    % NEURAL NETWORK FUNCTIONS
    % Critic neural network activation functions
    rho = @(e) [e(1); e(2); e(3); e(4);...
                e(1)^2; e(1)*e(2); e(1)*e(3); e(1)*e(4);...
                e(2)^2; e(2)*e(3); e(2)*e(4); e(3)^2; e(3)*e(4); e(4)^2];
    % Partial derivative of rho with respect to e
    drhode =@(e) [1, 0, 0, 0;
                  0, 1, 0, 0;
                  0, 0, 1, 0;
                  0, 0, 0, 1;
                  2*e(1), 0, 0, 0;
                  e(2), e(1), 0, 0;
                  e(3), 0, e(1), 0;
                  e(4), 0, 0, e(1);
                  0, 2*e(2), 0, 0;
                  0, e(3), e(2), 0;
                  0, e(4), 0, e(2);
                  0, 0, 2*e(3), 0;
                  0, 0, e(4), e(3);
                  0, 0, 0, 2*e(4)];

    % TOLERANCES
    % Convergence tolerance for control policy
    EpsilonPolicy = 0.1;
    % Convergence tolerance for critic neural network
    EpsilonWcritic = 0.1;

    % TRAINING PARAMETERS
    % Number of outer loop iterations
    outerLoopMax = 700;
    % Number of inner loop iterations
    innerLoopMax = 100;
    % Number of equations needed for training, number of sub-intervals
```

```
    % Number of training samples
    [~,nbar] = size(e_vec);

    % WEIGHT INITIALIZATION
    % Initialize the weights of the critic neural network to zero
    WcLast = zeros(length(rho(e_vec(:,1))),1);

    % LEAST-SQUARES COMPUTATION INITIALIZATION
    % Matrices required for computing least squares weights of the critic
    % neural networks -- EQ 20
    V = zeros(nbar,1);
    Lambda = zeros(nbar,length(rho(e_vec(:,1))));

    % Matrix to hold the derivative of the error model during policy
    % updating
    e_k_plus_1 = zeros(n,nbar);

    % Product of the least squares matrices must be invertible
    % Logic flag indicating if the critic weights are unsolvable
    % The weights are unsolvable because the least squares matrices have no
    % solution -- not invertible
    diverged = 0;

    % OUTER LOOP
    for i = 1:(outerLoopMax-1)
    % Determine if the least squares matrices are invertible
    if diverged == 0
        % For each of the data collection (discrete time index)
        for k = 1:nbar
            % Initialize the optimal inputs to zero
            uNew = [0; 0];
            % INNER LOOP
            for j = 1:(innerLoopMax-1)
                % Get the updated input value
                uLast = uNew;
                % Update the error model
                e_k_plus_1(:,k) = f(e_vec(:,k)) + g*uLast + h;
                % Compute the new optimal inputs
                uNew  = -0.5*(R^(-1))*g'*drhode(e_k_plus_1(:,k))'*WcLast;

                % Check convergence of the optimal inputs
                if norm(uNew - uLast) < EpsilonPolicy
                    break;
                end
            end

            % Update the values for the least-squares computation
            V(k,:) = Q(e_vec(:,k)) + uNew'*R*uNew + WcLast'*rho(e_k_plus_1
   (:,k));
            Lambda(k,:) = rho(e_vec(:,k))';
        end
    end

    % Verify the least square solution exists for the critic's weights
```

```
    % If the error data is consistent or there is no error, this will not
    % hold, so set the weights to zero
    if det(Lambda'*Lambda) == 0
        fprintf('AWESOME...YOU HAVE NO ERROR...I''M GOING TO SET THE WEIGHTS TO ZERO\n');
        weights = zeros(length(rho(e_vec(:,1))),1);
        break;
    end;

    % Calculate least squares solution of critic's weights -- EQ 20
    WcNew = (Lambda'*Lambda)^(-1)*Lambda'*V;
    % Make sure the weights did not diverge
    % If the weights are diverging, just set them to a large number
    if isnan(WcNew)
        fprintf('OOPS...DIVERGING WEIGHTS...I''M GOING TO USE LARGE WEIGHTS\n');
        weights = 1000*ones(length(rho(e_vec(:,1))),1);
        break;
    end;

    % Check for convergence of the critic weights
    if norm(WcNew - WcLast) < EpsilonWcritic
        weights = WcNew;
        fprintf('GREAT...THE WEIGHTS CONVERGED\n');
        break;
    end
    % If the weights did not converge, repeat the loop
    WcLast = WcNew;

    % If we reached the last iteration of the loop, just use the last
    % weights found
    if (i == (outerLoopMax-1))
        fprintf('OOPS...YOU REACHED THE END OF THE OUTER LOOP...I''M GOING TO USE THE LAST VALUE\n');
        weights = WcNew;
    end
    end
end

% CRITIC WEIGHT TUNING NEURAL NETWORK
% This function is specific to the helicopter because of the number of
% weights and error model
function weights = quanserAEROCriticTuning(A, B, e_vec,tau,R_Mat,Q_Mat,xd,wcInit)
    % REFERENCE DR. MIAH'S PAPER FOR EQUATION NUMBERS

    % SYSTEM PARAMETERS SPECIFIC TO THE 2-DOF QUANSER AERO
%     A = [0 0 1 0; 0 0 0 1; -1.7442 0 -0.3307 0; 0 0 0 -0.9283];
%     B = [0 0; 0 0; 0.0512 0.0977; -0.1139 0.0928];
    % System dimensions specific for our model
    [n,~] = size(B);

    % ERROR MODEL OF THE HELICOPTER
    % fbar and gbar -- EQ 8
```

```
    fbar = @(e) A*e;
    gbar = -B;
    hbar = -A*xd;

    % DISCRETE-TIME ERROR MODEL FOR TIME TAU
    % f and g -- EQ 10
    f = @(e) fbar(e)*tau + e;
    g = gbar*tau;
    h = hbar*tau;

    % COST FUNCTION PARAMETERS
    % State penalizing function in the continuous cost function
    Qbar = @(e) e'*Q_Mat*e;
    % Control penalizing matrix in the continuous cost function
    Rbar = R_Mat;
    % The discrete-time cost function will have terms:
    % Right after EQ 11 in paper
    % State penalizing function in the discretized cost function
    Q = @(e) Qbar(e)*tau;
    % Control penalizing matrix in the discretized cost function
    R = Rbar*tau;

    % NEURAL NETWORK FUNCTIONS
    % Critic neural network activation functions
    rho = @(e) [e(1); e(2); e(3); e(4);...
                e(1)^2; e(1)*e(2); e(1)*e(3); e(1)*e(4);...
                e(2)^2; e(2)*e(3); e(2)*e(4); e(3)^2; e(3)*e(4); e(4)^2];
    % Partial derivative of rho with respect to e
    drhode =@(e) [1, 0, 0, 0;
                  0, 1, 0, 0;
                  0, 0, 1, 0;
                  0, 0, 0, 1;
                  2*e(1), 0, 0, 0;
                  e(2), e(1), 0, 0;
                  e(3), 0, e(1), 0;
                  e(4), 0, 0, e(1);
                  0, 2*e(2), 0, 0;
                  0, e(3), e(2), 0;
                  0, e(4), 0, e(2);
                  0, 0, 2*e(3), 0;
                  0, 0, e(4), e(3);
                  0, 0, 0, 2*e(4)];

    % TOLERANCES
    % Convergence tolerance for control policy
    EpsilonPolicy = 0.1;
    % Convergence tolerance for critic neural network
    EpsilonWcritic = 0.1;

    % TRAINING PARAMETERS
    % Number of outer loop iterations
    outerLoopMax = 100;
    % Number of inner loop iterations
    innerLoopMax = 100;
```

```
    % Number of equations needed for training, number of sub-intervals
    % Number of training samples
    [~,nbar] = size(e_vec);

    % WEIGHT INITIALIZATION
    % Initialize the weights of the critic neural network to zero
    WcLast = zeros(length(rho(e_vec(:,1))),1);

    % LEAST-SQUARES COMPUTATION INITIALIZATION
    % Matrices required for computing least squares weights of the critic
    % neural networks -- EQ 20
    V = zeros(nbar,1);
    Lambda = zeros(nbar,length(rho(e_vec(:,1))));

    % Matrix to hold the derivative of the error model during policy
    % updating
    e_k_plus_1 = zeros(n,nbar);

    % Product of the least squares matrices must be invertible
    % Logic flag indicating if the critic weights are unsolvable
    % The weights are unsolvable because the least squares matrices have no
    % solution -- not invertible
    diverged = 0;

    % OUTER LOOP
    for i = 1:(outerLoopMax-1)
    % Determine if the least squares matrices are invertible
    if diverged == 0
        % For each of the data collection (discrete time index)
        for k = 1:nbar
            % Initialize the optimal inputs to zero
            uNew = [0; 0];
            % INNER LOOP
            for j = 1:(innerLoopMax-1)
                % Get the updated input value
                uLast = uNew;
                % Update the error model
                e_k_plus_1(:,k) = f(e_vec(:,k)) + g*uLast + h;
                % Compute the new optimal inputs
                uNew  = -0.5*(R^(-1))*g'*drhode(e_k_plus_1(:,k))'*WcLast;

                % Check convergence of the optimal inputs
                if norm(uNew - uLast) < EpsilonPolicy
                    break;
                end
            end

            % Update the values for the least-squares computation
            V(k,:) = Q(e_vec(:,k)) + uNew'*R*uNew + WcLast'*rho(e_k_plus_1
    (:,k));
            Lambda(k,:) = rho(e_vec(:,k))';
        end
    end

```

```
    % Verify the least square solution exists for the critic's weights
    % If the error data is consistent or there is no error, this will not
    % hold, so set the weights to what they were initially before the
    % simulation
    if det(Lambda'*Lambda) == 0
        %fprintf('AWESOME...YOU HAVE NO ERROR...I''M GOING TO USE THE ORIGINAL WEIGHTS\n');
        weights = wcInit;
        break;
    end;

    % Calulcate least squares solution of critic's weights -- EQ 20
    WcNew = (Lambda'*Lambda)^(-1)*Lambda'*V;
    % Make sure the weights did not diverge
    % If the weights diverged, set the weights to what they were initially
    % before the simulation
    if isnan(WcNew)
        %fprintf('OOPS...DIVERGING WEIGHTS...I''M GOING TO USE THE ORIGINAL WEIGHTS\n');
        weights = wcInit;
        break;
    end;

    % Check for convergence of the critic weights
    if norm(WcNew - WcLast) < EpsilonWcritic
        %fprintf('GREAT...THE WEIGHTS CONVERGED\n');
        weights = WcNew;
        break;
    end
    % If the weights did not converge, do another iteration of the loop
    WcLast = WcNew;

    % If we reach the last iteration of the loop, just use the last weights
    if (i == (outerLoopMax-1))
        %fprintf('OOPS...YOU REACHED THE END OF THE OUTER LOOP...I''M GOING TO USE THE LAST VALUE\n');
        weights = WcNew;
    end
    end
end
```

B.5 Motor Approach

B.5.1 Quanser AERO Base

```
-- Initialization ===============================================================
if (sim_call_type==sim.syscb_init) then
    simRemoteApi.start(19999)

  pJ = sim.getObjectHandle('pitchJoint');
  yJ = sim.getObjectHandle('yawJoint');
```

```
    prop0 = sim.getObjectHandle('motor0');
    prop1 = sim.getObjectHandle('motor1');
    timestep = sim.getSimulationTimeStep()
    print('timestep is : ', timestep)
    -- Get parameter values to work with
    Jy = sim.getScriptSimulationParameter(sim.handle_self, 'Jy')
    print('Jy is : ', Jy)
    Jp = sim.getScriptSimulationParameter(sim.handle_self, 'Jp')
    print('Jp is : ', Jp)
    Dp = sim.getScriptSimulationParameter(sim.handle_self, 'Dp')
    print('Dp is : ', Dp)
    Dy = sim.getScriptSimulationParameter(sim.handle_self, 'Dy')
    print('Dy is : ', Dy)
    Kpp = sim.getScriptSimulationParameter(sim.handle_self, 'Kpp')
    print('Kpp is : ', Kpp)
    Kyy = sim.getScriptSimulationParameter(sim.handle_self, 'Kyy')
    print('Kyy is : ', Kyy)
  Ksp = sim.getScriptSimulationParameter(sim.handle_self, 'Ksp')
    print('Ksp is : ', Ksp)

    -- Store current measured angular velocity to calculate angular acceleration
  prev_pJ_pose = sim.getJointPosition(pJ)
  prev_yJ_pose = sim.getJointPosition(yJ)

    sim.setJointTargetVelocity(pJ, 0)
    sim.setJointTargetVelocity(yJ, 0)
    sim.setJointTargetVelocity(prop0,0)
    sim.setJointTargetVelocity(prop1,0)
    pitchTarVel = 0
    yawTarVel = 0

    graphHandle=sim.getObjectHandle("Graph")

    PitchDesired = sim.getIntegerSignal("PitchDesired")
    lastPAcc = 0
    lastYAcc = 0

    sim.setJointTargetVelocity(prop0,1*0.8)
    sim.setJointTargetVelocity(prop1,-1*0.8)

end

-- Looping code ==================================================================================

if (sim_call_type==sim.syscb_actuation) then

        local t = sim.getSimulationTime()
      -- Receive velocities from server
    local packedData=sim.getStringSignal('MATLAB_SIG')
    if packedData then
        sim.clearStringSignal('MATLAB_SIG') -- Clear the signal
        local V=sim.unpackFloatTable(packedData,0,4,0)
```

```

        -- Measure pitch and yaw
    pJ_pose = sim.getJointPosition(pJ)
    yJ_pose = sim.getJointPosition(yJ)

        -- Calculate Velocites
        pJ_vel  = (pJ_pose - prev_pJ_pose)/timestep
        yJ_vel  = (yJ_pose - prev_yJ_pose)/timestep

        pAcc = -(Ksp/Jy)*math.sin(pJ_pose) + -(Dy/Jy)*pJ_vel + -(Kpp/Jy)*V
    [1] + (Kpp/Jy)*V[2]
        yAcc = -(Dp/Jp)*yJ_vel + -(Kyy/Jy)*V[1] + -(Kyy/Jy)*V[2]


        pitchTarVel = pitchTarVel + ((pAcc+lastPAcc)/2)*timestep
        yawTarVel = yawTarVel + ((yAcc+lastYAcc)/2)*timestep

        sim.setJointTargetVelocity(pJ, pitchTarVel)
        sim.setJointTargetVelocity(yJ, yawTarVel)

        prev_pJ_pose = pJ_pose
        prev_yJ_pose = yJ_pose
        lastPAcc = pAcc
        lastYAcc = yAcc
        sim.setGraphUserData(graphHandle,'PitchDesired',V[3])
        sim.setGraphUserData(graphHandle,'YawDesired',V[4])

       sim.setJointTargetVelocity(prop0,V[1]*0.8)
       sim.setJointTargetVelocity(prop1,-V[2]*0.8)

    end

end

function sysCall_sensing()
end

if (sim_call_type==sim.syscb_cleanup) then

end
```

B.5.2 Quanser AERO Motor#0

```
function getI(V)

  a = 0.0000277
  b = -0.00000966
  c = 0.0213
  d = -0.00319
  I = a*V*V*V + b*V*V + c*V + d;
  return I
end

if (sim_call_type==sim.syscb_init) then
```

```

    -- Get Handles for Propeller Objects and simulation timestep
    propeller_obj=sim.getObjectAssociatedWithScript(sim.handle_self)
    propeller_respondable=sim.getObjectHandle('propeller_respondable');
    timestep = sim.getSimulationTimeStep()

  -- Input voltage [Control]
    Vin = sim.getScriptSimulationParameter(sim.handle_self,'Vin')
  -- Constants from Quanser Model
  -- Thrust Coefficient in pitch
  Kpp = sim.getScriptSimulationParameter(sim.handle_self,'Kpp')
  -- Drag coefficient [m/(rad/s)]
  Kd = sim.getScriptSimulationParameter(sim.handle_self,'Kd')
  -- Motor internal resistance [ohms]
    Rm = sim.getScriptSimulationParameter(sim.handle_self,'Rm')
  -- Motor torque constant [same as Km] [N*m/A]
    Kt = sim.getScriptSimulationParameter(sim.handle_self,'Kt')
  -- Motor back EMF constant [same as Kt] [V/(rad/s)]
    Km = sim.getScriptSimulationParameter(sim.handle_self,'Km')
  -- Rotor Inertia [kg*m^2]
    Jm = sim.getScriptSimulationParameter(sim.handle_self,'Jm')
  -- Hub Inertia [kg*m^2]
  Jh = sim.getScriptSimulationParameter(sim.handle_self,'Jh')
  -- Propeller Inertia [kg*m^2]
  Jp = sim.getScriptSimulationParameter(sim.handle_self,'Jp')
  -- Inductance [H]
  L = sim.getScriptSimulationParameter(sim.handle_self,'L')
    --Propeller Center Radius
    R = sim.getScriptSimulationParameter(sim.handle_self,'R')

    momentOfInertia = sim.getScriptSimulationParameter(sim.handle_self,'
   motorMomentOfInertia')

    -- Store current measured angular velocity to calculate angular
   acceleration
  prev_i_m = 0

end

if (sim_call_type==sim.syscb_actuation) then

    -- Get the transformation matrix relative to global coordinates
    m=sim.getObjectMatrix(propeller_respondable,-1)

    -- assign Vin (this is set by the Aero script) [Volts]
    Vin=sim.getScriptSimulationParameter(sim.handle_self,'Vin')

  -- calc motor current [Amps]
  i_m = getI(Vin)

  -- estimate di_dt for inductance calc in KVL loop
  di_dt = (i_m - prev_i_m)/timestep

  -- estimate motor rpm [rad/s]
```

```
  omega_motor = (Vin - Rm*i_m - L*di_dt)/(Km)

  -- set the thrust equal to Kpp*Vin
  thrust = -Kpp*Vin/R

  --assign 3 dimensional force
    force={0.0,0.0,thrust}

    -- Get rotation matrix and its inverse
    m_rotation = m

    -- remove translational component from
    -- function return
    m_rotation[4]=0
    m_rotation[8]=0
    m_rotation[12]=0

  -- Calculate the coupling torque
  torque_m = Km*i_m

    -- Calculate Rotational Drag Torque (negative if clockwise and positive
   if cclockwise)
    torque_d = Kd*omega_motor

    -- Calculate total Torque
    torque = {0.0, 0.0, 0.0}
    torque[3] = torque_m + torque_d

    if(mccw) then
        torque[3] = -torque[3]
    end


    -- Apply force and torque onto the body
    force=sim.multiplyVector(m_rotation,force)
    torque=sim.multiplyVector(m_rotation,torque)

    sim.addForceAndTorque(propeller_respondable, force, torque)

    -- Store current
  prev_i_m = i_m
end

if (sim_call_type==sim.syscb_cleanup) then
    -- Do nothing
end
```

B.5.3 Quanser AERO Motor#1

```
function getI(V)

  a = 0.0000277
  b = -0.00000966
  c = 0.0213
```

```
  d = -0.00319
  I = a*V*V*V + b*V*V + c*V + d;
  return I
end

if (sim_call_type==sim.syscb_init) then

    -- Get Handles for Propeller Objects and simulation timestep
    propeller_obj=sim.getObjectAssociatedWithScript(sim.handle_self)
    propeller_respondable=sim.getObjectHandle('propeller_respondable');
    timestep = sim.getSimulationTimeStep()

  -- Input voltage [Control]
    Vin = sim.getScriptSimulationParameter(sim.handle_self,'Vin')

  -- Constants from Quanser Model
  -- Thrust Coefficient in pitch
  Kpp = sim.getScriptSimulationParameter(sim.handle_self,'Kpp')
  -- Drag coefficient [m/(rad/s)]
  Kd = sim.getScriptSimulationParameter(sim.handle_self,'Kd')
  -- Motor internal resistance [ohms]
    Rm = sim.getScriptSimulationParameter(sim.handle_self,'Rm')
  -- Motor torque constant [same as Km] [N*m/A]
    Kt = sim.getScriptSimulationParameter(sim.handle_self,'Kt')
  -- Motor back EMF constant [same as Kt] [V/(rad/s)]
    Km = sim.getScriptSimulationParameter(sim.handle_self,'Km')
  -- Rotor Inertia [kg*m^2]
    Jm = sim.getScriptSimulationParameter(sim.handle_self,'Jm')
  -- Hub Inertia [kg*m^2]
  Jh = sim.getScriptSimulationParameter(sim.handle_self,'Jh')
  -- Propeller Inertia [kg*m^2]
  Jp = sim.getScriptSimulationParameter(sim.handle_self,'Jp')
  -- Inductance [H]
  L = sim.getScriptSimulationParameter(sim.handle_self,'L')
    --Propeller Center Radius
    R = sim.getScriptSimulationParameter(sim.handle_self,'R')

    momentOfInertia = sim.getScriptSimulationParameter(sim.handle_self,'motorMomentOfInertia')

    -- Store current measured angular velocity to calculate angular acceleration
  prev_i_m = 0

end

if (sim_call_type==sim.syscb_actuation) then
    spinDir = mccw

    -- Get the transformation matrix relative to global coordinates
    m=sim.getObjectMatrix(propeller_respondable,-1)

    -- assign Vin (this is set by the Aero script) [Volts]
    Vin=sim.getScriptSimulationParameter(sim.handle_self,'Vin')
```

```

  -- calc motor current [Amps]
  i_m = getI(Vin)

  -- estimate di_dt for inductance calc in KVL loop
  di_dt = (i_m - prev_i_m)/timestep

  -- estimate motor rpm [rad/s]
  omega_motor = (Vin - Rm*i_m - L*di_dt)/(Km)

  -- set the thrust equal to Kpp*Vin
  thrust = -Kpp*Vin/R

  --assign 3 dimensional force
    force={0.0,0.0,thrust}

    -- Get rotation matrix and its inverse
    m_rotation = m

    -- remove translational component from
    -- function return
    m_rotation[4]=0
    m_rotation[8]=0
    m_rotation[12]=0

  -- Calculate the coupling torque
  torque_m = -Km*i_m

    -- Calculate Rotational Drag Torque (negative if clockwise and positive
   if cclockwise)
    torque_d = -Kd*omega_motor

    -- Calculate total Torque
    torque = {0.0, 0.0, 0.0}
    torque[3] = torque_m + torque_d



    -- Apply force and torque onto the body
    force=sim.multiplyVector(m_rotation,force)
    torque=sim.multiplyVector(m_rotation,torque)

    sim.addForceAndTorque(propeller_respondable, force, torque)

    -- Store current
  prev_i_m = i_m
end

if (sim_call_type==sim.syscb_cleanup) then
    --- Do nothing
end
```

B.5.4 Quanser AERO Motor PID Control

```
% Use exact model of the 2-DOF helicopter and the linearized model to find
% error data points
% Use the error data points directly in the neural network instead of
% randomizing the error
close all; clear; clc;

% Name of .mat file to save the data
testName = 'heli_joint_sqsq';

% LINEAR MODEL MATRIX PARAMETERS (2DoF Helicopter)
[ A, B ] = getAeroAB( 0 );

% 2-DOF QUANSER HELICOPTER PARAMETERS
% Maximum applied voltage for the rotor motors
maxVolt = 24;
% Number of state variables - theta, psi, dtheta, dpsi
n = 4;
theta = 1;    % Pitch
psi = 2;      % Yaw
thetaDot = 3; % Pitch angular velocity
psiDot = 4;   % Yaw angular velocity
% Number of inputs - Vp, Vy
m = 2;

% SIMULATION TIMING
% Initial and final simulation times [s]
t0 = 0; tf = 30;
% Sampling time [s]
tau = 0.05;
% Larger sampling time for updating the inputs [s]
T = 0.2;
% Number of time steps
tsteps = floor((tf-t0)/tau);
% Discrete time vecotor of sampling time (tau)
dt = tau*(0:tsteps);


% INITIAL STATES
% xInit = [deg2rad(0) deg2rad(0) (0*pi/180)*10 (0*pi/180)*7]';
%xInit = [0 0 0 0]';

% MATRIX INITIALIZATION
% Actual states - state variable x
pose = zeros(n,tsteps+1);

% State errors for entire simulation
error = zeros(n,tsteps+1);
% Actual inputs
u = zeros(m,tsteps+1);

% Desired states
% Uncomment for desired states of all zero
%xd = zeros(n,tsteps+1);
% Uncomment for desired states that switch constant values
```

```
%xd = [deg2rad(-30)*ones(1,tsteps+1); deg2rad(-80)*ones(1,tsteps+1); zeros
    (1,tsteps+1); zeros(1,tsteps+1)];
%xd = [deg2rad(10)*ones(1,1000), deg2rad(60)*ones(1,tsteps+1-1000); deg2rad
    (80)*ones(1,1500), deg2rad(-120)*ones(1,tsteps+1-1500); zeros(1,tsteps+1)
    ; zeros(1,tsteps+1)];
% Uncomment for desired states that are sine and cosine waves
 pitchSine = deg2rad(30)*sin(0.5*dt);
 yawCosine = deg2rad(60)*cos(0.5*dt);
%  pitchSquare = deg2rad(25)*square(0.5*dt);
%  yawSquare = deg2rad(-36)*square(0.25*dt);
xd = [pitchSine; yawCosine; zeros(1,tsteps+1); zeros(1,tsteps+1)];
% xd = [pitchSquare; yawSquare; zeros(1,tsteps+1); zeros(1,tsteps+1)];


%% connection routine
ip = '127.0.0.1';
port = 19999;

% create a handle to remote API
vrep = remApi('remoteApi'); % using the prototype file (remoteApiProto.m)
vrep.simxFinish(-1); % just in case, close all opened connections
clientID = vrep.simxStart(ip,port,true,true,5000,5);

%% simulation
if (clientID>-1) % if clientID exists
    disp('Connected to remote API server');

%Set up the handles
[rtn, aero] = vrep.simxGetObjectHandle(clientID,...
                                            'Aero',...
                                             vrep.simx_opmode_blocking)
[rtn, yJ] = vrep.simxGetObjectHandle(clientID,...
                                        'yawJoint',...
                                         vrep.simx_opmode_blocking)
[rtn, pJ] = vrep.simxGetObjectHandle(clientID,...
                                        'pitchJoint',...
                                        vrep.simx_opmode_blocking)

% setup joint position streaming
vrep.simxGetJointPosition(clientID,...
                             yJ,...
                             vrep.simx_opmode_streaming);
vrep.simxGetJointPosition(clientID,...
                             pJ,...
                             vrep.simx_opmode_streaming);

% add a delay to let the streaming operations initialize
pause(1);

[ret, pose(1,1)] = vrep.simxGetJointPosition(clientID,...
                                                  pJ,...
                                                  vrep.simx_opmode_buffer);

[ret, pose(2,1)] = vrep.simxGetJointPosition(clientID,...
```

```
                                                                   yJ ,...
                                                                   vrep.simx_opmode_buffer);


pose(3,1) = 0;
pose(4,1) = 0;
% Initialize the error
error(:,1) = xd(:,1) − pose(:,1);

k_d = 3;
k_i = 10;
k_p = 8;

k_1 = 4;
k_2 = 4;

e_theta_sum = 0;
e_psi_sum = 0;

tic;
% RUN THE SIMULATION
% Start at the next time tau after time zero
for k = 2:tsteps+1
    % Current time
    t = (k−1)*tau; % generate value discrete time index

    % Step 1 − get/store the state of the quadcopter from VREP
    [ret, pose(1,k)] = vrep.simxGetJointPosition(clientID,...
                                                                   pJ,...
                                                                   vrep.simx_opmode_buffer)
    ;

    [ret, pose(2,k)] = vrep.simxGetJointPosition(clientID,...
                                                                   yJ,...
                                                                   vrep.simx_opmode_buffer)
    ;

    pose(3,k) =  (pose(1,k) −  pose(1,k−1))/tau;
    pose(4,k) = (pose(2,k) −  pose(2,k−1))/tau;

    if(k>=3)
        pose(3,k) = (pose(3,k) + pose(3,k−1) + pose(3,k−2))/3;
        pose(4,k) = (pose(4,k) + pose(4,k−1) + pose(4,k−2))/3;
    end

    error(:,k) = xd(:,k) − pose(:,k);

    e_theta_sum = e_theta_sum + error(1,k)*tau;
    e_theta = k_p*error(1,k) + k_i*e_theta_sum + k_d*error(3,k);

    e_psi_sum = e_psi_sum + error(2,k)*tau;
    e_psi = k_p*error(2,k) + k_i*e_psi_sum + k_d*error(4,k);

    u(1,k) = −k_1*e_theta−k_2*e_psi;
```

```
    u(2,k) = -k_2*e_psi+k_1*e_theta;

    u(1,k) = sign(u(1,k))*min(abs(u(1,k)),maxVolt);
    u(2,k) = sign(u(2,k))*min(abs(u(2,k)),maxVolt);

    % Step 3 - Update Inputs
    Vin=[u(1,k) u(2,k)];
%   motorSpeeds=[20 0];
    packedData=vrep.simxPackFloats(Vin);
    vrep.simxSetStringSignal(clientID,...
                              'MATLAB_SIG',...
                              packedData,...
                              vrep.simx_opmode_oneshot);

   pause(tau);

   t = t + tau;


end
toc;

%ret = 1;
while(ret ~= 0)
    [ret]=vrep.simxClearStringSignal(clientID,...
                                      'MATLAB_SIG',...
                                      vrep.simx_opmode_blocking);
end

else
    disp('Failed connecting to remote API server');
end % if (clientID>-1)

disp('Sim ended');

figure
plot(dt,rad2deg(pose(1,:))); hold on;
plot(dt,rad2deg(xd(1,:))); hold off;
title('pitch')

figure
plot(dt,rad2deg(pose(2,:))); hold on;
plot(dt,rad2deg(xd(2,:))); hold off;
title('yaw')

figure
plot(dt,u(1,:)); hold on;
plot(dt,u(2,:)); hold off;
title('Inputs')
```

```
% SAVE OUTPUTS FOR PLOTTING
% Save the time, error, input, states and desired states vectors
save([testName,'.mat'],'dt', 'xd','tau','error','pose','u','k_p','k_i','k_d'
    );
% Notify that the simulation is complete
disp('SIMULATION COMPLETE');
```

Appendix C

QFLEX 2 USB

C.1 QFLEX 2 USB Panel (Tutorial)

1. Verify that all of the proper software is installed as mentioned in Section 6.2.
2. Install the QFLEX 2 USB panel to the Quanser AERO as seen in Figure C.1.

Figure C.1: QFLEX 2 USB panel installed on the Quanser AERO.

3. Open the Simulink model that you would like to use.
4. Power on the Quanser AERO, and connect your laptop to the QFLEX 2 USB panel with the provided cable. See Figure C.2.

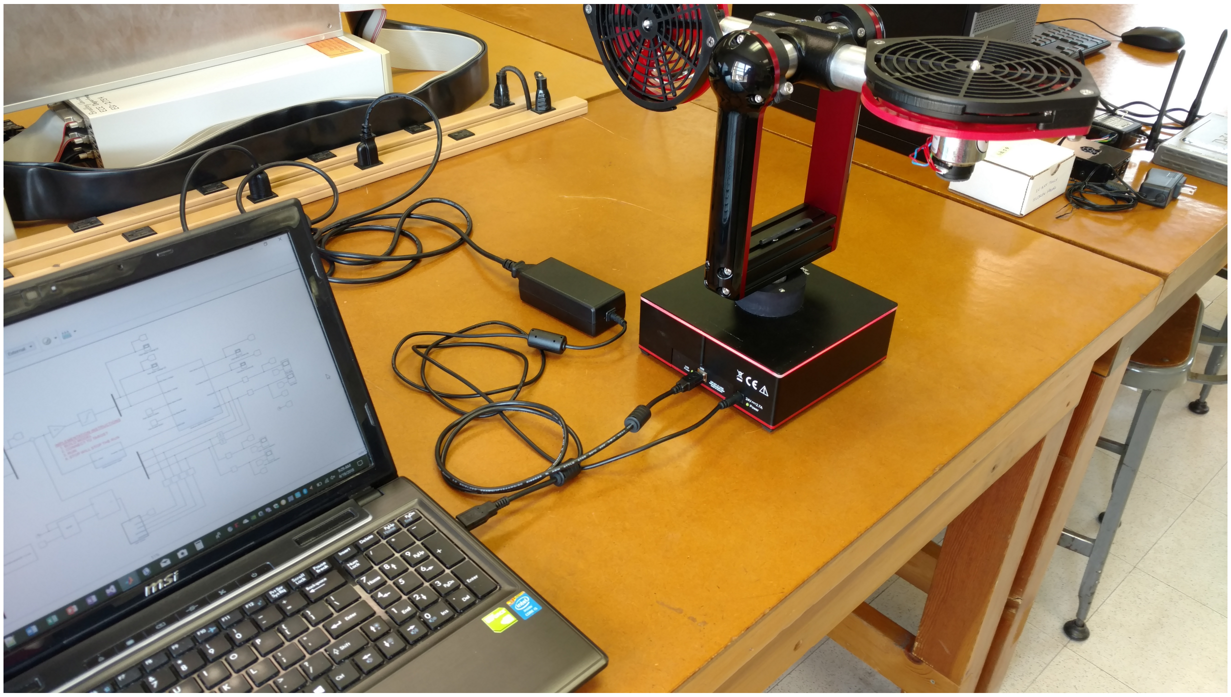


Figure C.2: Connections from your laptop to the Quanser AERO.

5. Build the Simulink model. See Figure C.3.

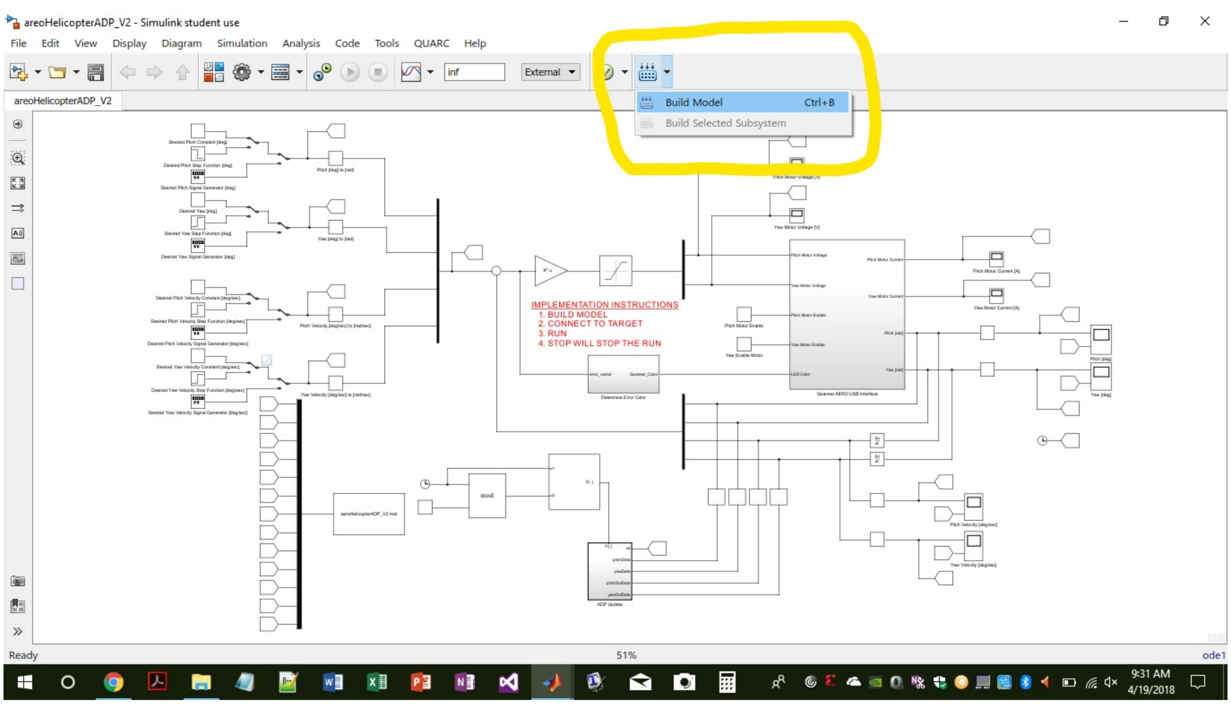


Figure C.3: Build the Simulink model.

6. Connect to the target. This starts the communication process between Simulink and

the Quanser AERO. See Figure C.4.

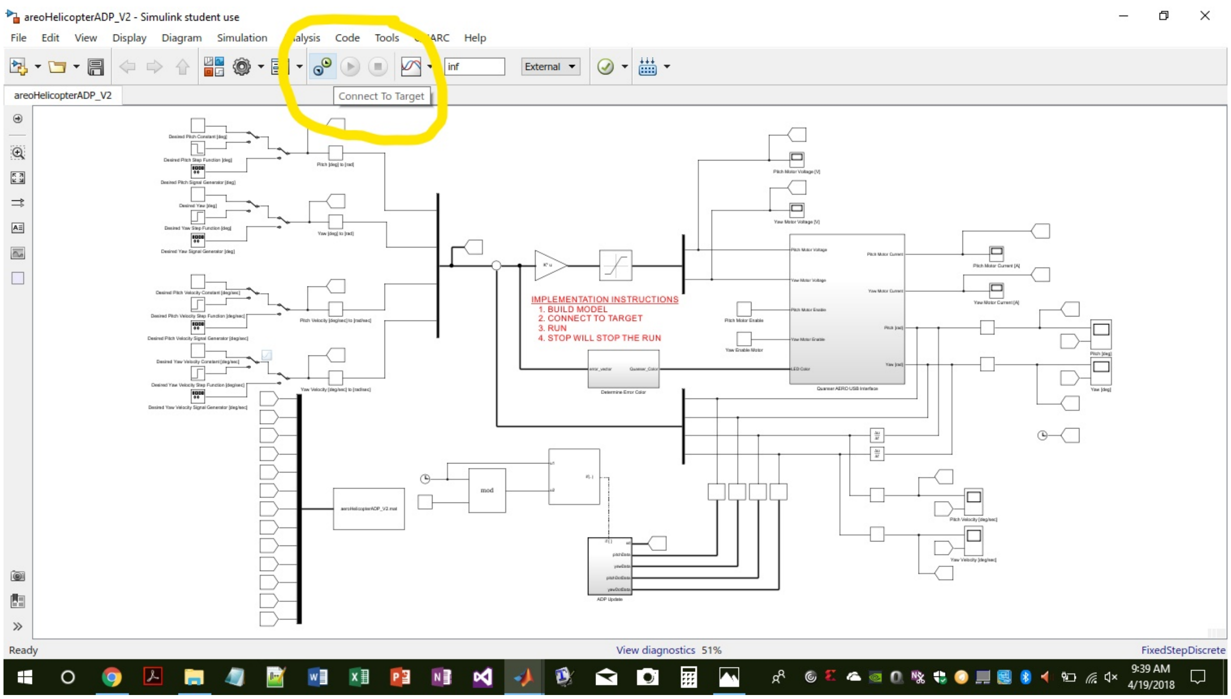


Figure C.4: Connect to the target.

7. Run the Simulink model. See Figure C.5.
8. Observe the Quanser AERO's motion control. See Figure C.6.
9. When you are done with the Quanser AERO, stop the model. See Figure C.7.
10. You may disconnect the Quanser AERO and close Simulink when finished.
11. **NOTES**
    - You should be able to partially adjust the desired positions while the Simulink model is running.
    - You really only need to rebuild the model if you make major changes to it which will affect the code generation.
    - After you stop the model, you can restart the model, but you have to connect to the target again.
    - You will be prompted with errors if you are not physically connected when you try to connect to the target.

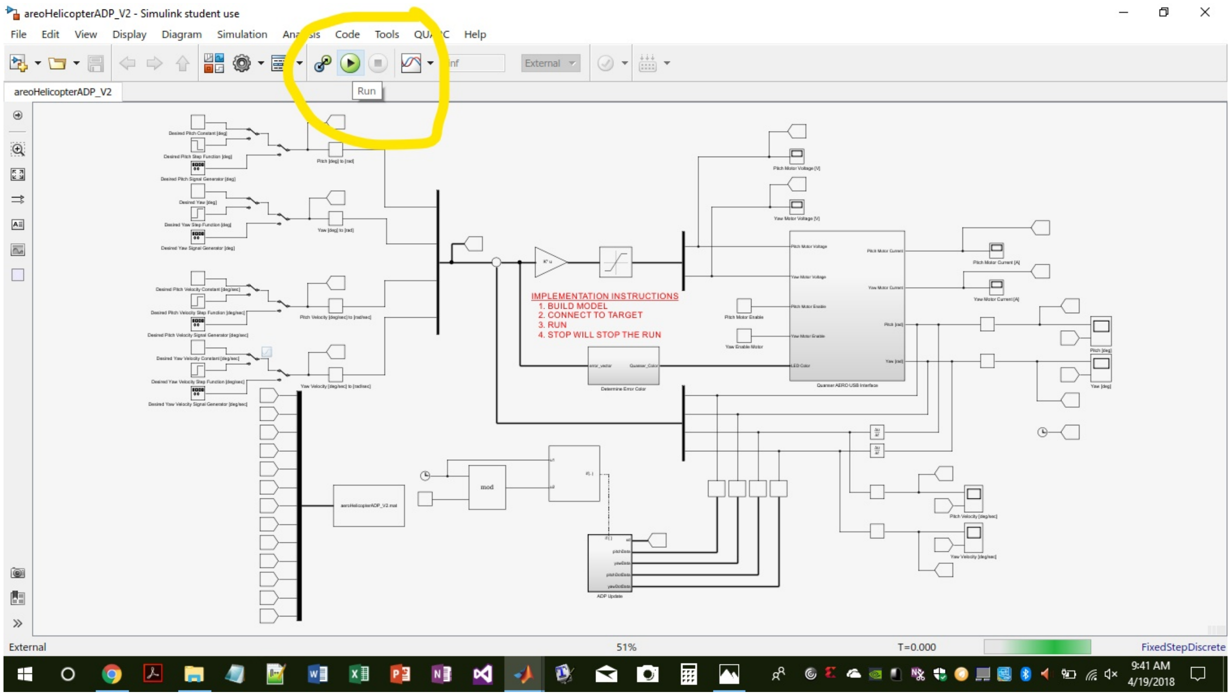


Figure C.5: Run the Simulink model.

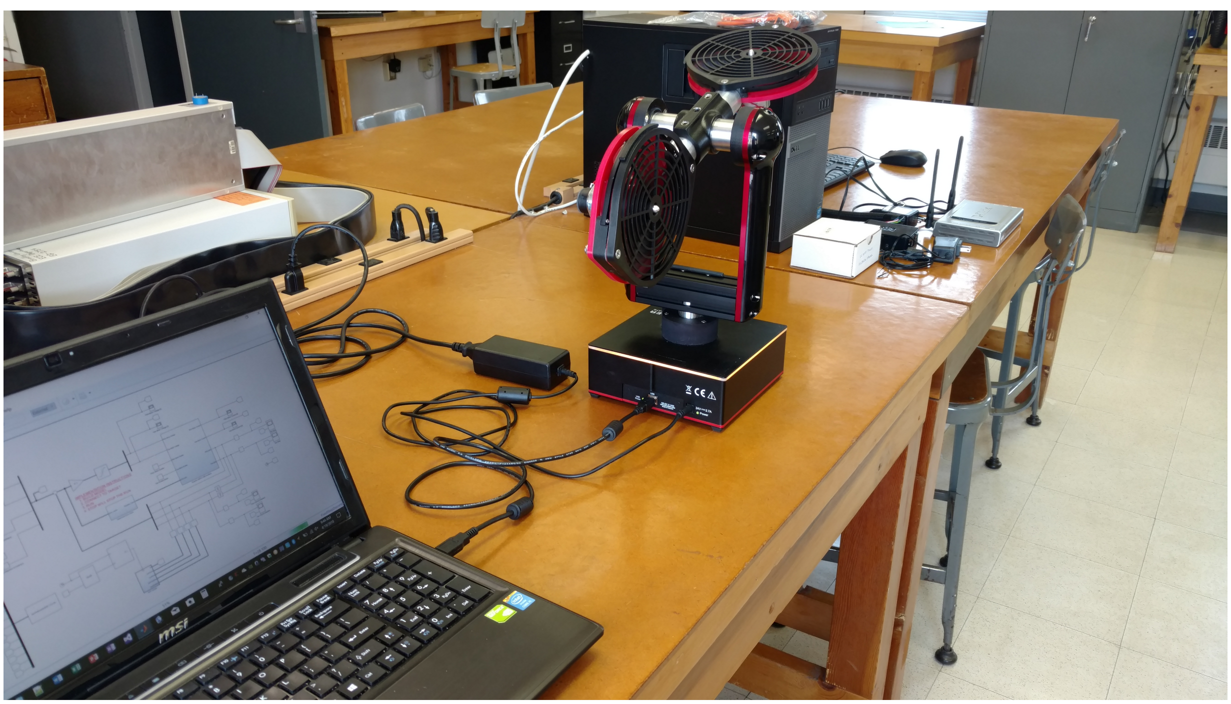


Figure C.6: Observe the Quanser AERO.

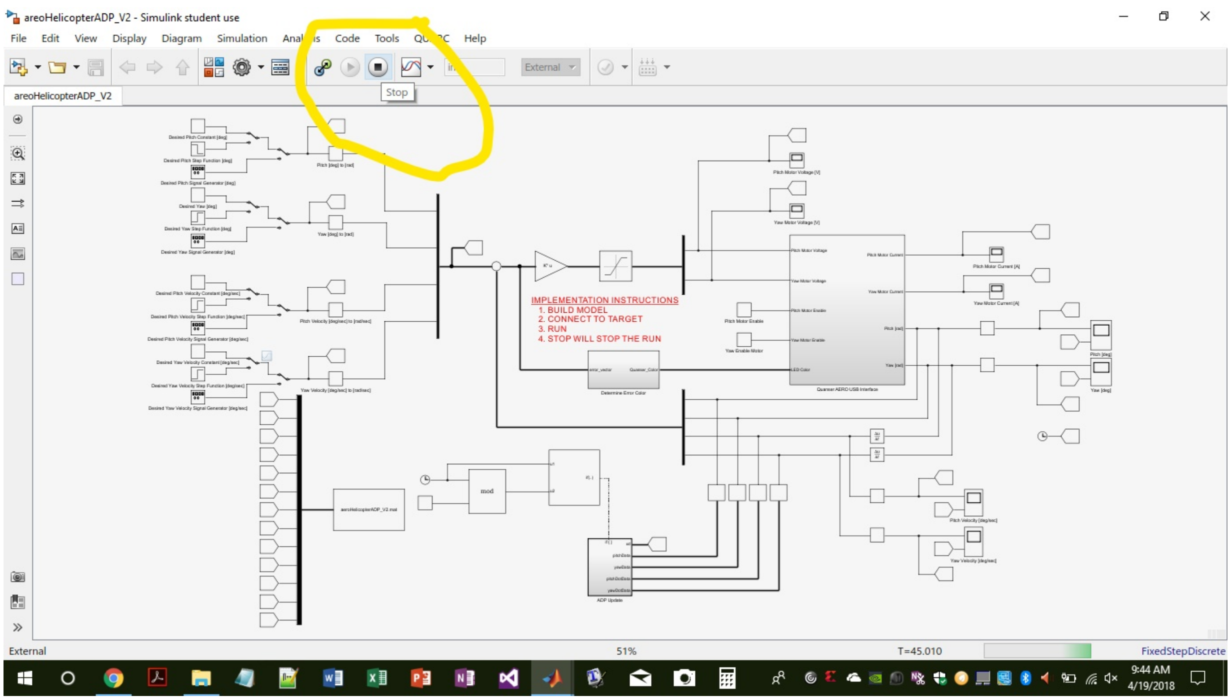


Figure C.7: Stop the Simulink model.

C.2 Initialization Code (MATLAB)

```
% Andrew Fandel
% - FIND THE CRITIC NEURAL NETWORK WEIGHTS BEFORE RUNNING THE SIMULATION
% USING RANDOM ERROR DATA
close all; clear; clc;

% Sampling time [s]
tau = 0.01;
% Update time [s]
T = 0.2;

% COST FUNCTION
% Q and R matrices used in the cost function
Q_Mat_ADP = diag([270 100 1 1]);
R_Mat_ADP = 0.005*diag([1 1]);

% CRITIC WEIGHT TUNING NEURAL NETWORK
% This function is specific to the helicopter because of the number of
% weights and error model
% REFERENCE DR. MIAH'S PAPER FOR EQUATION NUMBERS

% SYSTEM PARAMETERS SPECIFIC TO THE 2-DOF QUANSER AERO
A = [0 0 1 0; 0 0 0 1; -1.7442 0 -0.3307 0; 0 0 0 -0.9283];
B = [0 0; 0 0; -0.0149 0.0414; -0.0751 -0.1295];
% UNCOMMENT TO USE A STATE-SPACE MODEL THAT IS NOT THE CORRECT ONE DERIVED
% A = [0 0 1 0; 0 0 0 1; 0 0 -0.3307 0; 0 0 0 -0.9283];
% B = [0 0; 0 0; 0.1 0.1; -0.3 -0.3];
% System dimensions specific for our model
[n,~] = size(B);

% CREATE RANDOM ERROR DATA TO TRAIN THE CRITIC NEURAL NETWORK
% Number of equations needed for training, number of sub-intervals
% Number of training samples
nbar = 100;
% Create random error in the range of -pi to pi
e_vec = (2*(pi)).*rand(n,nbar) - (pi);

% ERROR MODEL OF THE HELICOPTER
% fbar and gbar -- EQ 8
% We do not have an xd term because we won't know the desired state vector
% when we execute this code
% This code can also use anonymous functions because this function is
% executed by MATLAB and not Simulink
fbar = @(e) A*e;
gbar = -B;

% DISCRETE-TIME ERROR MODEL FOR TIME TAU
% f and g -- EQ 10
f = @(e) fbar(e)*tau + e;
g = gbar*tau;

% COST FUNCTION PARAMETERS
```

```
% State penalizing function in the continuous cost function
Qbar = @(e) e'*Q_Mat_ADP*e;
% Control penalizing matrix in the continuous cost function
Rbar = R_Mat_ADP;
% The discrete-time cost function will have terms:
% Right after EQ 11 in paper
% State penalizing function in the discretized cost function
Q = @(e) Qbar(e)*tau;
% Control penalizing matrix in the discretized cost function
R = Rbar*tau;

% NEURAL NETWORK FUNCTIONS
% Critic neural network activation functions
rho = @(e) [e(1); e(2); e(3); e(4);...
            e(1)^2; e(1)*e(2); e(1)*e(3); e(1)*e(4);...
            e(2)^2; e(2)*e(3); e(2)*e(4); e(3)^2; e(3)*e(4); e(4)^2];
% Partial derivative of rho with respect to e
drhode =@(e) [1, 0, 0, 0;
              0, 1, 0, 0;
              0, 0, 1, 0;
              0, 0, 0, 1;
              2*e(1), 0, 0, 0;
              e(2), e(1), 0, 0;
              e(3), 0, e(1), 0;
              e(4), 0, 0, e(1);
              0, 2*e(2), 0, 0;
              0, e(3), e(2), 0;
              0, e(4), 0, e(2);
              0, 0, 2*e(3), 0;
              0, 0, e(4), e(3);
              0, 0, 0, 2*e(4)];

% TOLERANCES
% Convergence tolerance for control policy
EpsilonPolicy = 0.1;
% Convergence tolerance for critic neural network
EpsilonWcritic = 0.1;

% TRAINING PARAMETERS
% Number of outer loop iterations
outerLoopMax = 700;
% Number of inner loop iterations
innerLoopMax = 100;
% Number of equations needed for training, number of sub-intervals
% Number of training samples
[~,nbar] = size(e_vec);

% WEIGHT INITIALIZATION
% Initialize the weights of the critic neural network to zero
WcLast = zeros(length(rho(e_vec(:,1))),1);

% LEAST-SQUARES COMPUTATION INITIALIZATION
% Matrices required for computing least squares weights of the critic
% neural networks -- EQ 20
```

```
V = zeros(nbar,1);
Lambda = zeros(nbar,length(rho(e_vec(:,1))));

% Matrix to hold the derivative of the error model during policy
% updating
e_k_plus_1 = zeros(n,nbar);

% Product of the least squares matrices must be invertible
% Logic flag indicating if the critic weights are unsolvable
% The weights are unsolvable because the least squares matrices have no
% solution -- not invertible
diverged = 0;

% OUTER LOOP
for i = 1:(outerLoopMax-1)
% Determine if the least squares matrices are invertible
if diverged == 0
    % For each of the data collection (discrete time index)
    for k = 1:nbar
        % Initialize the optimal inputs to zero
        uNew = [0; 0];
        % INNER LOOP
        for j = 1:(innerLoopMax-1)
            % Get the updated input value
            uLast = uNew;
            % Update the error model
            e_k_plus_1(:,k) = f(e_vec(:,k)) + g*uLast;
            % Compute the new optimal inputs
            uNew  = -0.5*(R^(-1))*g'*drhode(e_k_plus_1(:,k))'*WcLast;

            % Check convergence of the optimal inputs
            if norm(uNew - uLast) < EpsilonPolicy
                break;
            end
        end

        % Update the values for the least-squares computation
        V(k,:) = Q(e_vec(:,k)) + uNew'*R*uNew + WcLast'*rho(e_k_plus_1(:,k))
    ;
        Lambda(k,:) = rho(e_vec(:,k))';
    end
end

% Verify the least square solution exists for the critic's weights
% If the error data is consistent or there is no error, this will not
% hold, so set the weights to zero
if det(Lambda'*Lambda) == 0
    weights = zeros(length(rho(e_vec(:,1))),1);
    break;
end;

% Calculate least squares solution of critic's weights -- EQ 20
WcNew = (Lambda'*Lambda)^(-1)*Lambda'*V;
% Make sure the weights did not diverge
```

```
% If the weights are diverging, just set them to a large number
if isnan(WcNew)
    weights = 1000*ones(length(rho(e_vec(:,1))),1);
    break;
end;

% Check for convergence of the critic weights
if norm(WcNew - WcLast) < EpsilonWcritic
    weights = WcNew;
    break;
end
% If the weights did not converge, repeat the loop
WcLast = WcNew;

% If we reached the last iteration of the loop, just use the last
% weights found
if (i == (outerLoopMax-1))
    weights = WcNew;
end
end

% Use the weights to determine the P matrix
P_Mat = [weights(5) weights(6) weights(7) weights(8);
         weights(6) weights(9) weights(10) weights(11);
         weights(7) weights(10) weights(12) weights(13);
         weights(8) weights(11) weights(13) weights(14)];

% Find the state-feedback gain EQ
K = 0.5*(R_Mat_ADP^-1)*B'*P_Mat;

% Save the initial weights
wcInit = weights;

% Keep only the ADP gain
clearvars -except K tau T wcInit;
```

C.3 Update ADP Gain (MATLAB)

```
% CRITIC WEIGHT TUNING NEURAL NETWORK
% This function is specific to the helicopter because of the number of
% weights and error model
% THIS FUNCTION CANNOT HAVE ANY ANONYMOUS FUNCTIONS DUE TO THE C CODE
% GENERATION UNSUPPORTING IT
function K = quanserAEROCriticTuning(xd,pitchData,yawData,pitchDotData,
    yawDotData,wcInit)
    % CREATE THE ERROR VECTOR MATRIX
    % Use the data from the tapped delay blocks to create the error state
    % vector matrix
    e_vec = [pitchData'; yawData'; pitchDotData'; yawDotData'];

    % REFERENCE DR. MIAH'S PAPER FOR EQUATION NUMBERS
    % Sampling time [s]
    tau = 0.01;
```

```

% COST FUNCTION
% Q and R matrices used in the cost function
Q_Mat = diag([270 100 1 1]);
R_Mat = 0.005*diag([1 1]);

% SYSTEM PARAMETERS SPECIFIC TO THE 2-DOF QUANSER AERO
A = [0 0 1 0; 0 0 0 1; -1.7442 0 -0.3307 0; 0 0 0 -0.9283];
B = [0 0; 0 0; -0.0149 0.0414; -0.0751 -0.1295];
% UNCOMMENT TO USE A STATE-SPACE MODEL THAT IS NOT THE CORRECT ONE
DERIVED
% A = [1 1 1 1; 1 1 1 1; 1 1 1 1; 1 1 1 1];
% B = [0 0; 0 0; 0.1 0.1; -0.3 -0.3];
% System dimensions specific for our model
[n,~] = size(B);

% ERROR MODEL OF THE HELICOPTER
% gbar and hbar -- EQ 8
% An anonymous function cannot be used for fbar
gbar = -B;
hbar = -A*xd;

% DISCRETE-TIME ERROR MODEL FOR TIME TAU
% g and h -- EQ 10
% An anonymous function cannot be used for f
g = gbar*tau;
h = hbar*tau;

% COST FUNCTION PARAMETERS
% The Q matrix cannot be treated as an anonymous function as in the
% MATLAB simulations
% Control penalizing matrix in the continuous cost function
Rbar = R_Mat;
% The discrete-time cost function will have terms:
% Right after EQ 11 in paper
% Control penalizing matrix in the discretized cost function
R = Rbar*tau;

% NEURAL NETWORK FUNCTIONS
% These functions cannot be written as anonymous functions as in the
% MATLAB simulations

% TOLERANCES
% Convergence tolerance for control policy
EpsilonPolicy = 0.1;
% Convergence tolerance for critic neural network
EpsilonWcritic = 0.1;

% TRAINING PARAMETERS
% Number of outer loop iterations
outerLoopMax = 700;
% Number of inner loop iterations
innerLoopMax = 100;
% Number of equations needed for training, number of sub-intervals
```

```
    % Number of training samples
    [~,nbar] = size(e_vec);

    % WEIGHT INITIALIZATION
    % Initialize the weights of the critic neural network to zero
    % Number of rows hard-coded for the helicopter
    WcLast = zeros(14,1);

    % LEAST-SQUARES COMPUTATION INITIALIZATION
    % Matrices required for computing least squares weights of the critic
    % neural networks -- EQ 20
    V = zeros(nbar,1);
    Lambda = zeros(nbar,14);

    % Matrix to hold the derivative of the error model during policy
    % updating
    e_k_plus_1 = zeros(n,nbar);

    % Product of the least squares matrices must be invertible
    % Logic flag indicating if the critic weights are unsolvable
    % The weights are unsolvable because the least squares matrices have no
    % solution -- not invertible
    diverged = 0;

    % OUTER LOOP
    for i = 1:(outerLoopMax-1)
    % Determine if the least squares matrices are invertible
    if diverged == 0
        % For each of the data collection (discrete time index)
        for k = 1:nbar
            % Initialize the optimal inputs to zero
            uNew = [0; 0];
            % INNER LOOP
            for j = 1:(innerLoopMax-1)
                % Get the updated input value
                uLast = uNew;
                % Update the error model
                % Because of no anonymous functions we have modified f
                % fbar = @(e) A*e;
                % f = @(e) fbar(e)*tau + e;
                e_k_plus_1(:,k) = (A*e_vec(:,k)*tau) + e_vec(:,k) + g*uLast
  + h;
                % Compute the new optimal inputs
                % Partial derivative of rho with respect to e
                drhode = [1, 0, 0, 0;
                          0, 1, 0, 0;
                          0, 0, 1, 0;
                          0, 0, 0, 1;
                          2*e_vec(1,k), 0, 0, 0;
                          e_vec(2,k), e_vec(1,k), 0, 0;
                          e_vec(3,k), 0, e_vec(1,k), 0;
                          e_vec(4,k), 0, 0, e_vec(1,k);
                          0, 2*e_vec(2,k), 0, 0;
                          0, e_vec(3,k), e_vec(2,k), 0;
```

```
                        0, e_vec(4,k), 0, e_vec(2,k);
                        0, 0, 2*e_vec(3,k), 0;
                        0, 0, e_vec(4,k), e_vec(3,k);
                        0, 0, 0, 2*e_vec(4,k)];
                uNew  = -0.5*(R^(-1))*g'*drhode'*WcLast;

                % Check convergence of the optimal inputs
                if norm(uNew - uLast) < EpsilonPolicy
                    break;
                end
            end

            % Update the values for the least-squares computation
            % Critic neural network activation functions
            rhoE = [e_vec(1,k); e_vec(2,k); e_vec(3,k); e_vec(4,k);...
                    e_vec(1,k)^2; e_vec(1,k)*e_vec(2,k);...
                    e_vec(1,k)*e_vec(3,k); e_vec(1,k)*e_vec(4,k);...
                    e_vec(2,k)^2; e_vec(2,k)*e_vec(3,k);...
                    e_vec(2,k)*e_vec(4,k); e_vec(3,k)^2;...
                    e_vec(3,k)*e_vec(4,k); e_vec(4,k)^2];
           rhoEK = [e_k_plus_1(1,k); e_k_plus_1(2,k); e_k_plus_1(3,k);...
                    e_k_plus_1(4,k); e_k_plus_1(1,k)^2;...
                    e_k_plus_1(1,k)*e_k_plus_1(2,k);...
                    e_k_plus_1(1,k)*e_k_plus_1(3,k);...
                    e_k_plus_1(1,k)*e_k_plus_1(4,k);...
                    e_k_plus_1(2,k)^2; e_k_plus_1(2,k)*e_k_plus_1(3,k);...
                    e_k_plus_1(2,k)*e_k_plus_1(4,k); e_k_plus_1(3,k)^2;...
                    e_k_plus_1(3,k)*e_k_plus_1(4,k); e_k_plus_1(4,k)^2];
            % State penalizing function in the continuous cost function
            % Qbar = @(e) e'*Q_Mat*e;
            % State penalizing function in the discretized cost function
            % Q = @(e) Qbar(e)*tau;
            V(k,:) = (e_vec(:,k)'*Q_Mat*e_vec(:,k)*tau) + uNew'*R*uNew +
    WcLast'*rhoEK;
            Lambda(k,:) = rhoE';
        end
    end

    % Verify the least square solution exists for the critic's weights
    % If the error data is consistent or there is no error, this will not
    % hold, so set the weights to what they were initially before the
    % simulation
    if det(Lambda'*Lambda) == 0
        weights = wcInit;
        break;
    end;

    % Calulcate least squares solution of critic's weights -- EQ 20
    WcNew = (Lambda'*Lambda)^(-1)*Lambda'*V;
    % Make sure the weights did not diverge
    % If the weights diverged, set the weights to what they were initially
    % before the simulation
    if isnan(WcNew)
        weights = wcInit;
```

```
            break;
        % Check for convergence of the critic weights
        elseif norm(WcNew - WcLast) < EpsilonWcritic
            weights = WcNew;
            break;
        % If we reach the last iteration of the loop, just use the last weights
        elseif (i == (outerLoopMax-1))
            weights = WcNew;
        % If all else fails, just set the weights to the initial weights
        else
            weights = wcInit;
        end
        % If the weights did not converge, do another iteration of the loop
        WcLast = WcNew;
    end

    % Use the weights to determine the P matrix
    P_Mat = [weights(5) weights(6) weights(7) weights(8);
             weights(6) weights(9) weights(10) weights(11);
             weights(7) weights(10) weights(12) weights(13);
             weights(8) weights(11) weights(13) weights(14)];

    % Find the state-feedback gain EQ
    K = 0.5*(R_Mat^-1)*B'*P_Mat;
end
```

C.4 Base Color (MATLAB)

```
% THIS FUNCTION DETERMINES THE PROPER COLOR MATRIX FOR THE QUANSER AERO
function colorSelect = colorSelector(u)

% If the magnitude is less than 1, output the color green
if (u < 1)
    colorSelect = [0 1 0];
% If the magnitude is between 3 and 10, output the color yellow
elseif (u < 10)
    colorSelect = [1 1 0];
% If the magnitude is greater than 10, output the color blue
else
    colorSelect = [0 0 1];
end
```

Appendix D

Raspberry Pi 3

D.1 Raspberry Pi 3 (Tutorial)

1. Verify that all of the proper software is installed as mentioned in Section 6.3.
2. Install the QFLEX 2 Embedded panel to the Quanser AERO as seen in Figure D.1.

Figure D.1: QFLEX 2 Embedded panel installed on the Quanser AERO.

3. Connect your laptop to the Raspberry Pi and power it on. Use an Ethernet cable to connect to the Raspberry Pi. See Figure D.2. You can connect the Raspberry Pi to the Quanser AERO, but I prefer to do that in a later step.

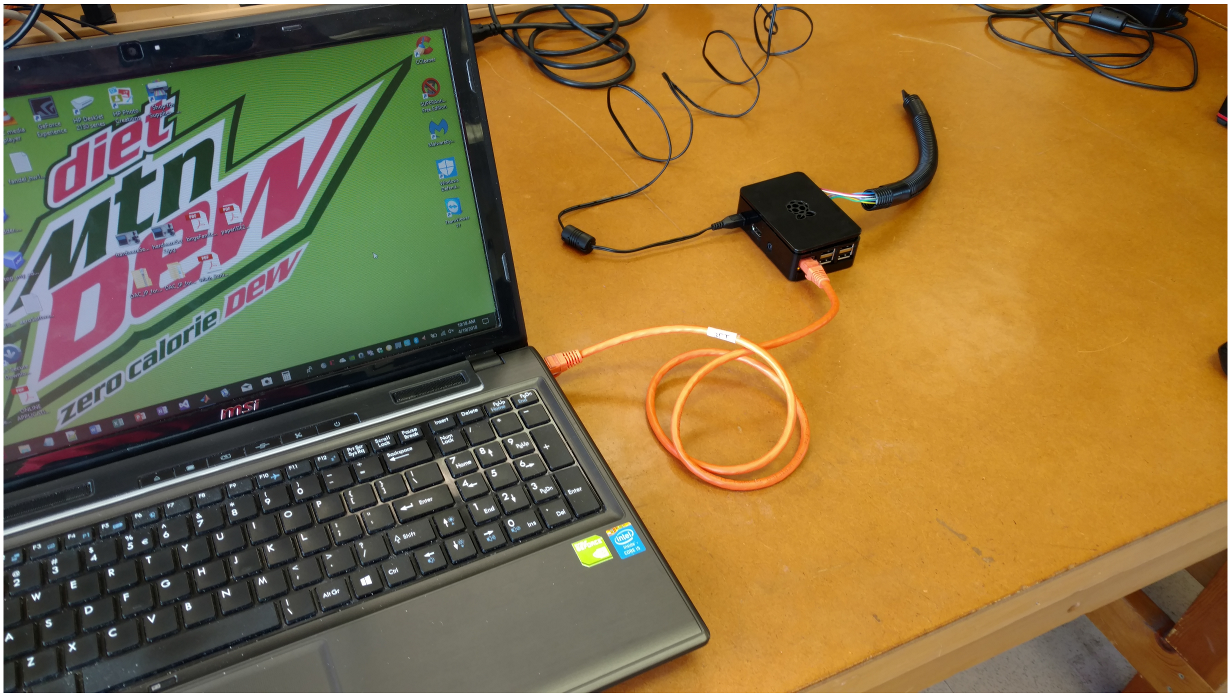


Figure D.2: Connect the Raspberry Pi to your laptop and power it on.

4. Wait a few moments and then connect to the Raspberry Pi using PuTTY. The IP address of our particular Raspberry Pi is 169.254.0.2. See Figure D.3.

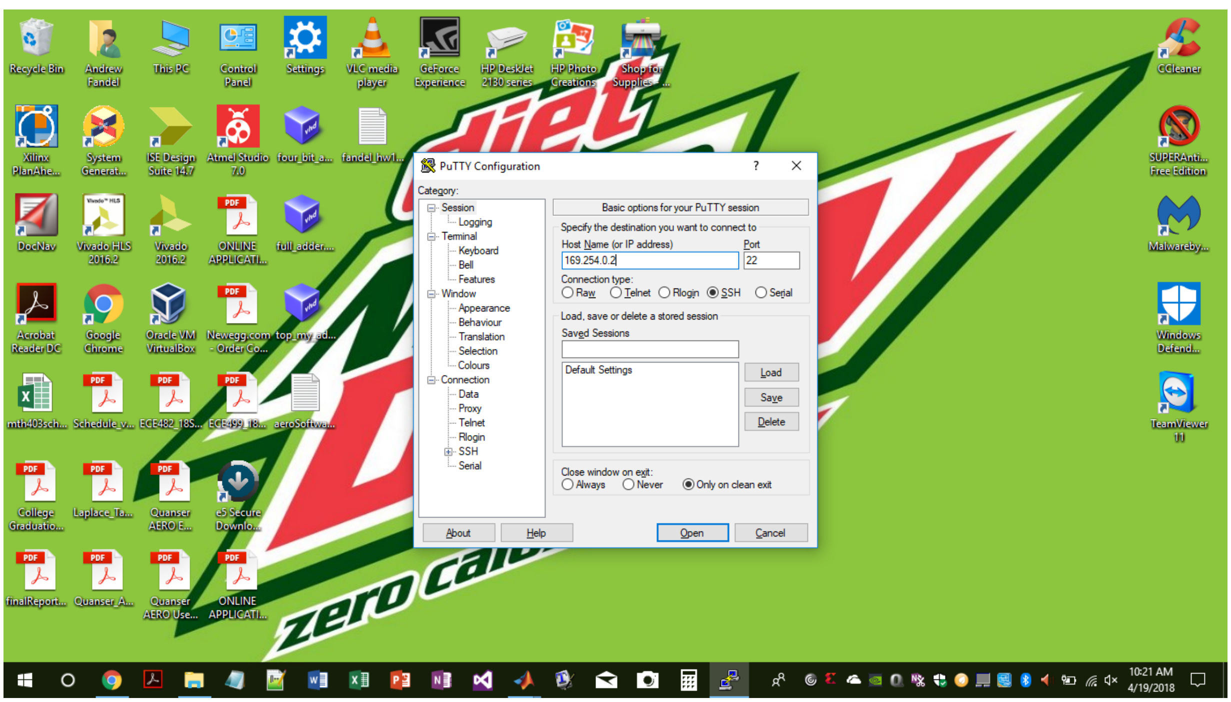


Figure D.3: Connect to the Raspberry Pi using PuTTY.

5. Once PuTTY is connected to the Raspberry Pi, login to it. Login as "pi" with the password "raspberry." See Figure D.4.

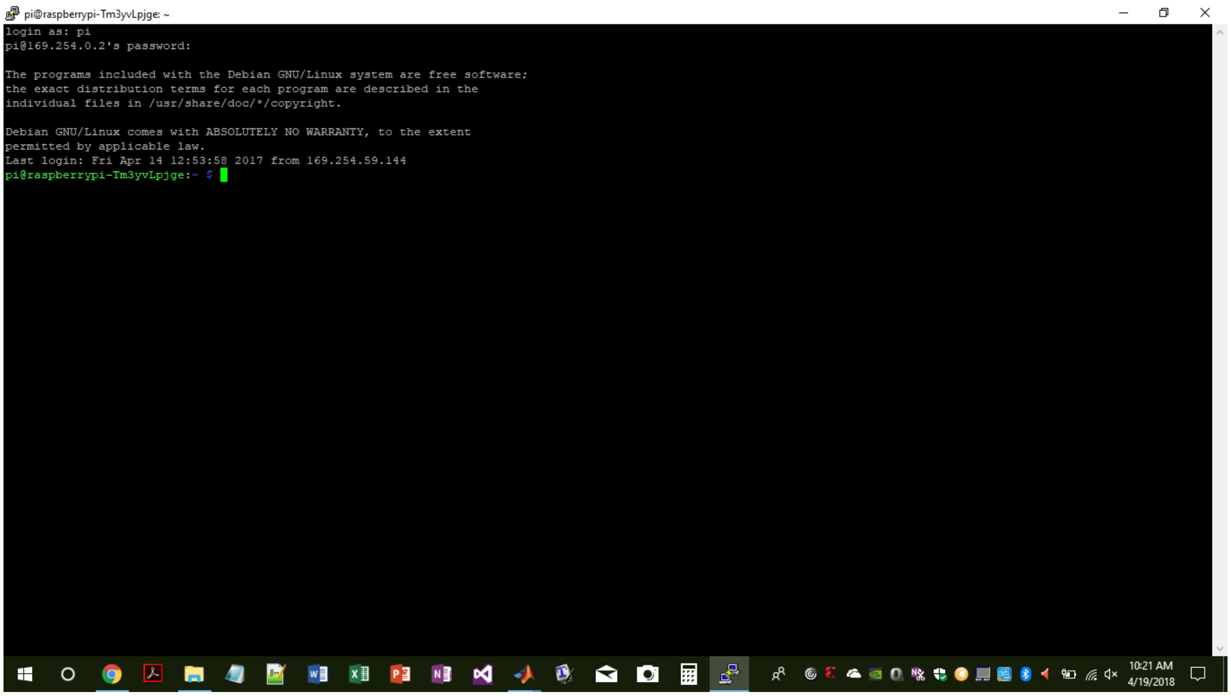


Figure D.4: Login to the Raspberry Pi.

6. We can see that this is the Linux version we installed on the SD card when configuring the support package. See Figure D.5. We can see where the code that will be generated by Simulink will be located.

7. Open the Simulink model that you would like to push to the Raspberry Pi.

8. Deploy the model to hardware. This will generate the C-code on the Raspberry Pi. See Figure D.6. This build may take a while.

9. **NOTE** - The Simulink model provided uses an initialization function to enable the SPI communication pins. If you just powered on the Raspberry Pi, you need to uncomment the line that disables SPI. If have already pushed to the Raspberry Pi while it has been on and want to do it again, you need to comment out the line that disables SPI. If you don't do this, there will be errors aborting the build.

10. Once the model is pushed to the Raspberry Pi, the model will actually begin running as a process.

11. Use the ps -A Linux command to see all the processes running. One should have a similar name of the model; find its process number.

12. Once you have the process number, use the sudo kill -9 1234 Linux command to kill the process. 1234 is the process number you currently found.

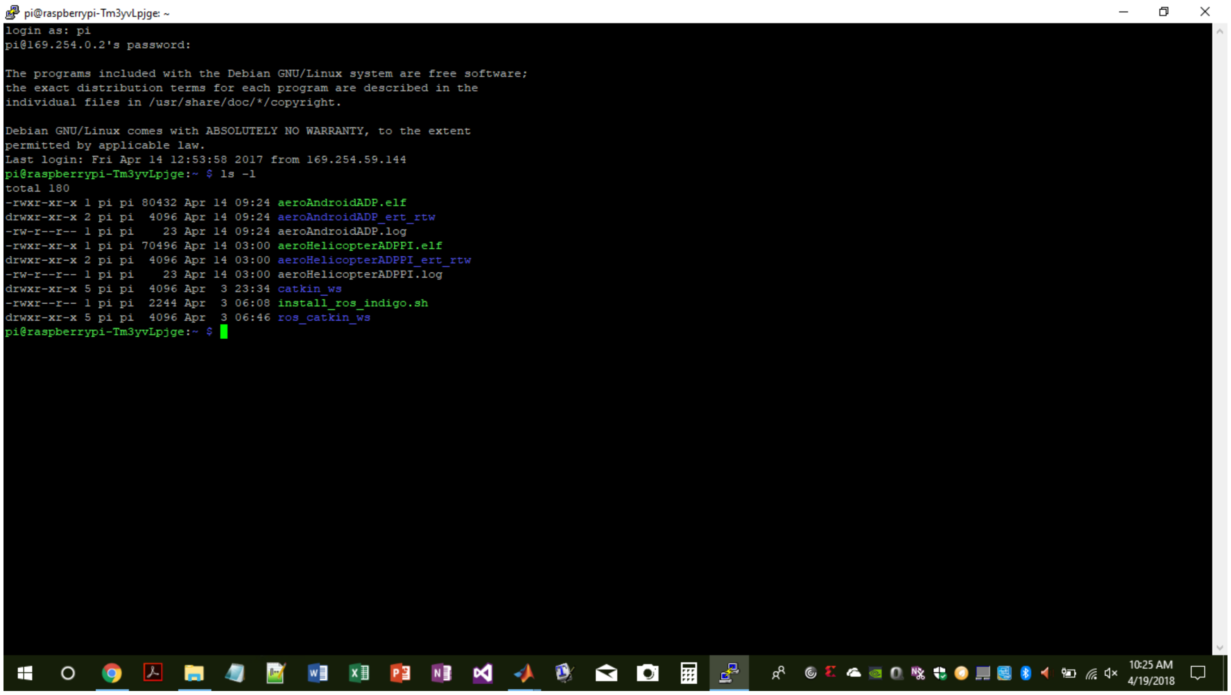


Figure D.5: Version of Linux installed on SD card by MATLAB.

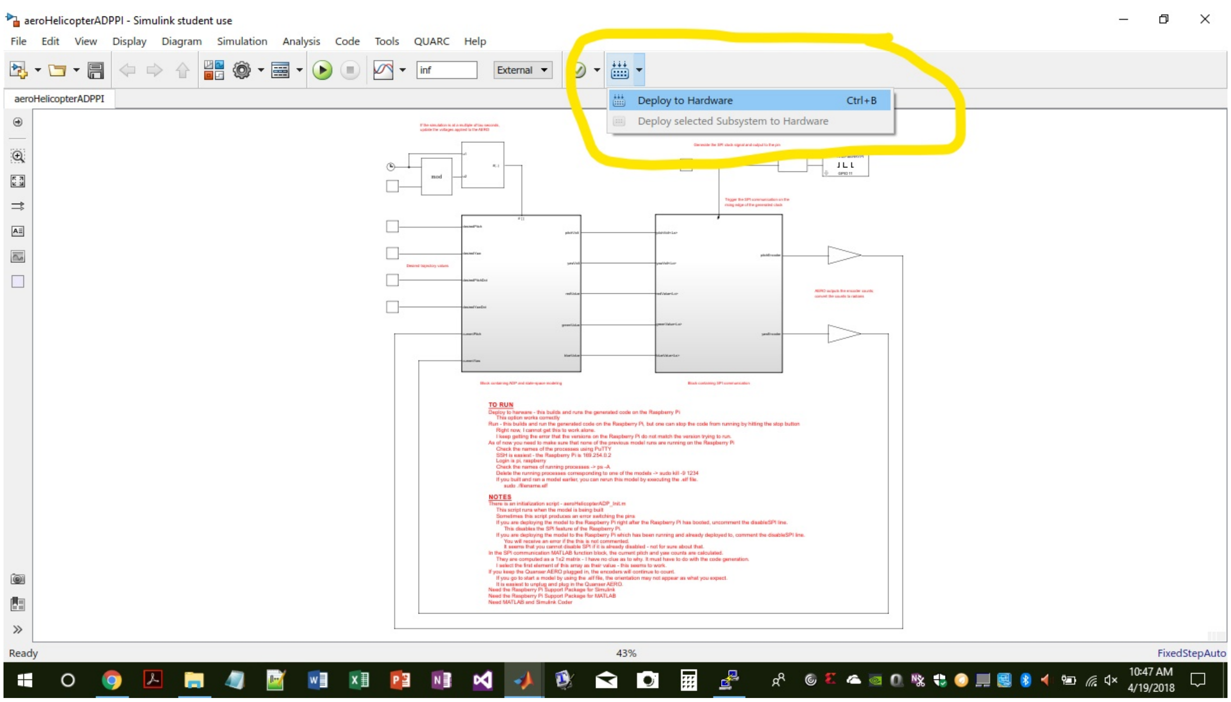


Figure D.6: Deploy model to hardware.

13. You now have the generated code on the Raspberry Pi, but it is not currently running.

14. Connect the Raspberry Pi to the Quanser AERO. See Figure D.7. The SPI wires

should already be connected to the Raspberry Pi. If not, you will need to consult the MATLAB initialization code and the Raspberry Pi data sheet.

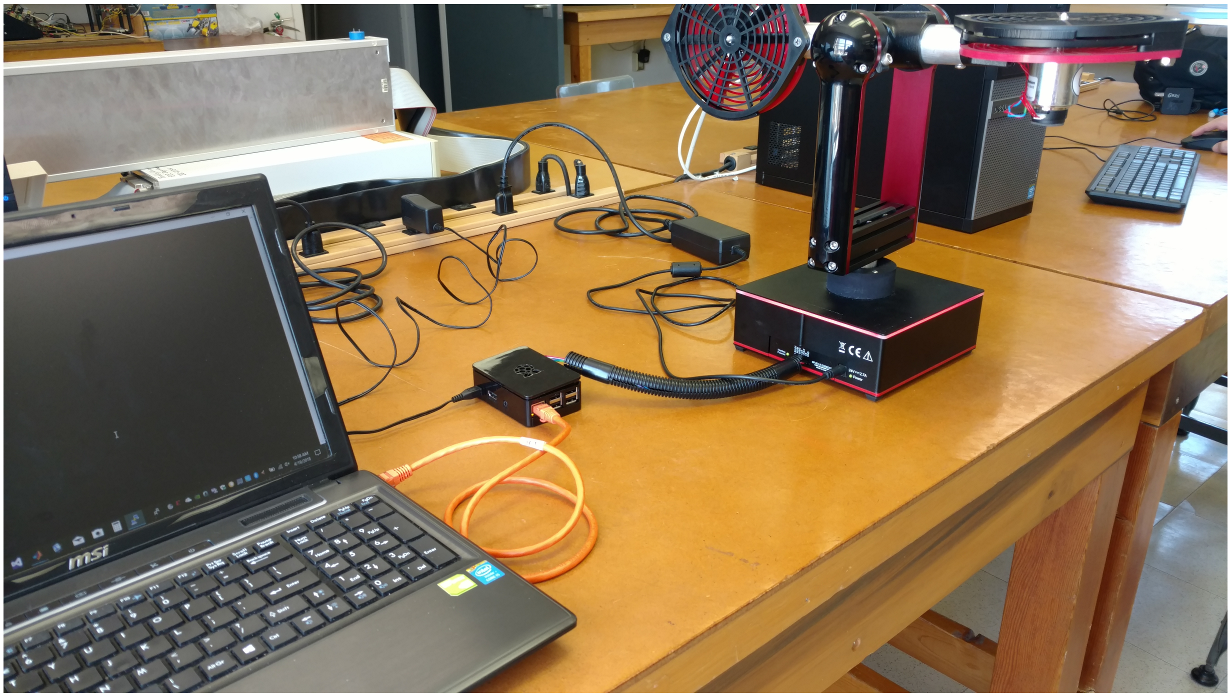


Figure D.7: Connect the Raspberry Pi to the Quanser AERO.

15. Execute the .elf file to run the generated code. Use the Linux command sudo ./file.elf. See Figure D.8.
16. The generated C-code should now be running on the Raspberry Pi.
17. When you are done running the code, repeat the steps used to kill the process. Sometimes you can also use Control-C to stop the running model.

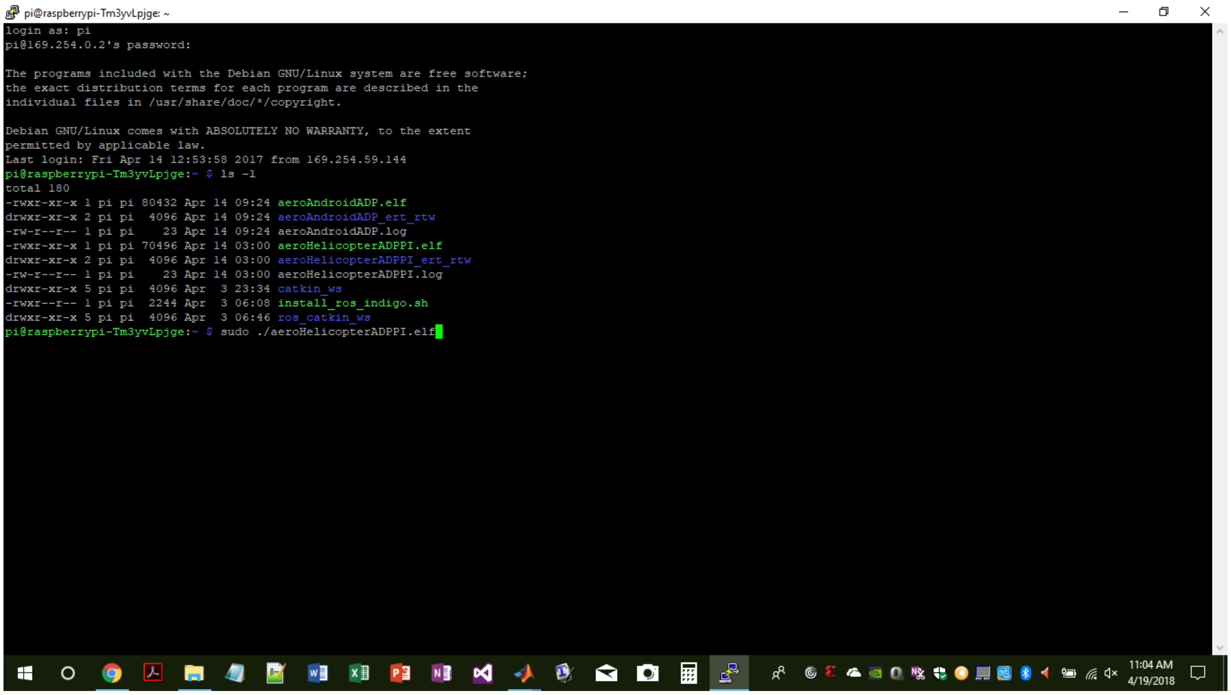


Figure D.8: Execute the code on the Raspberry Pi.

D.2 Initialization Code (MATLAB)

```
% Andrew Fandel
% - FIND THE CRITIC NEURAL NETWORK WEIGHTS BEFORE RUNNING THE SIMULATION
% USING RANDOM ERROR DATA
close all; clear all; clc;

% Sampling time [s]
tau = 0.05;
% Update time [s]
T = 1;

% COST FUNCTION
% Q and R matrices used in the cost function
Q_Mat_ADP = diag([270 100 1 1]);
R_Mat_ADP = 0.005*diag([1 1]);

% CRITIC WEIGHT TUNING NEURAL NETWORK
% This function is specific to the helicopter because of the number of
% weights and error model
% REFERENCE DR. MIAH'S PAPER FOR EQUATION NUMBERS

% SYSTEM PARAMETERS SPECIFIC TO THE 2-DOF QUANSER AERO
A = [0 0 1 0; 0 0 0 1; -1.7442 0 -0.3307 0; 0 0 0 -0.9283];
B = [0 0; 0 0; -0.0149 0.0414; -0.0751 -0.1295];
% System dimensions specific for our model
[n,~] = size(B);

```

```
% CREATE RANDOM ERROR DATA TO TRAIN THE CRITIC NEURAL NETWORK
% Number of equations needed for training, number of sub-intervals
% Number of training samples
nbar = 100;
% Create random error in the range of -pi to pi
e_vec = (2*(pi)).*rand(n,nbar) - (pi);

% ERROR MODEL OF THE HELICOPTER
% fbar and gbar -- EQ 8
% We do not have an xd term because we won't know the desired state vector
% when we execute this code
% This code can also use anonymous functions because this function is
% executed by MATLAB and not Simulink
fbar = @(e) A*e;
gbar = -B;

% DISCRETE-TIME ERROR MODEL FOR TIME TAU
% f and g -- EQ 10
f = @(e) fbar(e)*tau + e;
g = gbar*tau;

% COST FUNCTION PARAMETERS
% State penalizing function in the continuous cost function
Qbar = @(e) e'*Q_Mat_ADP*e;
% Control penalizing matrix in the continuous cost function
Rbar = R_Mat_ADP;
% The discrete-time cost function will have terms:
% Right after EQ 11 in paper
% State penalizing function in the discretized cost function
Q = @(e) Qbar(e)*tau;
% Control penalizing matrix in the discretized cost function
R = Rbar*tau;

% NEURAL NETWORK FUNCTIONS
% Critic neural network activation functions
rho = @(e) [e(1); e(2); e(3); e(4);...
            e(1)^2; e(1)*e(2); e(1)*e(3); e(1)*e(4);...
            e(2)^2; e(2)*e(3); e(2)*e(4); e(3)^2; e(3)*e(4); e(4)^2];
% Partial derivative of rho with respect to e
drhode =@(e) [1, 0, 0, 0;
              0, 1, 0, 0;
              0, 0, 1, 0;
              0, 0, 0, 1;
              2*e(1), 0, 0, 0;
              e(2), e(1), 0, 0;
              e(3), 0, e(1), 0;
              e(4), 0, 0, e(1);
              0, 2*e(2), 0, 0;
              0, e(3), e(2), 0;
              0, e(4), 0, e(2);
              0, 0, 2*e(3), 0;
              0, 0, e(4), e(3);
              0, 0, 0, 2*e(4)];

```

```
% TOLERANCES
% Convergence tolerance for control policy
EpsilonPolicy = 0.1;
% Convergence tolerance for critic neural network
EpsilonWcritic = 0.1;

% TRAINING PARAMETERS
% Number of outer loop iterations
outerLoopMax = 700;
% Number of inner loop iterations
innerLoopMax = 100;
% Number of equations needed for training, number of sub-intervals
% Number of training samples
[~,nbar] = size(e_vec);

% WEIGHT INITIALIZATION
% Initialize the weights of the critic neural network to zero
WcLast = zeros(length(rho(e_vec(:,1))),1);

% LEAST-SQUARES COMPUTATION INITIALIZATION
% Matrices required for computing least squares weights of the critic
% neural networks -- EQ 20
V = zeros(nbar,1);
Lambda = zeros(nbar,length(rho(e_vec(:,1))));

% Matrix to hold the derivative of the error model during policy
% updating
e_k_plus_1 = zeros(n,nbar);

% Product of the least squares matrices must be invertible
% Logic flag indicating if the critic weights are unsolvable
% The weights are unsolvable because the least squares matrices have no
% solution -- not invertible
diverged = 0;

% OUTER LOOP
for i = 1:(outerLoopMax-1)
% Determine if the least squares matrices are invertible
if diverged == 0
    % For each of the data collection (discrete time index)
    for k = 1:nbar
        % Initialize the optimal inputs to zero
        uNew = [0; 0];
        % INNER LOOP
        for j = 1:(innerLoopMax-1)
            % Get the updated input value
            uLast = uNew;
            % Update the error model
            e_k_plus_1(:,k) = f(e_vec(:,k)) + g*uLast;
            % Compute the new optimal inputs
            uNew  = -0.5*(R^(-1))*g'*drhode(e_k_plus_1(:,k))'*WcLast;

            % Check convergence of the optimal inputs
            if norm(uNew - uLast) < EpsilonPolicy
```

```
                break;
            end
        end

        % Update the values for the least-squares computation
        V(k,:) = Q(e_vec(:,k)) + uNew'*R*uNew + WcLast'*rho(e_k_plus_1(:,k))
    ;
        Lambda(k,:) = rho(e_vec(:,k))';
    end
end

% Verify the least square solution exists for the critic's weights
% If the error data is consistent or there is no error, this will not
% hold, so set the weights to zero
if det(Lambda'*Lambda) == 0
    weights = zeros(length(rho(e_vec(:,1))),1);
    break;
end

% Calculate least squares solution of critic's weights -- EQ 20
WcNew = (Lambda'*Lambda)^(-1)*Lambda'*V;
% Make sure the weights did not diverge
% If the weights are diverging, just set them to a large number
if isnan(WcNew)
    weights = 1000*ones(length(rho(e_vec(:,1))),1);
    break;
end

% Check for convergence of the critic weights
if norm(WcNew - WcLast) < EpsilonWcritic
    weights = WcNew;
    break;
end
% If the weights did not converge, repeat the loop
WcLast = WcNew;

% If we reached the last iteration of the loop, just use the last
% weights found
if (i == (outerLoopMax-1))
    weights = WcNew;
end
end

% Use the weights to determine the P matrix
P_Mat = [weights(5) weights(6) weights(7) weights(8);
         weights(6) weights(9) weights(10) weights(11);
         weights(7) weights(10) weights(12) weights(13);
         weights(8) weights(11) weights(13) weights(14)];

% Find the state-feedback gain EQ
K = 0.5*(R_Mat_ADP^-1)*B'*P_Mat;

% Save the initial weights
wcInit = weights;
```

```
% After much trial and error, it seems that when SPI is initiated, the
% channel remains high and there is no clock signal
% I will create a clock signal and issue the slave select
% Create a connection between MATLAB and the Raspberry Pi
aeroPi = raspi;
% Make sure that SPI is disable on the Raspberry Pi so we can use those
% same GPIO pins
% You may have to comment out this line if you are rebuilding a model and
% the SPI is already disabled
%disableSPI(aeroPi);
% Configure the pins for either inputs or outputs
% MOSI - output
configurePin(aeroPi,10,'DigitalOutput');
% MISO - input
configurePin(aeroPi,9,'DigitalInput');
% SPI Clock - output
configurePin(aeroPi,11,'DigitalOutput');
% SS - output
configurePin(aeroPi,8,'DigitalOutput');
% Specify the period of the SPI clockoutput
% 500 kHz = 2 us
spiPeriod = 0.0001;

% Keep only the variables pertaining to what we will need in the workspace
clearvars -except K tau T wcInit aeroPi spiPeriod;
```

D.3 SPI Communication (MATLAB)

```
% THIS FUNCTION CONTROLS THE SPI INTERFACING
% THIS FUNCTION DETERMINES WHAT DATA IS SENT TO THE QUANSER AERO AND WHAT
% DATA IS RETRIEVED FROM THE QUANSER AERO
function [MOSI,SS,pitchEncoder,yawEncoder,byteNumber,bitNumber,
    encoder2_23_16,encoder2_15_8,encoder2_7_0,encoder3_23_16,encoder3_15_8,
    encoder3_7_0] = fcn(MISO, pitchVolt,yawVolt,redValue,greenValue,blueValue
    ,byteNumber,bitNumber,encoder2_23_16,encoder2_15_8,encoder2_7_0,
    encoder3_23_16,encoder3_15_8,encoder3_7_0)
% Variables used in the function
complement = '0000000000000000';
pitchEncoderBin = '000000000000000000000000';
yawEncoderBin = '000000000000000000000000';
tempBin = '00000000000000000000000000000000';
tempVoltBin = '00000000';

% Output the value of the pitch encoder using the latest byte values
% Combine all of the encoder bytes
pitchEncoderIn = [encoder2_23_16 encoder2_15_8 encoder2_7_0];
% Convert the encoder binary vector into a character array
for i = 1:24
    if (pitchEncoderIn(i) == 1)
        pitchEncoderBin(i) = '1';
    else
        pitchEncoderBin(i) = '0';
```

```
    end
end
% Calculate the 2's complement value of the encoder
if(pitchEncoderBin(1) == '1')
    for i = 1:32
        if (i < 9)
            tempBin(i) = '1';
        else
            tempBin(i) = pitchEncoderBin(i - 8);
        end
    end
else
    for i = 1:32
        if (i < 9)
            tempBin(i) = '0';
        else
            tempBin(i) = pitchEncoderBin(i - 8);
        end
    end
end
pitchEncoderTemp = typecast(uint32(bin2dec(tempBin)),'int32');
pitchEncoderOut = cast(pitchEncoderTemp,'double');
pitchEncoder = pitchEncoderOut(1);
% pitchEncoder = cast(pitchEncoderTemp,'double');

% Output the value of the yaw encoder using the latest byte values
% Combine all of the encoder bytes
yawEncoderIn = [encoder3_23_16 encoder3_15_8 encoder3_7_0];
% Convert the encoder binary vector into a character array
for i = 1:24
    if (yawEncoderIn(i) == 1)
        yawEncoderBin(i) = '1';
    else
        yawEncoderBin(i) = '0';
    end
end
% Calculate the 2's complement value of the encoder
if(yawEncoderBin(1) == '1')
    for i = 1:32
        if (i < 9)
            tempBin(i) = '1';
        else
            tempBin(i) = yawEncoderBin(i - 8);
        end
    end
else
    for i = 1:32
        if (i < 9)
            tempBin(i) = '0';
        else
            tempBin(i) = yawEncoderBin(i - 8);
        end
    end
end
```

```
yawEncoderTemp = typecast(uint32(bin2dec(tempBin)),'int32');
yawEncoderOut = cast(yawEncoderTemp,'double');
yawEncoder = yawEncoderOut(1);
% yawEncoder = cast(yawEncoderTemp,'double');

% Determine which byte we are currently sending
% Specify the slave select value
% - 0 to activate the SPI on the Quanser AERO
% - 1 to de-activate the SPI on the Quanser AERO
% Determine the what the byte to be transmitted should be
switch byteNumber
    case 0
    % ** START OF BASE PACKET **
    % BYTE 0
    % MOSI DATA - BASE MODE (0X01)
    % MISO DATA - BASE ID MSB
    % Beginning of the transmission, so SS goes low
    SS = 0;
    readyMOSI = dec2bin(1,8);

    case 1
    % BYTE 1
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - BASE ID LSB
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 2
    % BYTE 2
    % MOSI DATA - BASE WRITE MASK
    % MISO DATA - ENCODER 2 (23-16)
    SS = 0;
    % Enable the overwriting of encoders 2 and 3 and the LED colors
    % Bit 4 - Set encoder 3 enable
    % Bit 3 - Set encoder 2 enable
    % Bit 2 - Write blue LED
    % Bit 1 - Write green LED
    % Bit 0 - Write red LED
    % Don't want to overwrite the encoder values
    readyMOSI = dec2bin(7,8);
    % Receive the MISO byte bit by bit and update the vector holding the
    % byte
    switch bitNumber
        case 1
            if (MISO == true)
                encoder2_23_16(1) = 1;
            else
                encoder2_23_16(1) = 0;
            end
        case 2
            if (MISO == true)
                encoder2_23_16(2) = 1;
            else
                encoder2_23_16(2) = 0;
```

```
            end
        case 3
            if (MISO == true)
                encoder2_23_16(3) = 1;
            else
                encoder2_23_16(3) = 0;
            end
        case 4
            if (MISO == true)
                encoder2_23_16(4) = 1;
            else
                encoder2_23_16(4) = 0;
            end
        case 5
            if (MISO == true)
                encoder2_23_16(5) = 1;
            else
                encoder2_23_16(5) = 0;
            end
        case 6
            if (MISO == true)
                encoder2_23_16(6) = 1;
            else
                encoder2_23_16(6) = 0;
            end
        case 7
            if (MISO == true)
                encoder2_23_16(7) = 1;
            else
                encoder2_23_16(7) = 0;
            end
        case 8
            if (MISO == true)
                encoder2_23_16(8) = 1;
            else
                encoder2_23_16(8) = 0;
            end
    end

    case 3
    % BYTE 3
    % MOSI DATA - RED LED MSB
    % MISO DATA - ENCODER 2 (15-8)
    SS = 0;
    readyMOSI = dec2bin(redValue(1),8);
    % Receive the MISO byte bit by bit and update the vector holding the
    % byte
    switch bitNumber
        case 1
            if (MISO == true)
                encoder2_15_8(1) = 1;
            else
                encoder2_15_8(1) = 0;
            end
```

```
        case 2
            if (MISO == true)
                encoder2_15_8(2) = 1;
            else
                encoder2_15_8(2) = 0;
            end
        case 3
            if (MISO == true)
                encoder2_15_8(3) = 1;
            else
                encoder2_15_8(3) = 0;
            end
        case 4
            if (MISO == true)
                encoder2_15_8(4) = 1;
            else
                encoder2_15_8(4) = 0;
            end
        case 5
            if (MISO == true)
                encoder2_15_8(5) = 1;
            else
                encoder2_15_8(5) = 0;
            end
        case 6
            if (MISO == true)
                encoder2_15_8(6) = 1;
            else
                encoder2_15_8(6) = 0;
            end
        case 7
            if (MISO == true)
                encoder2_15_8(7) = 1;
            else
                encoder2_15_8(7) = 0;
            end
        case 8
            if (MISO == true)
                encoder2_15_8(8) = 1;
            else
                encoder2_15_8(8) = 0;
            end
    end

    case 4
    % BYTE 4
    % MOSI DATA - RED LED LSB
    % MISO DATA - ENCODER 2 (7-0)
    SS = 0;
    readyMOSI = dec2bin(redValue(2),8);
    % Receive the MISO byte bit by bit and update the vector holding the
    % byte
    switch bitNumber
        case 1
```

```
                if (MISO == true)
                    encoder2_7_0(1) = 1;
                else
                    encoder2_7_0(1) = 0;
                end
            case 2
                if (MISO == true)
                    encoder2_7_0(2) = 1;
                else
                    encoder2_7_0(2) = 0;
                end
            case 3
                if (MISO == true)
                    encoder2_7_0(3) = 1;
                else
                    encoder2_7_0(3) = 0;
                end
            case 4
                if (MISO == true)
                    encoder2_7_0(4) = 1;
                else
                    encoder2_7_0(4) = 0;
                end
            case 5
                if (MISO == true)
                    encoder2_7_0(5) = 1;
                else
                    encoder2_7_0(5) = 0;
                end
            case 6
                if (MISO == true)
                    encoder2_7_0(6) = 1;
                else
                    encoder2_7_0(6) = 0;
                end
            case 7
                if (MISO == true)
                    encoder2_7_0(7) = 1;
                else
                    encoder2_7_0(7) = 0;
                end
            case 8
                if (MISO == true)
                    encoder2_7_0(8) = 1;
                else
                    encoder2_7_0(8) = 0;
                end
        end

        case 5
        % BYTE 5
        % MOSI DATA - GREEN LED MSB
        % MISO DATA - ENCODER 3 (23-16)
        SS = 0;
```

```
    readyMOSI = dec2bin(greenValue(1),8);
    % Receive the MISO byte bit by bit and update the vector holding the
    % byte
    switch bitNumber
        case 1
            if (MISO == true)
                encoder3_23_16(1) = 1;
            else
                encoder3_23_16(1) = 0;
            end
        case 2
            if (MISO == true)
                encoder3_23_16(2) = 1;
            else
                encoder3_23_16(2) = 0;
            end
        case 3
            if (MISO == true)
                encoder3_23_16(3) = 1;
            else
                encoder3_23_16(3) = 0;
            end
        case 4
            if (MISO == true)
                encoder3_23_16(4) = 1;
            else
                encoder3_23_16(4) = 0;
            end
        case 5
            if (MISO == true)
                encoder3_23_16(5) = 1;
            else
                encoder3_23_16(5) = 0;
            end
        case 6
            if (MISO == true)
                encoder3_23_16(6) = 1;
            else
                encoder3_23_16(6) = 0;
            end
        case 7
            if (MISO == true)
                encoder3_23_16(7) = 1;
            else
                encoder3_23_16(7) = 0;
            end
        case 8
            if (MISO == true)
                encoder3_23_16(8) = 1;
            else
                encoder3_23_16(8) = 0;
            end
    end

```

```
case 6
% BYTE 6
% MOSI DATA - GREEN LED LSB
% MISO DATA - ENCODER 3 (15-8)
SS = 0;
readyMOSI = dec2bin(greenValue(2),8);
% Receive the MISO byte bit by bit and update the vector holding the
% byte
switch bitNumber
    case 1
        if (MISO == true)
            encoder3_15_8(1) = 1;
        else
            encoder3_15_8(1) = 0;
        end
    case 2
        if (MISO == true)
            encoder3_15_8(2) = 1;
        else
            encoder3_15_8(2) = 0;
        end
    case 3
        if (MISO == true)
            encoder3_15_8(3) = 1;
        else
            encoder3_15_8(3) = 0;
        end
    case 4
        if (MISO == true)
            encoder3_15_8(4) = 1;
        else
            encoder3_15_8(4) = 0;
        end
    case 5
        if (MISO == true)
            encoder3_15_8(5) = 1;
        else
            encoder3_15_8(5) = 0;
        end
    case 6
        if (MISO == true)
            encoder3_15_8(6) = 1;
        else
            encoder3_15_8(6) = 0;
        end
    case 7
        if (MISO == true)
            encoder3_15_8(7) = 1;
        else
            encoder3_15_8(7) = 0;
        end
    case 8
        if (MISO == true)
            encoder3_15_8(8) = 1;
```

```
            else
                encoder3_15_8(8) = 0;
            end
    end

    case 7
    % BYTE 7
    % MOSI DATA - BLUE LED MSB
    % MISO DATA - ENCODER 3 (7-0)
    SS = 0;
    readyMOSI = dec2bin(blueValue(1),8);
    % Receive the MISO byte bit by bit and update the vector holding the
    % byte
    switch bitNumber
        case 1
            if (MISO == true)
                encoder3_7_0(1) = 1;
            else
                encoder3_7_0(1) = 0;
            end
        case 2
            if (MISO == true)
                encoder3_7_0(2) = 1;
            else
                encoder3_7_0(2) = 0;
            end
        case 3
            if (MISO == true)
                encoder3_7_0(3) = 1;
            else
                encoder3_7_0(3) = 0;
            end
        case 4
            if (MISO == true)
                encoder3_7_0(4) = 1;
            else
                encoder3_7_0(4) = 0;
            end
        case 5
            if (MISO == true)
                encoder3_7_0(5) = 1;
            else
                encoder3_7_0(5) = 0;
            end
        case 6
            if (MISO == true)
                encoder3_7_0(6) = 1;
            else
                encoder3_7_0(6) = 0;
            end
        case 7
            if (MISO == true)
                encoder3_7_0(7) = 1;
            else
```

```
                encoder3_7_0(7) = 0;
            end
        case 8
            if (MISO == true)
                encoder3_7_0(8) = 1;
            else
                encoder3_7_0(8) = 0;
            end
    end

    case 8
    % BYTE 8
    % MOSI DATA - BLUE LED LSB
    % MISO DATA - TACHOMETER 2 (23-16)
    SS = 0;
    readyMOSI = dec2bin(blueValue(2),8);

    case 9
    % BYTE 9
    % MOSI DATA - SET ENCODER 2 (23-16)
    % MISO DATA - TACHOMETER 2 (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 10
    % BYTE 10
    % MOSI DATA - SET ENCODER 2 (15-8)
    % MISO DATA - TACHOMETER 2 (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 11
    % BYTE 11
    % MOSI DATA - SET ENCODER 2 (7-0)
    % MISO DATA - TACHOMETER 3 (23-16)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 12
    % BYTE 12
    % MOSI DATA - SET ENCODER 3 (23-16)
    % MISO DATA - TACHOMETER 3 (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 13
    % BYTE 13
    % MOSI DATA - SET ENCODER 3 (15-8)
    % MISO DATA - TACHOMETER 3 (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 14
    % BYTE 14
```

```
    % MOSI DATA - SET ENCODER 3 (7-0)
    % MISO DATA - RESERVED (0X00)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 15
    % ** START OF CORE PACKET **
    % BYTE 15
    % MOSI DATA - CORE MODE (0X01)
    % MISO DATA - CORE ID MSB
    SS = 0;
    readyMOSI = dec2bin(1,8);

    case 16
    % BYTE 16
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - CORE ID LSB
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 17
    % BYTE 17
    % MOSI DATA - CORE WRITE MASK
    % MISO DATA - CURRENT SENSE 0 (15-8)
    SS = 0;
    % Do not enable the overwriting of encoders 0 and 1, but do enable the
    % overwriting of the motor voltages
    % Bit 5 - Set encoder 1 enable
    % Bit 4 - Set encoder 0 enable
    % Bit 3 - Write motor 1 voltage
    % Bit 2 - Write motor 1 enable
    % Bit 1 - Write motor 0 voltage
    % Bit 0 - Write motor 0 enable
    readyMOSI = dec2bin(15,8);

    case 18
    % BYTE 18
    % MOSI DATA - MOTOR 0 COMMAND (15-8)
    % MISO DATA - CURRENT SENSE 0 (7-0)
    SS = 0;
    % Convert the desired voltage to a value between -999 and 999
    % 24 is the saturation level in the model
    pitchVoltTemp = ceil((999*pitchVolt)/24);
    % If the desired voltage is positive, concatenate a '1' to the front of
    % the voltage value
    % Pass the MSB of the new value
    if (sign(pitchVoltTemp) == 1)
        temp = dec2bin(pitchVoltTemp,15);
        for i = 1:8
            if (i == 1)
                tempVoltBin(i) = '1';
            else
                tempVoltBin(i) = temp(i - 1);
            end
```

```
        end
        readyMOSI = tempVoltBin;
    % If the desired voltage is negative, find the 2's complement value of
    % the voltage
    elseif (sign(pitchVoltTemp) == -1)
        temp = dec2bin(-1*pitchVoltTemp,15);
        % Find the complement
        for i = 1:15
            if (temp(i) == '1')
                complement(i) = '0';
            else
                complement(i) = '1';
            end
        end
        temp1 = bin2dec(complement) + 1;
        temp2 = dec2bin(temp1,15);
        for i = 1:8
            if (i == 1)
                tempVoltBin(i) = '1';
            else
                tempVoltBin(i) = temp2(i - 1);
            end
        end
        readyMOSI = tempVoltBin;
    % If the desired voltage is zero, don't activate the motor
    else
        readyMOSI = dec2bin(0,8);
    end

    case 19
    % BYTE 19
    % MOSI DATA - MOTOR 0 COMMAND (7-0)
    % MISO DATA - CURRENT SENSE 1 (15-8)
    SS = 0;
    % Convert the desired voltage to a value between -999 and 999
    % 24 is the saturation level in the model
    pitchVoltTemp = ceil((999*pitchVolt)/24);
    % If the voltage is positive, pass the LSB
    if (sign(pitchVoltTemp) == 1)
        temp = dec2bin(pitchVoltTemp,15);
        readyMOSI = temp(8:15);
    % If the voltage is negative, find the 2's complement value and then
    % pass the LSB
    elseif (sign(pitchVoltTemp) == -1)
        temp = dec2bin(-1*pitchVoltTemp,15);
        % Find the complement
        for i = 1:15
            if (temp(i) == '1')
                complement(i) = '0';
            else
                complement(i) = '1';
            end
        end
        temp1 = bin2dec(complement) + 1;
```

```
        temp2 = dec2bin(temp1,15);
        readyMOSI = temp2(8:15);
    % If the desired voltage is zero, do not activate the motor
    else
        readyMOSI = dec2bin(0,8);
    end

    case 20
    % BYTE 20
    % MOSI DATA - MOTOR 1 COMMAND (15-8)
    % MISO DATA - CURRENT SENSE 1 (7-0)
    SS = 0;
     % Convert the desired voltage to a value between -999 and 999
    % 24 is the saturation level in the model
    yawVoltTemp = ceil((999*yawVolt)/24);
    % If the desired voltage is positive, concatenate a '1' to the front of
    % the voltage value
    % Pass the MSB of the new value
    if (sign(yawVoltTemp) == 1)
        temp = dec2bin(yawVoltTemp,15);
        for i = 1:8
            if (i == 1)
                tempVoltBin(i) = '1';
            else
                tempVoltBin(i) = temp(i - 1);
            end
        end
        readyMOSI = tempVoltBin;
    % If the desired voltage is negative, find the 2's complement value of
    % the voltage
    elseif (sign(yawVoltTemp) == -1)
        temp = dec2bin(-1*yawVoltTemp,15);
        % Find the complement
        for i = 1:15
            if (temp(i) == '1')
                complement(i) = '0';
            else
                complement(i) = '1';
            end
        end
        temp1 = bin2dec(complement) + 1;
        temp2 = dec2bin(temp1,15);
        for i = 1:8
            if (i == 1)
                tempVoltBin(i) = '1';
            else
                tempVoltBin(i) = temp2(i - 1);
            end
        end
        readyMOSI = tempVoltBin;
    % If the desired voltage is zero, don't activate the motor
    else
        readyMOSI = dec2bin(0,8);
    end
```

```

    case 21
    % BYTE 21
    % MOSI DATA - MOTOR 1 COMMAND (7-0)
    % MISO DATA - TACHOMETER 0 (23-16)
    SS = 0;
    % Convert the desired voltage to a value between -999 and 999
    % 24 is the saturation level in the model
    yawVoltTemp = ceil((999*yawVolt)/24);
    % If the voltage is positive, pass the LSB
    if (sign(yawVoltTemp) == 1)
        temp = dec2bin(yawVoltTemp,15);
        readyMOSI = temp(8:15);
    % If the voltage is negative, find the 2's complement value and then
    % pass the LSB
    elseif (sign(yawVoltTemp) == -1)
        temp = dec2bin(-1*yawVoltTemp,15);
        for i = 1:15
            if (temp(i) == '1')
                complement(i) = '0';
            else
                complement(i) = '1';
            end
        end
        temp1 = bin2dec(complement) + 1;
        temp2 = dec2bin(temp1,15);
        readyMOSI = temp2(8:15);
    % If the desired voltage is zero, do not activate the motor
    else
        readyMOSI = dec2bin(0,8);
    end

    case 22
    % BYTE 22
    % MOSI DATA - SET ENCODER 0 (23-16)
    % MISO DATA - TACHOMETER 0 (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 23
    % BYTE 23
    % MOSI DATA - SET ENCODER (7-0)
    % MISO DATA - TACHOMETER 0 (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 24
    % BYTE 24
    % MOSI DATA - SET ENCODER 0 (7-0)
    % MISO DATA - TACHOMETER 1 (23-16)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 25
```

```
    % BYTE 25
    % MOSI DATA - SET ENCODER 1 (23-16)
    % MISO DATA - TACHOMETER 1 (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 26
    % BYTE 26
    % MOSI DATA - SET ENCODER 1 (15-8)
    % MISO DATA - TACHOMETER 1 (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 27
    % BYTE 27
    % MOSI DATA - SET ENCODER 1 (7-0)
    % MISO DATA - STATUS
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 28
    % BYTE 28
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ENCODER 0 (23-16)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 29
    % BYTE 29
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ENCODER 0 (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 30
    % BYTE 30
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ENCODER 0 (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 31
    % BYTE 31
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ENCODER 1 (23-16)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 32
    % BYTE 32
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ENCODER 1 (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);
```

```

    case 33
    % BYTE 33
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ENCODER 1 (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 34
    % BYTE 34
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ACCELEROMETER X (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 35
    % BYTE 35
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ACCELEROMETER X (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 36
    % BYTE 36
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ACCELEROMETER Y (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 37
    % BYTE 37
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ACCELEROMETER Y (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 38
    % BYTE 38
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ACCELEROMETER Z (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 39
    % BYTE 39
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - ACCELEROMETER Z (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 40
    % BYTE 40
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - GYROSCOPE X (15-8)
```

```
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 41
    % BYTE 41
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - GYROSCOPE X (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 42
    % BYTE 42
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - GYROSCOPE Y (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 43
    % BYTE 43
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - GYROSCOPE Y (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 44
    % BYTE 44
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - GYROSCOPE Z (15-8)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 45
    % BYTE 45
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - GYROSCOPE Z (7-0)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 46
    % BYTE 46
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - RESERVED (0X00)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 47
    % BYTE 47
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - RESERVED (0X00)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 48
    % BYTE 48
```

```
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - RESERVED (0X00)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 49
    % BYTE 49
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - RESERVED (0X00)
    SS = 0;
    readyMOSI = dec2bin(0,8);

    case 50
    % BYTE 50
    % MOSI DATA - PADDING BYTE (0X00)
    % MISO DATA - RESERVED (0X00)
    readyMOSI = dec2bin(0,8);
    % Reset the the slave select to begin the process again
    % Maybe not needed
    SS = 1;

    otherwise
    % Just in case
    readyMOSI = dec2bin(0,8);
    SS = 0;
end
% Pass the byte we have derived bit by bit
% Use the character array to determine the bit value
switch bitNumber
    case 1
        if (readyMOSI(1) == '1')
            MOSI = 1;
        else
            MOSI = 0;
        end
    case 2
        if (readyMOSI(2) == '1')
            MOSI = 1;
        else
            MOSI = 0;
        end
    case 3
        if (readyMOSI(3) == '1')
            MOSI = 1;
        else
            MOSI = 0;
        end
    case 4
        if (readyMOSI(4) == '1')
            MOSI = 1;
        else
            MOSI = 0;
        end
    case 5
```

```
        if (readyMOSI(5) == '1')
            MOSI = 1;
        else
            MOSI = 0;
        end
    case 6
        if (readyMOSI(6) == '1')
            MOSI = 1;
        else
            MOSI = 0;
        end
    case 7
        if (readyMOSI(7) == '1')
            MOSI = 1;
        else
            MOSI = 0;
        end
    case 8
        if (readyMOSI(8) == '1')
            MOSI = 1;
        else
            MOSI = 0;
        end
        % If we have just sent the last bit, update the byte number
        byteNumber = byteNumber + 1;
        % Reset back to zero if we have finished one transfer
        if (byteNumber == 51)
            byteNumber = 0;
        end
    otherwise
        MOSI = 0;
end
% Update the bit number that we are sending
bitNumber = bitNumber + 1;
% Reset back to 1 if we have finished one byte
if (bitNumber == 9)
    bitNumber = 1;
end
end
```

Appendix E

Android Application

E.1 Android Application (Tutorial)

1. Verify that all of the proper software is installed as mentioned in Section 6.4.
2. Repeat the steps used in Appendix D.1 to push the Simulink model onto the Raspberry Pi. The Simulink model is specific for Android communication.
3. Open the Simulink model for the Android smart phone application.
4. Connect your Android smart phone to your laptop using a USB cable. Mine was provided with the phone. See Figure E.1.
5. Deploy the model to the hardware. This is the same as that in Appendix D.1, but this time the hardware is your smart phone.
6. You need to have your phone connected to internet in order for the build to complete.
7. Once the build is complete, you should see an application with the name of your model. See Figure E.2.
8. Connect your smart phone to the local network you set up with static IPs. This was discussed in Section 6.4.
9. Open the application and the model should begin to run. See Figure E.3.
10. Execute the code on the Raspberry Pi for its corresponding model. This is the same as in Appendix D.1.
11. If both models are setup correctly and the IP addresses are consistent with the models, there should be communication working between the two devices.

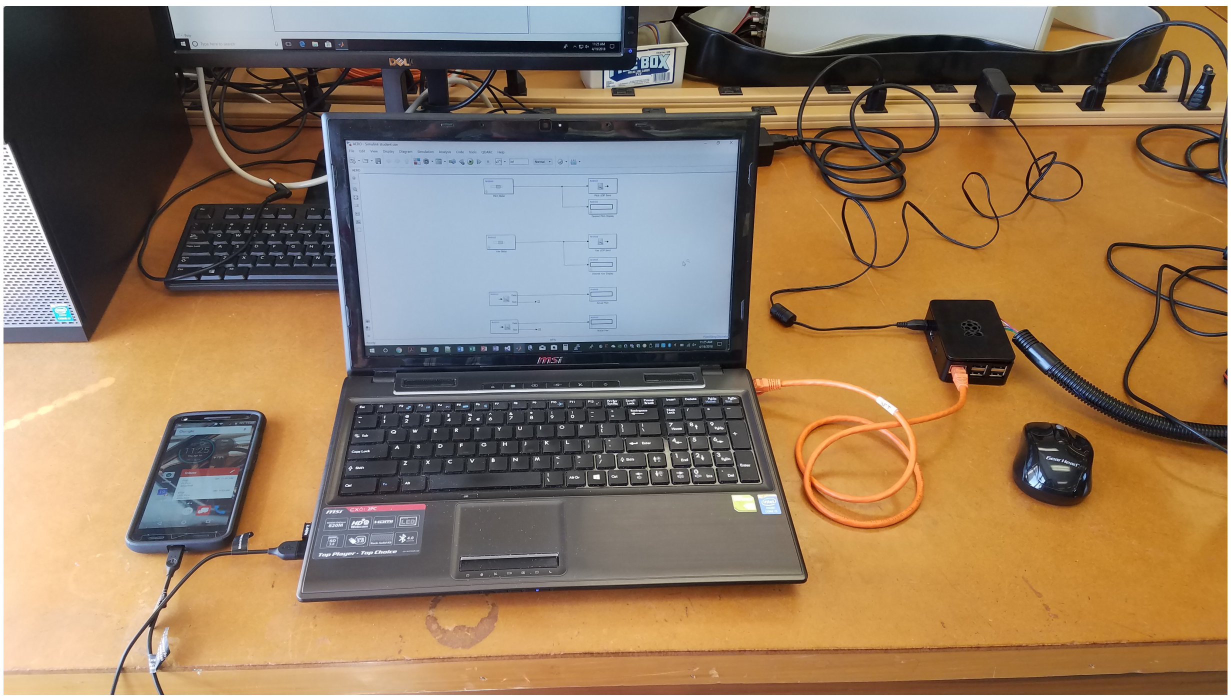


Figure E.1: Connect your Android smart phone to your laptop.

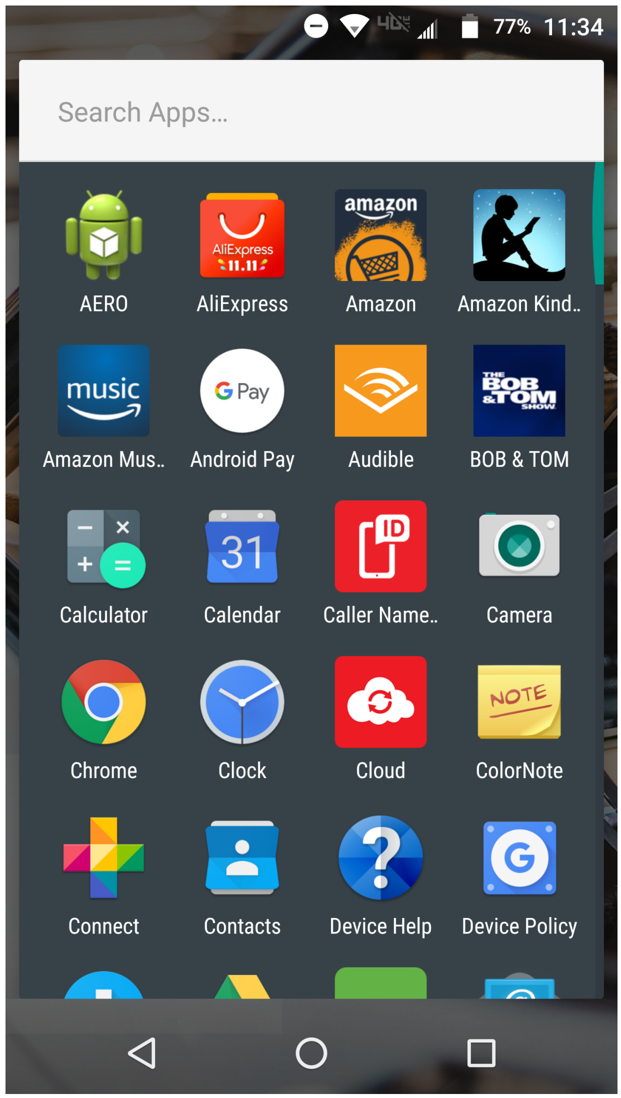


Figure E.2: Android smart phone application.

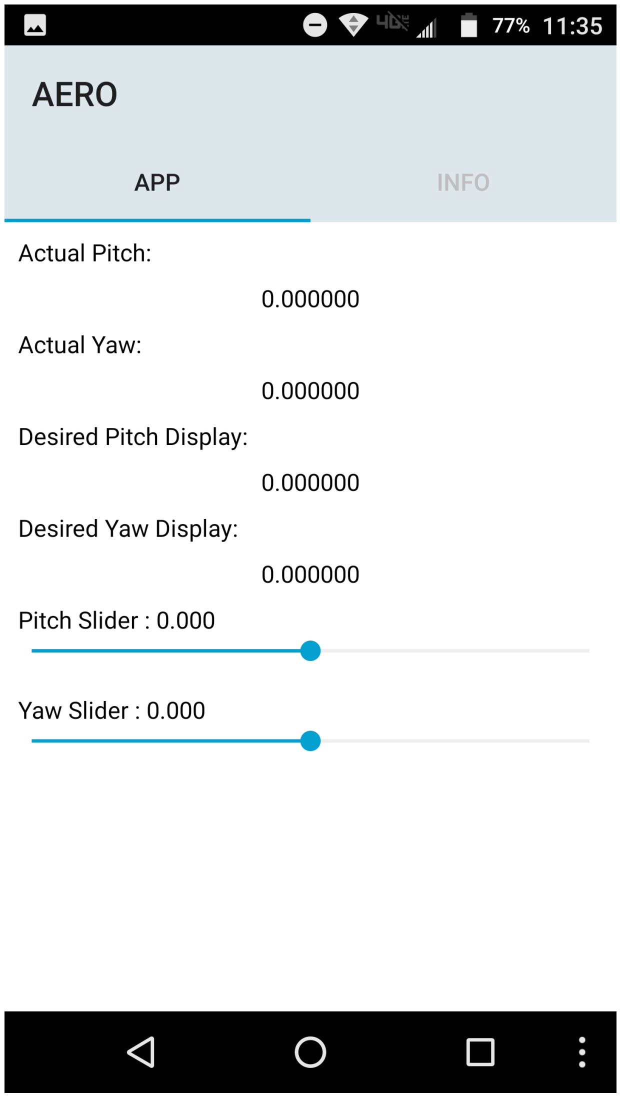


Figure E.3: Android smart phone application.

Appendix F

Quanser AERO Data Sheets

F.1 Quanser AERO Product Specifications

QUANSER AERO

Dual-rotor aerospace experiment with reconfigurable dynamic components for mechatronics exploration and controls.

See system specifications on reverse.

FLEXIBLE PLATFORM FOR MECHATRONICS AND CONTROLS

The Quanser AERO is a fully integrated lab experiment, designed for teaching mechatronics and control concepts at the undergraduate level, as well as for advanced aerospace research applications.

The experiment is reconfigurable for various aerospace systems, from 1 DOF attitude control and 2 DOF helicopter to half-quadrotor. Integrating Quanser-developed QFLEX 2 computing interface technology, the Quanser AERO also offers flexibility in lab configurations, using a PC, or microcontrollers, such as NI myRIO, Arduino and Raspberry Pi. With the comprehensive course materials included, you can build a state-of-the-art teaching lab for your mechatronics or control courses, engage students in various design and capstone projects, and validate your research concepts on a high-quality, robust, and precise platform.

HOW IT WORKS

The Quanser AERO consists of two propellers, powered by DC motors. Combined with the light-weight design of the experiment, this makes the system capable of highly responsive movements. The Quanser AERO's compact base includes a built-in amplifier with an integrated current sensor, built-in data acquisition device, and an interchangeable QFLEX 2 interface panel. The experiment comes with additional propellers to illustrate the efficiency of different propeller designs and effects of cross-coupling.

The propeller motors are equipped with optical encoders. The motor current and voltage sensors can be used to monitor the power consumption of the experiment.

The slip ring mechanism allows for continuous 360° yaw rotation. The angles of the pitch and yaw axes are measured using high-resolution optical encoders. The pitch and yaw axis can be independently locked, and the angle of the propeller assemblies can be adjusted between horizontal and vertical positions. This allows users to reconfigure the Quanser AERO for various aerospace systems (1 DOF attitude control, 2 DOF helicopter, half-quadrotor) and experiments (e.g. pitch-only system modeling).

The Inertial Measurement Unit (IMU) board includes accelerometer and gyroscope sensors, which can be used for attitude and yaw estimation and verification against the direct position measurements from the encoders.

The Quanser AERO also has a user-controllable tri-color LED strip. It can be programmed to indicate state, power, or other control performance characteristics of the Quanser AERO.

QUANSER AERO SOLUTION COMPONENTS

- Quanser AERO with QFLEX 2 interface panel of your choice *(USB or Embedded)*
  Optional: Additional QFLEX 2 interface panel
- Quanser Control Software (required for Quanser AERO USB experiment): QUARC for MATLAB/Simulink or QRCP for LabVIEW*
- Complete dynamic model and pre-build controllers
- ABET[1]-aligned, flexible digital media courseware (for Quanser AERO USB experiment)
- Arduino examples and interfacing datasheet (for Quanser AERO Embedded experiment)

**MATLAB/Simulink and LabVIEW licenses not included*

QUANSER AERO INTERFACE OPTIONS

The Quanser AERO is available with two different, easily interchangeable interface panels:

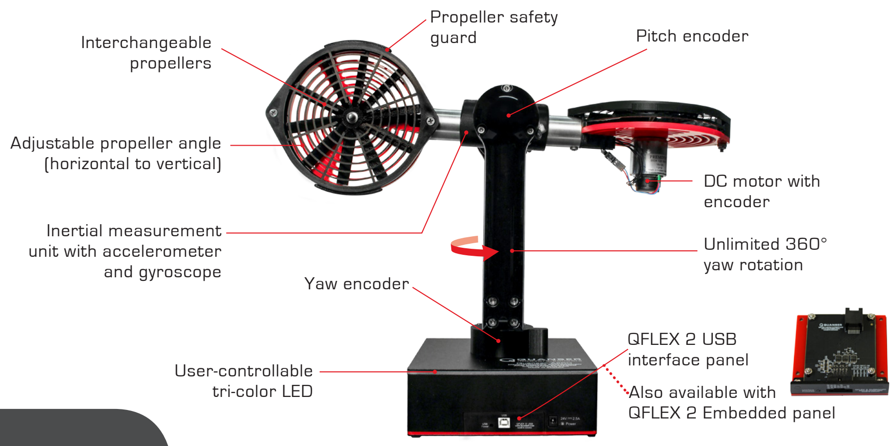
- Quanser AERO USB experiment (with QFLEX 2 USB panel) interfaces to Quanser's control software running on your lab's PC via a standard USB 2.0 connection. The Quanser AERO USB can be used with MATLAB®/ Simulink® and Quanser QUARC software, or with LabVIEW™ using the Quanser RCP software. With the USB version of the experiment, you can take full advantage of the comprehensive course materials and lab experiments for your controls-based courses and projects.
- Quanser AERO Embedded experiment (with QFLEX 2 Embedded panel) interfaces to your microcontroller (not included with the experiment) via SPI connection. The Quanser AERO Embedded does not require any additional software. This option is ideal to expose students to various microcontroller techniques, as well as for final (capstone) projects in mechatronics, control, or other similar programs.

Note: *The Quanser AERO experiment includes one interface panel of your choice. Additional interface panel(s) can be purchased separately.*

[1] ABET, Inc. is the recognized accreditor for college and university programs in applied science, computing, engineering, and technology.

SYSTEM SPECIFICATIONS

Quanser AERO



FEATURES

- Compact and integrated system
- High-efficiency coreless DC motors
- High resolution optical encoders
- Pitch & yaw axes and DC motors/rotors speed measurements through digital tachometer
- Built-in voltage amplifier with integrated current sensor
- Integrated data acquisition(DAQ) device
- User-controllable tri-color LED
- Easy-connect cables and connectors
- Flexible QFLEX 2 computing interface for USB and SPI connections
- Open architecture design, allowing users to design their own controller
- Fully compatible with MATLAB®/Simulink® and LabVIEW™
- Fully documented system models and parameters provided for MATLAB®/Simulink® and LabVIEW™
- ABET-aligned, modular, digital media courseware provided for the Quanser AERO USB
- Microcontroller examples and interfacing datasheet provided for the Quanser AERO Embedded
- Additional community-created resources available on www.QuanserShare.com

COURSEWARE TOPICS COVERED

The Quanser AERO USB courseware includes ABET[1]-aligned **Instructor and Student Workbooks** with complete lab exercises, covering topics:

- Hardware integration
- Single propeller speed control

1 DOF attitude control configuration:

- PID control
- Introduction to IMU
- Modeling using transfer function
- System identification
- Gain scheduling

Laboratory Guides with modeling and control design examples:

2 DOF helicopter configuration

- Modeling
- Linear state-space representation
- State-feedback control
- Coupled dynamics

Half-quadrotor configuration

- Modeling
- Simple yaw control
- Kalman filter

DEVICE SPECIFICATION

Base dimensions (W x H x D)	17.8 cm x 17.8 cm x 7 cm
Device height (with propeller in horizontal position)	35.6 cm
Device length	51 cm
Device mass	3.6 kg
Propeller diameter	12.7 cm
Yaw angle range	360°
Elevation angle range	124° (± 62° from horizontal) half-quadrotor configuration
Pitch encoder resolution (in quadrature)	512 counts/revolution
Yaw/travel encoder resolution (in quadrature)	1024 counts/revolution
Motor current torque constant	57.7 mN.m/A
Tri-axis gyroscope range	± 245 dps
Tri-axis accelerometer range	± 2g
Interfaces available: QFLEX 2 USB	USB 2.0
QFLEX 2 Embedded	SPI

About Quanser:
Quanser is the world leader in education and research for real-time control design and implementation. We specialize in outfitting engineering control laboratories to help universities captivate the brightest minds, motivate them to success and produce graduates with industry-relevant skills. Universities worldwide implement Quanser's open architecture control solutions, industry-relevant curriculum and cutting-edge work stations to teach Introductory, Intermediate or Advanced controls to students in Electrical, Mechanical, Mechatronics, Robotics, Aerospace, Civil, and various other engineering disciplines.

v. 1.1

F.2 Quanser AERO User Manual

USER MANUAL

Quanser AERO Experiment

Set Up and Configuration

CAPTIVATE. MOTIVATE. GRADUATE.

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For more information on the solutions Quanser Inc. offers, please visit the web site at:
http://www.quanser.com

FCC Notice This device complies with Part 15 of the FCC rules. Operation is subject to the following two conditions: (1) this device may not cause harmful interference, and (2) this device must accept any interference received, including interference that may cause undesired operation.

Industry Canada Notice This Class A digital apparatus complies with Canadian ICES-003. Cet appareil numérique de la classe A est conforme à la norme NMB-003 du Canada.

Japan VCCI Notice This is a Class A product based on the standard of the Voluntary Control Council for Interference (VCCI). If this equipment is used in a domestic environment, radio interference may occur, in which case the user may be required to take corrective actions.

この装置は、クラス A 情報技術装置です。この装置を家庭環境で使用すると電波妨害を引き起こすことがあります。この場合には使用者が適切な対策を講ずるよう要求されることがあります。 VCCI–A

Korea Communications Comission (KCC) Notice This equipment is Industrial (Class A) electromagnetic wave suitability equipment and seller or user should take notice of it, and this equipment is to be used in the places except for home.

이 기기는 업무용(A급) 전자파적합기기로서 판매자 또는 사용자는 이 점을 주의하시기 바라며, 가정외의 지역에서 사용하는 것을 목적으로 합니다.

Waste Electrical and Electronic Equipment (WEEE)

This symbol indicates that waste products must be disposed of separately from municipal household waste, according to Directive 2002/96/EC of the European Parliament and the Council on waste electrical and electronic equipment (WEEE). All products at the end of their life cycle must be sent to a WEEE collection and recycling center. Proper WEEE disposal reduces the environmental impact and the risk to human health due to potentially hazardous substances used in such equipment. Your cooperation in proper WEEE disposal will contribute to the effective usage of natural resources. For information about the available collection and recycling scheme in a particular country, go to ni.com/citizenship/weee.

电子信息产品污染控制管理办法 （中国 RoHS）

中国客户 National Instruments 符合中国电子信息产品中限制使用某些有害物质命令 （RoHS）。
关于National Instruments 中国 RoHS合规性信息，请登录 ni.com/environment/rohs_china
(For information about China RoHS compliance, go to ni.com/environment/rohs_china)

CE Compliance

This product meets the essential requirements of applicable European Directives as follows:

- 2006/95/EC; Low-Voltage Directive (safety)
- 2004/108/EC; Electromagnetic Compatibility Directive (EMC)

Warning: This is a Class A product. In a domestic environment this product may cause radio interference, in which case the user may be required to take adequate measures

CONTENTS

1 PRESENTATION

The Quanser Quanser Aero, pictured in Figure 1.1, is a compact dual-rotor two degree-of-freedom aerospace system that can be used to perform a variety of actuator and flight control based experiments. The Quanser Aero can be configured with either the QFLEX 2 USB or QFLEX 2 Embedded interface modules. The QFLEX 2 USB allows control by a computer via USB connection. The QFLEX 2 Embedded allows for control by a microcontroller device such as an Arduino via a 4-wire SPI interface.

For all versions, the system is driven using two direct-drive 18V brushed DC motors. The motors are powered by a built-in PWM amplifier with built-in current sense. Single-ended rotary encoders are used to measure the angular position of the DC motors, and the speed of the motors can be measured with a software tachometer.

Main Quanser Aero features:

- Compact and complete aerospace control system
- 18V direct-drive brushed DC motors
- Encoders mounted on DC motors and support yolk
- DC motor tachometer output
- Built-in PWM amplifier with integrated current sense
- Built-in data acquisition (DAQ) device
- Low and High-efficiency propellers
- Lockable pitch and yaw axes
- Tri-color LED indicator lights

Figure 1.1: Quanser Aero

Caution: This equipment is designed to be used for educational and research purposes and is not intended for use by the general public. The user is responsible to ensure that the equipment will be used by technically qualified personnel only.

2 SYSTEM HARDWARE

2.1 System Schematic

The Quanser Aero can be configured with one of two different I/O interfaces: the QFLEX 2 USB, and the QFLEX 2 Embedded. The QFLEX 2 USB provides a USB interface for use with a computer. The QFLEX 2 Embedded provides a 4-wire SPI interface for use with an external microcontroller board.

The interaction between the different system components on the Quanser Aero is illustrated in Figure 2.1. On the data acquisition (DAQ) device block, the motor position encoders are connected to Encoder Input (EI) channels #0 and #1. EI2 reads the pitch angle of the Aero body, and EI3 reads the yaw angle of the yoke. The Analog Output (AO) channels are connected to the power amplifier command, which then drives the DC fan motors. The DAQ Analog Input (AI) channels are connected to the PWM amplifier current sense circuitry. The DAQ also controls the integrated tri-colour LEDs via an internal serial data bus. The DAQ can be interfaced to the PC or laptop via USB link in the QFLEX 2 USB, or to an external microcontroller via SPI in the QFLEX 2 Embedded.

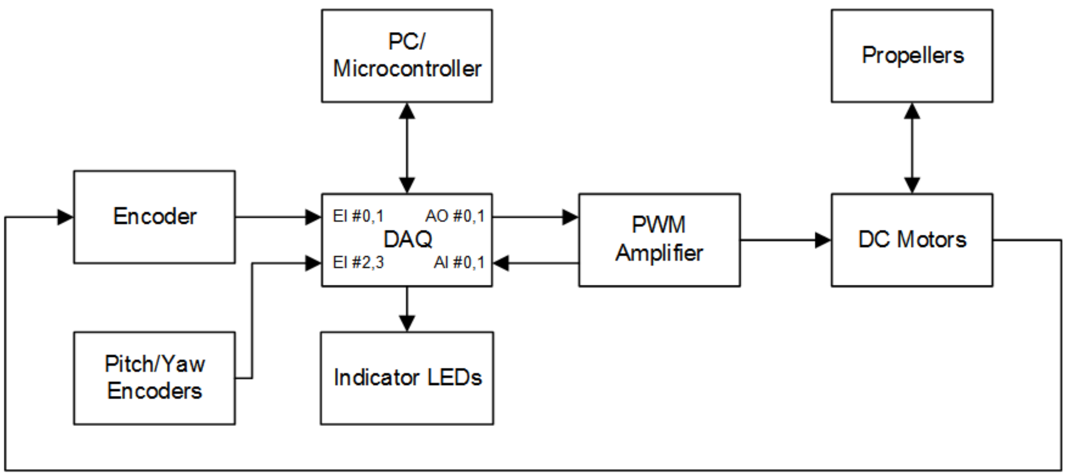


Figure 2.1: Interaction between Quanser Aero components.

The schematic given in Figure **??** illustrates the main Quanser Aero components and how they interact with a data acquisition (DAQ) device.

2.2 Hardware Components

The main Quanser Aero components - for the USB and SPI embedded interfaces - are listed in Table 2.1. The components on the QFLEX 2 USB are labeled in Figure 2.2c, the components on the QFLEX 2 Embedded are shown in Figure 2.2d.

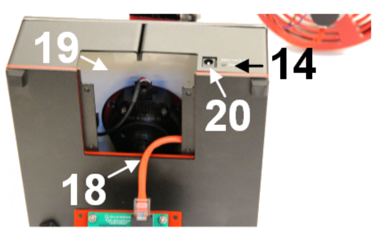
ESD Warning: **Quanser Aero internal components are sensitive to electrostatic discharge. Before handling the Quanser Aero ensure that you have been properly grounded.**

ID	Component	ID	Component
1	Aero base	11	Propeller guard screw
2	Yaw pivot	12	Propeller attachment hub
3	Support yolk	13	IMU (Internal to aero body)
4	Aero body	14	System Power LED
5	Pitch pivot	15	Interface Power LED
6	Pitch lock screws	16	SPI Data Connector*
7	Status LED strip	17	USB connector†
8	Thruster rotation locks	18	Quanser Aero internal data bus
9	Thruster 0	19	Quanser Aero DAQ/amplifier board
10	Thruster 1	20	Power Connector

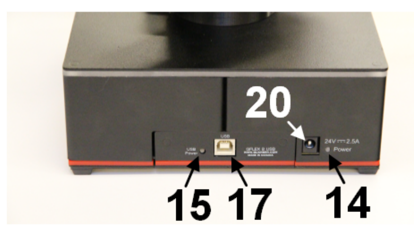
Table 2.1: Quanser Aero Components

† only on QFLEX 2 USB
*only on QFLEX 2 Embedded

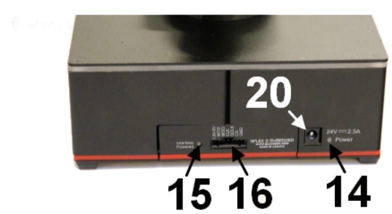
(a) Quanser Aero Top View



(b) Quanser Aero Interior



(c) Quanser Aero with QFLEX 2 USB



(d) Quanser Aero with QFLEX 2 Embedded

Figure 2.2: Quanser Aero components

2.2.1 DC Motor

The Quanser Aero includes two direct-drive 18V brushed DC motors. The motor specifications are given in Table 2.2.

The Quanser Aero incorporates the Allied Motion CL40 Series Coreless DC Motor model 16705. The complete specification sheet of the motor is included at: http://alliedmotion.com/Products/Series.aspx?s=29.

Caution: Max motor input ± 18 V, 2 A peak, 0.5 A continuous.

Caution: Exposed moving parts.

Caution: Holding the motor in a stalled position for a prolonged period of time at applied voltages of over 5V can result in permanent damage.

2.2.2 Thruster Assemblies

The Quanser Aero has two identical thruster assemblies which are attached to the Aero body. Thruster 0 can be identified by locating the pitch lock screws or the yaw lock magnets. Both of these items are on the side of the yolk facing thruster 0.

2.2.3 High-efficiency Propeller

The Quanser Aero ships with two counter-rotating APC 5.0x4.6 propellers, models LP05046E/EP. More information on the propellers can be found on the Advanced Precision Composites website (www.apcprop.com).

2.2.4 Low-efficiency Propeller

The Quanser Aero ships with two eight-vane counter-rotating 3D-printed propellers.

2.2.5 Propeller Hub

The Quanser Aero propellers are connected to the DC motors with aluminum prop adapters with collets. The propeller adaptors are E-flite part number EFLM1922.

2.2.6 Pitch and Motor Position Encoders

The encoders used to measure the pitch of the Aero body and the angular position of the DC motorson the Quanser Aero is a single-ended optical shaft encoder. It outputs 2048 counts per revolution in quadrature mode (512 lines per revolution).

The encoders used to measure the pitch of the Aero body, and angular position of the DC motors on the Quanser Aero is the US Digital E8P-512-118 single-ended optical shaft encoder. The complete specification sheet of the E8P optical shaft encoder is given in E8P Data Sheet.

2.2.7 Yaw Encoder

The encoders used to measure the yaw of the support yolk on the Quanser Aero is an optical encoder. It outputs 4096 counts per revolution in quadrature mode (1024 lines per revolution).

The encoders used to measure the yaw of the support yolk on the Quanser Aero is the US Digital E3-1024-984 optical encoder. The complete specification sheet of the E3 optical shaft encoder is given in E3 Data Sheet.

2.2.8 Inertial Measurement Unit

The Quanser Aero includes an integrated IMU mounted within the Aero body. This module allows for real-time measurement of the angular position and velocity along all three of the primary axes of the Aero body.

The IMU incorporated into the Quanser Aero is the STMicroelectronics LSM6DS0 iNEMO intertial module. Further information on the module can be found in the LSM6DS0 data sheet.

2.2.9 Data Acquisition (DAQ) Device

The Quanser Aero includes an integrated data acquisition device with four 16-bit encoder channels with quadrature decoding and two PWM analog output channels. The DAQ also incorporates a 12-bit ADC which provides current sense feedback for the motors. The current feedback is used to detect motor stalls and will disable the amplifier if a prolonged stall is detected.

2.2.10 Power Amplifier

The Quanser Aero circuit board includes a PWM voltage-controlled power amplifier capable to providing 2 A peak current and 0.5 A continuous current (based on the thermal current rating of the motor). The output voltage range to the load is between ±24 V.

2.2.11 Power Supply

The Quanser Aero is equipped with an external DC power supply which provides power for the sensors and motors. This supply is intended for use with 100-240 VAC at 50-60 Hz.

Only the provided power supply and AC cord should be used with the Quanser Aero. The included supply is an Adapter Technology Co Ltd model ATS065-P241.

2.2.12 Embedded System Connector

The SPI data connector pictured on the QFLEX 2 Embedded in Figure 2.2d allows an external microcontroller to set motor voltage and LED brightnesses, read and set encoder counters, and read motor speed and current flow. See the QFLEX 2 Embedded data sheet for information on connecting the SPI interface.

2.3 Environmental

The Quanser Aero is designed to function under the following environmental conditions:

- Standard rating
- Indoor use only
- Temperature 5°C to 40°C
- Altitude up to 2000 m
- Maximum relative humidity of 80% up to 31°C decreasing linearly to 50% relative humidity at 40°C
- Pollution Degree 2
- Mains supply voltage fluctuations up to ± 10% of nominal voltage
- Maximum transient overvoltage 2500 V

- Marked degree of protection to IEC 60529: Ordinary Equipment (IPX0)

2.4 System Parameters

Table 2.2 lists and characterizes the main parameters associated with the Quanser Aero.

Symbol	Description	Value
DC Motor		
V_{nom}	Nominal input voltage	18.0 V
τ_{nom}	Nominal torque	22.0 mN-m
ω_{nom}	Nominal speed	3050 RPM
I_{nom}	Nominal current	0.540 A
R_m	Terminal resistance	8.4 Ω
k_t	Torque constant	0.042 N-m/A
k_m	Motor back-emf constant	0.042 V/(rad/s)
J_m	Rotor inertia	4.0×10^{-6} kg-m^2
L_m	Rotor inductance	1.16 mH
Aero Body		
M_b	Mass of body	1.075 kg
D_m	Center of mass	-7.59 mm
J_p	Pitch inertia	2.15×10^{-2} kg-m^2
J_y	Yaw inertia	2.37×10^{-2} kg-m^2
D_t	Thrust displacement	15.8 cm
Motor and Pitch Encoders		
	Encoder line count	512 lines/rev
	Encoder line count in quadrature	2048 lines/rev
	Encoder resolution (in quadrature)	0.176 deg/count
Yaw Encoder		
	Encoder line count	1024 lines/rev
	Encoder line count in quadrature	4096 lines/rev
	Encoder resolution (in quadrature)	0.088 deg/count
Amplifier		
	Amplifier type	PWM
	Peak Current	2 A
	Continuous Current	0.5 A
	Output voltage range (recommended)	$\pm$18 V
	Output voltage range (maximum)	$\pm$24 V

Table 2.2: Quanser Aero System Parameters

3 SYSTEM SETUP

Caution: If the equipment is used in a manner not specified by the manufacturer, the protection provided by the equipment may be impaired.

3.1 Components

To setup the Quanser Aero system, you need the following components:

1. Quanser Aero (USB or Embedded version)
2. High and low-efficiency propellers
3. Propeller adapters
4. Power supply with the following ratings:
   - Input Rating: 100-240 V AC, 50-60 Hz, 1.4 A
   - Output Rating: 24 V DC, 2.71 A

   Note: Only the power supply provided (AC-DC adapter by Adapter Technology Co Ltd, model ATS065-P241) should be used with the Quanser Aero
5. Power cable

   Note: Only the power cable provided should be used with the Quanser Aero

   Note: Make sure that the power cable is accessible for disconnection in case of emergency.

   Caution: Precaution must be taken during the connection of this equipment to the AC outlet to make sure the grounding (earthing) is in place, and that the ground wire is not disconnected
6. USB 2.0 A/B cable (for QFLEX 2 USB) or jumper wires (for QFLEX 2 Embedded)

3.2 QFLEX 2 USB Hardware Setup

To setup the QFLEX 2 USB follow these steps:

1. The Quanser Aero should have one of the included sets of propellers installed. If the other propellers are required, follow the procedure for exchanging propellers in Section **?? before** connecting power.
2. Connect USB 2.0 cable from back cover of Quanser Aero to an enabled USB 2.0 port on your desktop PC or laptop.
3. Connect the **Power** connector on the Quanser Aero to the power supply. Ensure the power supply is connected to a wall outlet using the appropriate power cable.
4. The QFLEX 2 USB driver should install automatically. If not, then you may not have installed all the required software to support the device including either QUARC®or Quanser Rapid Control Prototyping Toolkit®.

3.3 QFLEX 2 Embedded Hardware Setup

This section describes how to connect the QFLEX 2 Embedded to an external microcontroller board. The connection procedure is given below, and summarized in Table 3.1. The wires required to connect the QFLEX 2 Embeddedare not included with the unit, connections may be made with jumper wires or a custom wiring solution dependent on the external controller being used.

Follow these steps to connect the QFLEX 2 Embedded to your microcontroller device:

1. Before proceeding make sure your microcontroller device has been setup and successfully tested. Refer to the documentation supplied with your control system for set up and testing instructions.
2. Make sure the everything is powered off before making any of these connections. This includes turning off the external microcontroller board.
3. Connect the GND pin on the QFLEX 2 Embedded to a digital ground connection on the microcontroller board.
4. Connect the MOSI, MISO, and CLK pins on the QFLEX 2 Embedded to the microcontroller board as outlined in the SPI interface documentation for your controller.
5. Connect the CS pin on the QFLEX 2 Embedded to a digital output on the microcontroller board.
6. Connect the 1.8V-5V pin on the QFLEX 2 Embedded to a signal level power pin on the microcontroller board in the 1.8V to 5V range.

Caution: **Applying voltages in excess of 5V to the 1.8V-5V input on the QFLEX 2 Embedded may result in damage to the QFLEX 2 Embedded.**

Cable #	From Microcontroller	To QFLEX 2 Embedded	Signal
1	VCC/VDD(1.8V-5V)	1.8V-5V	QFLEX 2 Embedded interface power.
2	MOSI/SDO/SO	MOSI	SPI master out, slave in data line.
3	MISO/SDI/SI	MISO	SPI master in, slave out data line.
4	SCLK/SCK	CLK	SPI clock line.
5	Digital output line	CS	SPI slave select line.
6	GND/DGND	GND	SPI digital signal ground.

Table 3.1: QFLEX 2 Embedded wiring summary

3.4 Exchanging QFLEX 2 Panels

Follow these steps to install the QFLEX 2 USB or QFLEX 2 Embedded panel in your Quanser Aero.

1. Disconnect the 24VDC power input from the Quanser Aero.
2. Disconnect any connections between the currently installed QFLEX panel and the computer or microcontroller board.

ESD Warning: **The interior of the Quanser Aero contains components which are sensitive to electrostatic discharge. Before opening the Quanser Aero case, ensure that both you and the workspace are properly grounded.**

3. Remove the four screws at the corners of the QFLEX panel to release the panel from the Aero chassis. The Quanser Aero is shown in Figure 3.1 below with the screws removed.

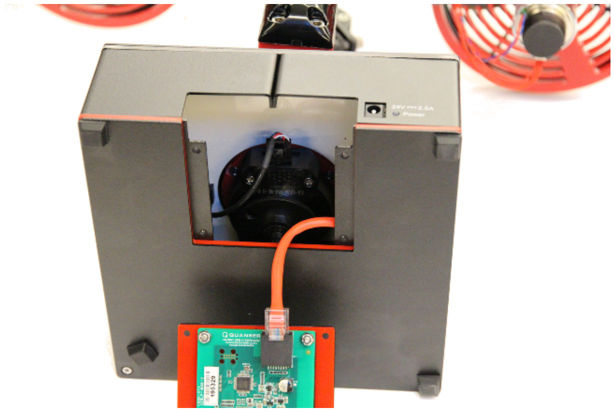


Figure 3.1: Quanser Aero with QFLEX panel detached.

4. Disconnect the Aero internal data cable from the QFLEX panel by depressing the latching tab.
5. Connect the Aero internal data cable to the QFLEX panel to be installed, pressing the connector into the socket until a click is heard and the connector latches in place.
6. Anchor the QFLEX panel in place using the four screws removed earlier.

   Caution: Ensure that the Quanser Aero is completely reassembled, with all screws in place before connecting power or attempting operation.

3.5 Changing Thruster Orientation

Follow these steps to change the orientation of the thrusters on the Quanser Aero.

1. Use the included hex key to loosen the thruster rotation lock set-screw on the thruster you wish to rotate by one quarter turn as shown in Figure 3.2a

   Caution: Do not loosen the set screws more than one half turn to prevent accidentally detaching the thruster assembly.

2. Rotate the propeller to the desired angle as shown in Figure 3.2b

   Note: Each thruster has a 90 degree range motion and will only rotate in one direction from either the vertical or horizontal positions.

3. Tighten the thruster rotation lock set-screw.

3.6 Exchanging Propellers

Follow these steps to change the propellers in the Quanser Aero.

Caution: The Quanser Aero is intended only for use with the included propellers. Operating the Quanser Aero with any other propellers may result in damage and/or injury.

1. Disconnect the 24VDC power input from the Quanser Aero.
2. Disconnect any connections between the currently installed QFLEX panel and the computer or microcontroller board.
3. Unfasten the propeller guard screws and remove the propeller guard.

(a) Hex key inserted in thruster rotation lock (b) Thruster rotated

Figure 3.2: Quanser Aero propeller change steps

4. Insert a small hex key or similar object (not provided) through the cap of the propeller hub as shown in Figure 3.3a.
5. While holding the propeller still, loosen the collet in the propeller hub slightly by turning the cap counter-clockwise.
6. Pull gently on the propeller hub to slide the assembly off the motor shaft.
7. Disassemble the propeller hub as shown in Figure 3.3b

(a) Propeller with guard removed (b) Quanser Aero prop assembly

Figure 3.3: Quanser Aero propeller change steps

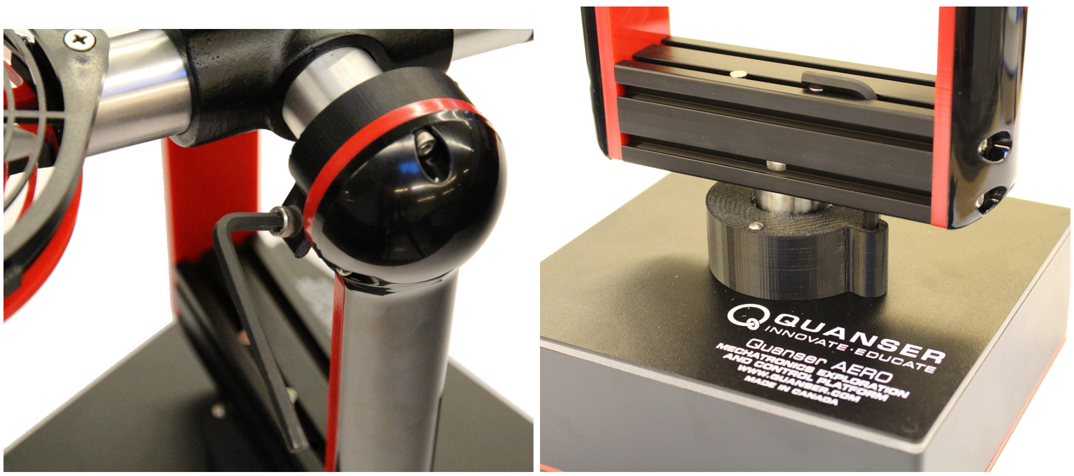
8. Identify the correct propeller from the counter-rotating pairs for the thruster being swapped. Under positive voltage, viewed from above, thruster 0 rotates counter-clockwise and thruster 1 rotates clockwise. Select the propeller such that positive voltage results in downward thrust. In the case of the high-efficiency propellers, the prop labeled 5x4.6E is intended for thruster 0, and that labeled 5x4.6EP is intended for thruster 1.
9. Slide the propeller assembly on to the motor shaft and tighten the cap.
10. Place the propeller cover back in position and replace the screws holding it in place.

Caution: **Ensure that the Quanser Aero is completely reassembled, with all screws in place before connecting power or attempting operation. The outer propeller guard screw must be fastened with the included lock nut. Improper assembly may result in damage and/or injury.**

3.7 Pitch and Yaw Locks

To lock the pitch of the Aero body, use the included hex key to tighten the pitch lock screws as shown in Figure 3.4a

To lock the yaw of the yoke, remove the hex key from its storage location in the bottom of the yoke and reinsert it with the long arm of the key down as shown in Figure 3.4b. Once the key is inserted all the way and protruding from the bottom of the yoke, rotate the yoke clockwise until the key comes in contact with the magnetized stop on the yaw pivot.



(a) Locking Pitch (b) Locking Yaw

Figure 3.4: Quanser Aero attitude locks

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3 DOF Helicopter

2 DOF Helicopter

3 DOF Hover

QBall 2

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F.3 Quanser AERO Embedded Data Sheet

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QFLEX 2 Embedded for Quanser AERO

For additional resources visit www.quanser.com/courseware

Features

Control DC motor voltages
Read encoder feedback from DC motors
Read encoder value from pitch and yaw pivots
Read current sense feedback from PWM amplifier
Read acceleration and velocity information from IMU
Set encoder reference points
Control tri-colour LED output

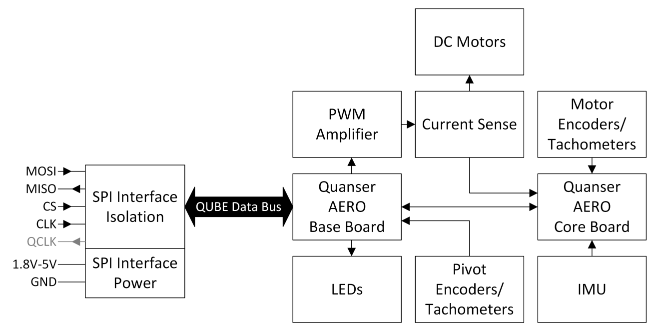

Figure 1: *System Diagram*

System Overview

The QFLEX 2 Embedded provides a standard SPI interface for interacting with the Quanser AERO. The QFLEX 2 Embedded panel is powered via the 1.8V-5V and GND input pins and provides signal isolation between the external connector and the internal QUBE data bus. Data signals in the SPI interface will operate at the voltage provided on the 1.8V-5V pin. The Quanser AERO controller receives the data from the SPI interface and sets the motor PWM duty cycle, LED brightness, and encoder reference. The controller collects current sense and encoder read values and feeds this information back over the data bus, back to the SPI interface. The full system diagram is shown in Figure 1.

Connections

The QFLEX 2 Embedded has a 7-pin connector (shown in Figure 2) which mates with a TE Connectivity 104257-6 rectangular connector. Table 1 outlines the IO connection pins

Table 1: *QFLEX 2 Embedded IO connections*

#	Pin Name	Description
1	1.8V-5V	Controller Vcc
2	MOSI	AERO data IN
3	MISO	AERO data OUT
4	CLK	SPI data clock IN
5	QCLK	QBUS clock OUT
6	CS	AERO slave chip select
7	GND	Controller ground

Communication

The QFLEX 2 Embedded operates as a standard SPI slave with SPI mode 2. Note that the complete data packet is comprised of two sub-packets, the first intended for the AERO base board, the second intended for the core board. When SPI communication is initiated by pulling the CS line low, the first byte of data received by the QFLEX 2 Embedded, referred to as the mode byte, will determine the communication mode to follow.

Mode = 0 Only read ID (no settings changed)

Mode = 1 A full command packet is transmitted

Concurrently, as the mode byte is being received, the Quanser AERO will respond with the upper byte of the base board ID. The next byte sent to the QFLEX 2 Em-

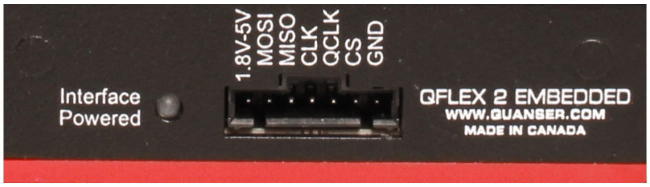

Figure 2: *QFLEX 2 Embedded Panel*

bedded is a padding byte with a value of 0, the data returned during this transmission will be the lower byte of the base board ID. At this point if Mode = 0, the next expected byte will be the mode byte for the core board, or byte 15 in a normal packet. If Mode = 1, the command communication packet continues as outlined in Table 2. Similarly, if mode 0 is selected for the core board, a padding byte will be expected and the lower byte of the core board ID will be returned, after which communication is complete and the CS line can be returned high. Otherwise if Mode = 1 for the core board, communication continues as outlined in Table 2.

Transmit/Receive Bytes

The expected values and description for each of the data bytes are as follows.

Base Mode (Transmit byte 0)

Expected value 0x00-0x01. A value of 0 will result in only the Base ID being returned. A value of 1 will initiate the transmission of a full command packet.

Base Write Mask (Transmit byte 2)

Expected value 0x00-0x7F. This byte controls what values will be overwritten on the AERO base board. The mapping of the bits in the mask to written values is shown in Table 3 To zero the pivot encoders, the write mask must be 0b00011xxx. To set the LED values, the mask must be 0b000xx111.

LED Values (Transmit bytes 3-8)

Expected value 0x0000-0x03E7. The brightness of the LEDs is controlled on a scale from 0 to 999 (decimal) this value is transmitted over two bytes for each LED with the MSB preceding the LSB.

Set Encoder Values (Transmit bytes 9-14,22-27)

These bytes allow for the encoder counts value to be set as desired. The most likely application for these bytes is to send 0x00 for all bytes to zero the encoder counts.

Table 2: *QFLEX 2 Embedded Command Packet Structure*

B	MOSI Data	MISO Data
	Start of Base Packet	
0	Base Mode (0x01)	Base ID MSB
1	Padding byte (0x00)	Base ID LSB
2	Base Write Mask	Encoder 2 (23-16)
3	Red LED MSB	Encoder 2 (15-8)
4	Red LED LSB	Encoder 2 (7-0)
5	Green LED MSB	Encoder 3 (23-16)
6	Green LED LSB	Encoder 3 (15-8)
7	Blue LED MSB	Encoder 3 (7-0)
8	Blue LED LSB	Tachometer 2 (23-16)
9	Set Encoder 2 (23-16)	Tachometer 2 (15-8)
10	Set Encoder 2 (15-8)	Tachometer 2 (7-0)
11	Set Encoder 2 (7-0)	Tachometer 3 (23-16)
12	Set Encoder 3 (23-16)	Tachometer 3 (15-8)
13	Set Encoder 3 (15-8)	Tachometer 3 (7-0)
14	Set Encoder 3 (7-0)	Reserved (0x00)
	Start of Core Packet	
15	Core Mode (0x01)	Core ID MSB
16	Padding byte (0x00)	Core ID LSB
17	Core Write Mask	Current Sense 0 (15-8)
18	Motor 0 Command (15-8)	Current Sense 0 (7-0)
19	Motor 0 Command (7-0)	Current Sense 1 (15-8)
20	Motor 1 Command (15-8)	Current Sense 1 (7-0)
21	Motor 1 Command (7-0)	Tachometer 0 (23-16)
22	Set Encoder 0 (23-16)	Tachometer 0 (15-8)
23	Set Encoder 0 (15-8)	Tachometer 0 (7-0)
24	Set Encoder 0 (7-0)	Tachometer 1 (23-16)
25	Set Encoder 1 (23-16)	Tachometer 1 (15-8)
26	Set Encoder 1 (15-8)	Tachometer 1 (7-0)
27	Set Encoder 1 (7-0)	Status
28	Padding byte (0x00)	Encoder 0 (23-16)
29	Padding byte (0x00)	Encoder 0 (15-8)
30	Padding byte (0x00)	Encoder 0 (7-0)
31	Padding byte (0x00)	Encoder 1 (23-16)
32	Padding byte (0x00)	Encoder 1 (15-8)
33	Padding byte (0x00)	Encoder 1 (7-0)
34	Padding byte (0x00)	Accelerometer X (15-8)
35	Padding byte (0x00)	Accelerometer X (7-0)
36	Padding byte (0x00)	Accelerometer Y (15-8)
37	Padding byte (0x00)	Accelerometer Y (7-0)
38	Padding byte (0x00)	Accelerometer Z (15-8)
39	Padding byte (0x00)	Accelerometer Z (7-0)
40	Padding byte (0x00)	Gyroscope X (15-8)
41	Padding byte (0x00)	Gyroscope X (7-0)
42	Padding byte (0x00)	Gyroscope Y (15-8)
43	Padding byte (0x00)	Gyroscope Y (7-0)
44	Padding byte (0x00)	Gyroscope Z (15-8)
45	Padding byte (0x00)	Gyroscope Z (7-0)
46	Padding byte (0x00)	Reserved (0x00)
47	Padding byte (0x00)	Reserved (0x00)
48	Padding byte (0x00)	Reserved (0x00)
49	Padding byte (0x00)	Reserved (0x00)
50	Padding byte (0x00)	Reserved (0x00)

Table 3: *Base Write Mask Bit Mapping*

b	Action Enabled
7	-
6	-
5	-
4	Set Encoder 3
3	Set Encoder 2
2	Write Blue LED
1	Write Green LED
0	Write Red LED

Core Mode (Transmit byte 15)

Expected value 0x00-0x01. A value of 0 will result in only the Core ID being returned, ending communication after byte 16. A value of 1 will continue the transmission of a full command packet.

Core Write Mask (Transmit byte 2)

Expected value 0x00-0x7F. This byte controls what values will be overwritten on the AERO base board. The mapping of the bits in the mask to written values is shown in Table 3 To set the motor encoders, the write mask must be 0b0011xxxx. To enable the motors and allow a command value to be written the mask must be 0b00xx1111.

Table 4: *Core Write Mask Bit Mapping*

b	Action Enabled
7	-
6	-
5	Set Encoder 1
4	Set Encoder 0
3	Write Motor 1 Voltage
2	Write Motor 1 Enable
1	Write Motor 0 Voltage
0	Write Motor 0 Enable

Motor Command (Transmit bytes 18-21)

Bit 15 of the motor commands control whether the PWM amplifier in the Quanser AERO for that motor is activated. Thus the upper byte of the motor command must be 0b1xxxxxxx in order for the motor to be enabled. The expected value of the bits 14-0 of the motor command is a value between -999 and 999 (decimal) formatted in 2's complement and represents a value equal to 10 times the desired percentage duty cycle of the PWM amplifier. For example, writing the motor command value 0x81F4 would activate the PWM amplifier and apply a 50% duty cycle, equivalent to approximately 12VDC.

Padding Bytes (Transmit bytes 28-50)

The value of these bytes are ignored by the QFLEX 2 Embedded, serving only to provide clock cycles for additional MISO data. By convention these bytes are usually zero.

Base Module ID (Receive bytes 0-1)

Expected Value of these bytes for the Quanser AERO Base Module is always 0x304 or 772 decimal. Any other value indicates a fault in communication between the QFLEX 2 Embedded and the AERO base board.

Read Encoder Values (Receive bytes 2-7,28-33)

These bytes represent the current value of the encoder counts, represented in 2's complement. Each value consists of three bytes and begins with the most significant byte. Encoders 0 and 1 are the encoders indicating the position of the DC motors. Encoder 2 is the encoder connected to the shoulder "pitch" pivot of the AERO body. Encoder 3 is the encoder connected to the base "yaw" pivot of the support yolk. Note that encoders 0-2 are 2048 quadrature counts per rotation, while encoder 3 is 4096 quadrature counts per rotation.

Tachometer Value (Receive bytes 8-13,21-26)

These bytes represent the current value read from the Quanser AERO internal tachometer, consisting of three bytes and beginning with the most significant byte. Tachometer values are calculated for each encoder. The most significant bit represents the direction of rotation with a value of zero indicating counter-clockwise rotation. Bits 23-0 represent the number of clock cycles (at 40MHz) between rising edges of the encoder *A* signal line. The encoder value can be converted to encoder counts per second with the following equation:

$$\text{Counts/Sec} = \frac{4}{Tach \cdot (25 \cdot 10^{-9})} \quad (1)$$

Note that when the motor/pivot velocity is 0 the tachometer value will indicate the maximum value of 0x7FFFFF indicating a reading of approximately 20 counts/s.

Core Module ID (Receive bytes 15-16)

Expected Value of these bytes for the Quanser AERO Core Module is always 0x307 or 775 decimal. Any other value indicates a fault in communication between the QFLEX 2 Embedded and the AERO core board.

Current Sense (Receive bytes 17-20)

These bytes represent the measured current draw of the DC motors, represented in two's complement. The current can be calculated using the equation:

$$\text{Current (mA)} = \frac{I_{sense} - 8190}{9828} \quad (2)$$

Status (Receive byte 27)

The three least significant bits of this byte represent various warning or error states as outlined in Table 5. A status byte value of 0x00 indicates normal operation.

Table 5: *Status Bits*

b	Indicated Status
7	-
6	-
5	Motor 1 Stall Error
4	Motor 1 Stall Detected
3	Motor 0 Stall Error
2	Motor 0 Stall Detected
1	Amplifier 1 Over-Current Fault
0	Amplifier 0 Over-Current Fault

Accelerometer Readings (Receive bytes 34-39)

There are three accelerometer readings each represented by a 16-bit signed integer in two's complement. The full scale reading is $\pm 2g$ so the acceleration can be calculated from the measured value using the following formula:

$$\text{Acceleration} = \frac{2 \cdot Acc_n \cdot 9.81}{32768} \quad (3)$$

Note that only the X axis (extending down the center of the AERO body towards thruster 0) and Z axis (extending vertically when the AERO body is horizontal) will be of interest. Under normal operation the Y-axis acceleration should always be approximately zero.

Gyroscope Readings (Receive bytes 40-45)

There are three accelerometer readings each represented by a 16-bit signed integer in two's complement. The full scale reading is ± 245 degrees/s so the angular velocity can be calculated from the measured value using the following formula:

$$\omega_n = \frac{Gyro_n \cdot 245}{32768} \quad (4)$$

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